Ann Oper Res
DOI 10.1007/s10479-015-1838-0

Joint procurement and demand-side bidding strategies under price volatility

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Abstract We consider a firm buying a commodity from a spot market as raw material and selling a final product by submitting bids. Bidding opportunities (i.e., demand arrivals) are random, and the likelihood of winning bids (i.e., selling the product) depends on the bid price. The price of the commodity raw material is also stochastic. The objective of the firm is to jointly decide on the procurement and bidding strategies to maximize its expected total discounted profit in the face of this demand and supply randomness. We model the commodity prices in the spot market as a Markov chain and the bidding opportunities as a Poisson process. Subsequently, we formulate the decision-making problem of the firm as an infinite-horizon stochastic dynamic program and analytically characterize its structural properties. We prove that the optimal procurement strategy follows a price-dependent base-stock policy and the optimal bidding price is decreasing with respect to the inventory level. We also formulate and analyze three intuitively appealing heuristic strategies, which either do not allow for carrying inventory or adopt simpler bidding policies (e.g., a constant bid price or myopically set bid prices). Using historical daily prices of several commodities, we then calibrate our models and conduct an extensive numerical study to compare the performances of the different

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Published online: 26 March 2015

strategies. Our study reveals the importance of adopting the optimal integrative procurement and bidding strategy, which is particularly rewarding when the raw material prices are more volatile and/or when there is significant competition on the demand side (the probability of winning is much smaller when submitting the same bid price). We establish that the relative performances of the three heuristic strategies depend critically on the holding cost of raw material inventory and the competitive environment, and identify conditions under which the shortfalls in profits from adopting such strategies are relatively less significant.

Keywords Supply chain management · Procurement · Bidding · Supply risk · Price volatility · Price-dependent base-stock policy

1 Research motivation

The price volatility of raw materials, especially commodity items (e.g., copper, oil, and steel), has become one of the most important concerns for supply chain managers. According to the Economist Intelligence Unit, such price volatility has been a major culprit for disruptions in supply chains, and it will remain so in the near future (Economist Intelligence Unit 2009). It is therefore considered as one of the top two supply chain risk elements, and its importance is on the rise as reported by a recent survey by the consultancy firm AMR Research (O'Marah 2009). Actual commodity price volatility data corroborate this fact. Crude oil is one of the most well-known examples. During a mere two-year span from mid-2007 to mid-2009, the price per barrel of (light) crude oil first went up by 100 % (from \$70 to \$140), then went down by 70 % (to \$40), only to rise again by 75 % (to \$70). Naturally, price fluctuations are not singular to crude oil; many commodities have witnessed such volatility in the last few years. Figure 1 illustrates the daily copper prices from January 2006 to October 2009. During that time period, the lowest price per metric tonne (MT) of copper was \$2667.5, while the highest price was \$8982.5—more than *three* times higher than the lowest price. Inevitably, such price changes have immediate cost implications for firms. For example, experts estimate that if oil prices rise by about 50 % per barrel, total supply chain costs could rise by 3 % (Economist Intelligence Unit 2009). In a global environment where firms face increasing competition and tightening margins, it is not difficult to imagine the catastrophic ramifications of these wide swings in raw material prices. The effects are especially pronounced for firms whose major cost item is commodity-type raw materials.[1]

Our study in this paper is motivated by a company who faces the brunt of such volatility in commodity prices. The company is a medium-sized one (approximately 250 employees and annual sales of around \$30 million) specializing in the manufacturing of wires and towers for electrical transmission systems. Labor and raw material constitute the primary cost elements, with purchased raw material, in particular copper, accounting for almost 50 % of the costs. The company's revenue stream comes from the projects it secures from public and private utility companies for setting up electric power transmission lines.

The modus operandi of the company is as follows. Utility companies announce their projects and requests for bids (RFBs). The company deals with many different utility companies in both Canada and the US; so the timing of these RFBs is essentially random. The nature of these projects, however, is relatively standard, involving the construction of transmission wires, posts, and related equipment. Once an RFB is received, the *estimation* department prepares a bid based on the job description. The company has over 50 years of experience in

[1] Data about oil and copper prices are from http://www.datastream.com.

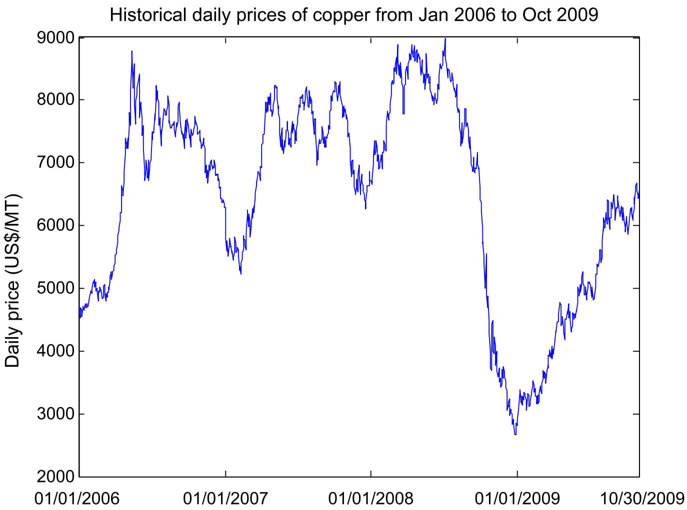

Fig. 1 Historical daily prices of copper

dealing with similar projects; so the estimation of the material and labor requirements of a project is done quite accurately (that is, the actual labor and material requirements are close to what is estimated). The bid price is then set to be approximately equal to the sum of the cost of the estimated raw material amount in the spot market, the estimated labor cost, and a mark-up to account for overheads and a profit margin.

Utility companies collect competitive bids for each project; so whether the company wins the project bid or not depends (mostly) on the bid prices of all other firms. If the company wins the bid, the *procurement* department (separate from the estimation department) initiates the procurement of the required amount of material, primarily from the spot market, via wholesalers. Although there is a time lag between submitting the bid and actually starting the project (if the company wins the bid), this lag is relatively short. The implication is that the company can procure the material requirements of the project from the spot market at essentially the current spot market price. The company foresees that, at least in the near future, the same policy will be employed, making the spot market the primary procurement channel. This is partly due to the size of the firm, which precludes it from getting preferential treatment from much larger commodity producers and striking longer-term forward contracts for raw materials.

Based on the above discussion, it is clear that, in the status quo, bidding and procurement decisions for the company are made separately and both of them are done in an intuitive, but somewhat naive, manner. That is, the bid price for a project is set myopically based solely on the spot market price for the raw material (and labor) at that time, and the procurement policy is just to procure for the current project. The company comprehends that there are potential benefits in making the two decisions in a more integrated and sophisticated manner. For example, if the current spot market procurement cost is low, the firm can bid lower to increase its chance of winning the project, and utilize a forward-looking procurement policy to buy more at the low price for future projects.[2] However, implementing such strategies may require the firm to alter its organization structure, train employees involved with bidding and procurement, and incur some coordination costs. The challenge therefore remains in assessing

[2] Since the majority of the projects are rather similar in nature, indeed the company can buy for future projects.

the potential values of *integrating bidding and procurement decisions* and *optimizing the procurement policy* so that efforts in this direction can be justified.

Our objective in this paper is to address the above challenge. Although the motivation stems from the specific company described above, the issues are pertinent to any firm that faces supply-side risk in the form of raw material price volatility, and demand-side risk in terms of customer arrivals and converting them into sales (depending on bid prices). Specifically, we seek answers to the following research questions:

– Assuming that the bid price is set myopically like in the status quo, what is the *optimal procurement policy* and what is the value of adopting such a policy compared to the status quo?
– What is the structure of the optimal policy when procurement and bidding policies are decided jointly, but the bid price does not change over time? What is the value of such *integrative decision-making* compared to the status quo?
– What is the optimal joint procurement and bidding policy when the bid price can change over time? What is the value of adopting such an *integrative and dynamic policy* compared to the status quo?
– How do price volatility and other operating factors impact the potential benefits associated with each policy? Under what conditions is it ideal to use each strategy?

In order to address these questions, we develop a stylized framework capturing all the salient characteristics of the decision-making environment described above. Specifically, we model a single firm that bids for projects of a standard size and procures a commodity material from a spot market. The project arrivals follow a Poisson process, while the spot market price of the commodity follows a discrete-state continuous-time Markov chain (DSCTMC). When a project arrives, the firm submits a bid with a bid price. The chance of winning the bid depends on competitors' bidding strategies and auction rules. To abstract away from the details of auction mechanisms, we assume that the winning probability is characterized by a decreasing function of the bid price. In this setting, we formulate and analyze the following four models of increasing order of sophistication, each corresponding to a different strategy. We start with the status quo strategy, which we call a *zero-inventory* (ZI) strategy. Then, we formulate and analyze a *myopic bidding* (MB) strategy. Under the MB strategy, the firm still makes bidding decisions based solely on the current commodity price, but the procurement policy is optimized by allowing the firm to procure for future projects. The comparison of MB and ZI strategies enables us to assess the value of an *optimal procurement* strategy. In order to assess the value of *integrating bidding and procurement*, we formulate and analyze two strategies where the two decisions are made jointly. Under a *static bidding* (SB) strategy, the firm determines a constant bid price, while under a *dynamic bidding* (DB) strategy, the bid price varies over time based on the current raw material price and the available inventory. We provide a *full analytical characterization* of the optimal bidding and procurement policy under each strategy.

Subsequently, we calibrate the model parameters using real historical data for different commodities with varying degrees of price volatility, and compare the performances of the above strategies under a variety of business settings. This enables us to generate a number of managerial insights. First, our numerical study demonstrates that the potential gains from dynamically integrating bidding and procurement strategies (i.e., the DB strategy) can be very significant. We find that the benefits increase as the raw material prices become more volatile and/or there is a significant amount of competition on the demand side. Second, the inventory holding cost is the primary driver of the relative performances of the three heuristics—ZI, MB, and SB. When the holding cost is low, ZI is the worst among all three of them, while SB is the best. This signifies that the firm can garner additional profits by strategically maintaining raw material inventory to hedge against raw material price risk, even if the bidding decisions are made in a simplified suboptimal manner. On the other hand, when the holding cost is high, ZI can perform as well as MB and they both perform better than SB. In such business environments, it is better for the firm to limit the amount of inventory but instead utilize more adaptive bidding strategies to mitigate raw material price risk. Third, we find that the gap between ZI and MB gets smaller as the probability of winning the project becomes more sensitive to the bid price. Consequently, when the demand market is highly competitive and inventory costs are high, a strategy that does not keep inventory and procures from the spot market only as needed (i.e., ZI), such as the one currently practiced by the firm in the motivating example, can perform just as good as more sophisticated heuristics and reasonably well compared to even the optimal integrated procurement and bidding strategy.

The remainder of the paper is structured as follows. In Sect. 2, we review the related literature. Subsequently, in Sect. 3, we develop a framework to analyze the four policies indicated above, and characterize their structural properties in Sect. 4. Section 5 provides an extensive numerical study to demonstrate the value of dynamic and/or integrative decision-making. Our concluding remarks are presented in Sect. 6. All technical proofs and supplementary tables are provided in appendices.

2 Literature review

Uncertainty in market demand has long been established in the academic literature as the primary risk factor that supply chain managers have to deal with. In recent years, with increasing global competition and outsourcing, however, supply risk has emerged as another crucial risk factor. Consequently, managing supply risk has received a lot of attention lately (Vakharia and Yenipazarlı 2009). In general, managing supply risk belongs to a broader field called supply chain risk management. For a detailed literature review on supply chain risk management, please refer to Tang (2006).

The literature on supply risk can be categorized into two streams: one dealing with uncertainty in terms of quantity and the other uncertainty in terms of price. In this paper, we are interested in the second stream where the buyer's risk arises (primarily) due to volatility in purchase prices.[3] The majority of the existing literature in this stream has focused on developing optimal procurement (inventory management) strategies for the buyer under such risky conditions. Models in this domain can be further categorized depending on whether the spot market is the only channel used for procurement or other channels, e.g., forward/option contracts, are also considered. Table 1 provides a sample of recent related work based on this classification.

The *single-channel* literature dates back to the 1950's when Fabian et al. (1959) develop a dynamic programming model in a periodic-review setting and propose heuristics in order to achieve a minimum average expected cost per period. Kalymon (1971) also investigates an inventory model with fixed ordering costs where the purchasing price follows a Markovian stochastic process, and establishes the optimality of a price-dependent (s, S) policy. Kingsman (1986) provides an early review on this category. A number of papers analyze variants of the problem in a periodic-review setting, differing mainly in terms of how the purchase price process is modeled (e.g., Golabi 1985; Özekici and Parlar 1999). More recently, Wang

[3] We refer the readers to Gümüş et al. (2012) and Vakharia and Yenipazarlı (2009) for detailed reviews of papers dealing with quantity risk.

Table 1 Literature on procurement under price risk

Single channel	Periodic review	Continuous review
	Özekici and Parlar (1999)	Yang and Xia (2009)
	Wang (2001)	Berling and Martínez de Albéniz (2011)
	Gavirneni (2004)	Guo et al. (2011)
Multiple channels	Single period	Multiple periods
	Akella et al. (2002)	Cohen and Agrawal (1999)
	Seifert et al. (2004)	Martínez de Albéniz and Simchi-Levi (2005)
	Fu et al. (2010)	Goel and Gutierrez (2007)

(2001) considers stochastic and decreasing purchase prices and provides conditions under which myopic stocking policies are optimal. Lastly, in Gavirneni (2004) the purchase price follows a discrete-state Markov chain but there are no fixed ordering costs. He shows that a price-dependent base-stock policy is optimal and provides conditions under which the optimal base-stock levels are decreasing in prices.

We use the spot market as the only procurement channel like the papers referred above, but in a continuous-review framework. For this reason, single-channel papers that assume continuous review for replenishment decisions are more directly related to our study. In this domain, Yang and Xia (2009) analyze a continuous-review version of Kalymon (1971), show that a price-dependent base-stock policy is optimal, and establish monotonicity properties of the base-stock levels with respect to price levels. On the other hand, Berling and Martínez de Albéniz (2011) focus on optimal inventory policies when the purchasing price follows the one-factor model of Schwartz (1997) and demand follows a Poisson process. They explicitly characterize the optimal base-stock level via a series of threshold prices. In contrast, Guo et al. (2011) consider an inventory model where there is random demand at the end of a single period with random length, and during this period the firm can buy or sell in the spot market governed by a geometric Brownian motion price process. They formulate the model as a two-dimensional singular control problem and show that a simple price-dependent two-threshold policy is optimal.

A natural extension of the models discussed above is to allow for multiple channels for procurement. In certain real-life situations, firms can not only use spot market contracts for procuring raw materials, but also other longer-term alternatives like forward and option contracts. Haksöz and Seshadri (2007) present a detailed review of optimal procurement strategies using multiple supply channels. Particularly relevant studies in this area are those that consider raw material price risk. Akella et al. (2002), Seifert et al. (2004), and Fu et al. (2010) are examples of multi-channel procurement papers in *single-period* settings. Specifically, Akella et al. (2002) show that, for a risk-neutral buyer, a strategy that uses both a long-term contract and the spot market is optimal. When the buyer is risk-averse, Seifert et al. (2004) demonstrate that procuring a moderate fraction via the spot market and the rest via a forward contract can lead to significant profit improvements. Fu et al. (2010) consider the optimal procurement strategy using a combination of option contracts and the spot market. In a *multiple-period* environment, Cohen and Agrawal (1999) propose a stochastic dynamic program to determine the optimal contract type for each review period. Martínez de Albéniz and Simchi-Levi (2005) use a combination of option contracts, termed as a portfolio contract, for procurement purposes. Goel and Gutierrez (2007) extend the work of Seifert et al. (2004)

to a multi-period setting, and develop optimal procurement policies for both with and without transaction costs.

Another natural extension to single-channel procurement models under raw material fluctuations is to include the buyer's pricing (bidding) decision and understand the interaction between procurement and bidding decisions. To the best of our knowledge, there is no explicit effort in the literature exploring this extension. Our paper makes a distinct contribution in this direction in the context of continuous-review models.

3 Model framework

We start by providing a formal description of the spot market price process and the demand arrival process. Consistent with our motivating application, we assume that the spot market is the unique procurement channel. The firm can purchase the commodity from the spot market with instantaneous delivery and no transaction costs. The firm does not sell any commodity back to the spot market for speculative purposes.

The commodity price in the spot market is modeled as a DSCTMC with K states. Let $p = [p_i]_{i=1}^K$ denote the prices, where $i \in \{1, 2, \ldots, K\}$. Without loss of generality, we order the prices such that $p_1 < p_2 < \cdots < p_K$. A DSCTMC model can be fully described by a vector $\mu = [\mu_i]_{i=1}^K$ and a matrix $\gamma = [\gamma_{ij}]_{i,j=1}^K$, where μ_i^{-1} is the average time in state i and γ_{ij} is the transition probability from state i to state j.

We assume that projects arrive randomly over time, following a Poisson process with rate λ. In accordance with the business environment of the motivating firm, we assume that the projects are homogeneous, i.e., each project requires the winning firm to use the same number of inventory units. Without loss of generality, we normalize the size of each project to 1. Upon a project arrival, the firm submits a bid that includes a bid price $b \geq 0$. The chance of winning each bid depends on competitors' bidding strategies and auction rules, which can vary from project to project. In order to abstract our modeling framework away from the details of auction mechanisms, we assume that if a bid price b is offered, the winning probability is given by $P(b)$, which is a decreasing function of the bid price.

We start with analyzing the status quo strategy of the firm where it does not hold inventory and only procures from the spot market to satisfy demand from each winning bid. We call this a ZI strategy. Since demand is satisfied by procuring from the spot market, it is easy to see that the optimal bid price for ZI is given by

$$b_{\mathrm{ZI}}(i) = \arg\max_{b \geq 0} P(b)(b - p_i), \tag{1}$$

when the current spot market price is p_i. Hence, the bid price fluctuates with the spot market price. In this sense, the bidding strategy behaves myopically.

The ZI strategy is easy to compute and easy to implement because the firm does not need to hold inventory. It is also provably optimal when the firm does not hold any raw material inventory.[4] But how does such a strategy perform in general? To answer this question, we consider strategies which optimize procurement policies and/or integrate bidding and procurement decisions.

We first consider a potential improvement of the ZI strategy where bids are determined myopically, as in ZI, but inventory holding is allowed and the associated procurement policy is optimized. We call this strategy an MB strategy. In order to determine the optimal

[4] When the firm does not hold any inventory, the planning periods are decoupled, and hence a myopic bidding strategy is indeed optimal.

inventory/procurement strategy under MB, we formulate the problem faced by the firm as an infinite-horizon continuous-time Markov decision process. The objective is to maximize the expected total discounted profit. The system state is given by (x, i), where x is the on-hand inventory and i means that the current price level is p_i. The bid price is given by $b_{\mathrm{ZI}}(i)$ for each price level i. Following established results in the dynamic programming literature (e.g., Puterman 1994), the problem can be cast as a discrete-time Markov decision process. This implies that we can restrict attention to epochs where a transition occurs either due to a demand arrival or a price change.

Let $h(x)$ be the inventory holding cost rate when the inventory level is x. We assume that all costs and revenues are discounted continuously with discount rate α. Given the current price level p_i, let T_i be the time until either a demand arrival or a price change from level i to another level. Clearly, T_i is exponentially distributed with rate $\lambda + \mu_i$. Moreover, with probability $\frac{\lambda}{\lambda+\mu_i}$ a potential project arrives before a price change, and with probability $\frac{\mu_i}{\lambda+\mu_i}$ a price change occurs before a project arrival.

Let $V_{\mathrm{MB}}(x, i)$ be the expected total discounted profit given state (x, i) under MB. The optimality equations are given by

$$\begin{aligned} V_{\mathrm{MB}}(x, i) = E\bigg[& -\int_0^{T_i} h(x) e^{-\alpha t} dt + e^{-\alpha T_i} \frac{\lambda}{\lambda + \mu_i} \Big(P(b_{\mathrm{ZI}}(i))(F V_{\mathrm{MB}}(x, i) + b_{\mathrm{ZI}}(i)) \\ & + (1 - P(b_{\mathrm{ZI}}(i))) V_{\mathrm{MB}}(x, i) \Big) + e^{-\alpha T_i} \frac{\mu_i}{\lambda + \mu_i} \sum_{j \neq i} \gamma_{ij} H V_{\mathrm{MB}}(x, j) \bigg], \quad \forall x, i, \end{aligned} \tag{2}$$

where F and H are functional operators defined as follows:

$$F v(x, i) = \begin{cases} \max\big\{v(x, i) - p_i, v(x-1, i)\big\}, & \text{if } x \geq 1 \\ v(x, i) - p_i, & \text{if } x = 0 \end{cases} \tag{3}$$

and

$$H v(x, j) = \max_{q \geq 0} \Big\{ v(x+q, j) - p_j q \Big\}. \tag{4}$$

Before proceeding with the other policies, we briefly discuss terms inside the square bracket in (2). The first term is the total discounted cost of holding x units of inventory for T_i units of time. Due to the exponential assumptions, the probability of the next event being a demand arrival is $\frac{\lambda}{\lambda+\mu_i}$, whereas the likelihood of a price change from state i to state j is $\frac{\mu_i \gamma_{ij}}{\lambda+\mu_i}$. The second and third terms capture, respectively, the profit contributions of demand arrival and price change events. Upon a demand arrival, the firm submits a bid. The functional operator F captures the decision to be made if the firm wins the bid. It will either buy one unit from the spot market at the price p_i or use one unit of on-hand inventory, if available, to satisfy the demand. If the firm loses the bid, it will do nothing. The functional operator H determines the optimal procurement quantity q at the new price p_j.

The MB strategy, discussed above, optimizes the procurement decisions of the firm, but it does so in a sequential manner (i.e., given an independently and myopically set bidding price). The first integrated bidding and procurement strategy we consider is an SB strategy. Under SB, the bid price stays constant over time but the procurement policy is optimized. The model is again formulated as a Markov decision process. To this end, let $V_{\mathrm{SB}}(x, i; b)$ denote the expected total discounted profit given state (x, i) and a static bid price b. We can write

$$V_{\text{SB}}(x,i;b) = E\bigg[-\int_0^{T_i} h(x)e^{-\alpha t}dt + e^{-\alpha T_i}\frac{\lambda}{\lambda+\mu_i}\Big(P(b)(FV_{\text{SB}}(x,i;b)+b) + (1-P(b))V_{\text{SB}}(x,i;b)\Big) + e^{-\alpha T_i}\frac{\mu_i}{\lambda+\mu_i}\sum_{j\neq i}\gamma_{ij}HV_{\text{SB}}(x,j;b)\bigg], \quad \forall x,i. \tag{5}$$

The best static bid price is given by

$$b_{\text{SB}}(x,i) = \arg\max_{b\geq 0} V_{\text{SB}}(x,i;b). \tag{6}$$

With slight abuse of notation, the optimal value function is given by $V_{\text{SB}}(x,i) = V_{\text{SB}}(x,i;b_{\text{SB}})$.

We remark that MB and SB bear resemblance to several pricing heuristics proposed in the joint dynamic pricing and inventory control literature. Among them are static pricing (see, e.g., Federgruen and Heching 1999) and environment-dependent pricing (see, e.g., Gayon et al. 2009b). Static pricing utilizes a fixed price (as in SB), while environment-dependent pricing offers prices according to an environmental variable such as the current price level (as in MB).

It is possible to improve over MB and SB because both strategies offer bid prices that do not directly take into account inventory information. In the following, we consider a DB strategy where bidding and procurement strategies are jointly optimized. Let $V_{\text{DB}}(x,i)$ denote the expected total discounted profit under DB given state (x,i). We can express the optimality equations as follows:

$$V_{\text{DB}}(x,i) = E\bigg[-\int_0^{T_i} h(x)e^{-\alpha t}dt + e^{-\alpha T_i}\frac{\lambda}{\lambda+\mu_i}\max_{b\geq 0}\Big\{P(b)(FV_{\text{DB}}(x,i)+b) + (1-P(b))V_{\text{DB}}(x,i)\Big\} + e^{-\alpha T_i}\frac{\mu_i}{\lambda+\mu_i}\sum_{j\neq i}\gamma_{ij}HV_{\text{DB}}(x,j)\bigg], \quad \forall x,i. \tag{7}$$

In (7), the firm determines an optimal bid price upon a project arrival; see the second term inside the square bracket. Therefore, the optimal bid price depends on the inventory level x and the price level i. This should be contrasted with MB and SB.

In the next section, we unify the value functions defined in Eqs. (2)–(7) in a common framework, and characterize structural properties of the optimal procurement and bidding decisions under each strategy.

4 Structural results

In this section, we study structural properties of the dynamic programs for MB, SB, and DB.[5] Recall that the only difference among these strategies is the way that bidding decisions are determined. In order to treat all of them in a unified framework, we introduce the following notation for the value functions: $V_{\text{I}}(x,i)$, where I $\in$ {MB, SB, DB}. Without loss of generality, we restrict prices to the unit range, i.e., $0 < p_i < 1$ for all i. We also assume that the

[5] The optimal policy for ZI has already been discussed in the previous section.

probability of winning the bid is linearly decreasing in the bid price. For ease of exposition, we let $P(b) = 1 - b$.

Define $\overline{\mu} = \max\{\mu_1, \mu_2, \ldots, \mu_K\}$. Following Lippman (1975), we apply a uniformization technique to transform our models into ones that have uniform jump rates, that is, $\lambda + \overline{\mu}$. For a detailed illustration of the uniformization procedure, please refer to Chapter 14 of Porteus (2002). We also rescale time such that $\alpha + \lambda + \overline{\mu} = 1$. The optimality equations for MB, SB, and DB policies can then be rewritten as

$$V_{\mathrm{I}}(x, i) = T_{\mathrm{I}} V_{\mathrm{I}}(x, i), \tag{8}$$

where the functional operator T_{I} is defined as follows:

$$\begin{aligned} T_{\mathrm{I}} v(x, i) = {} & -h(x) + \lambda \Big\{ P(b_{\mathrm{I}})(F v(x, i) + b_{\mathrm{I}}) + (1 - P(b_{\mathrm{I}})) v(x, i) \Big\} + \mu_i \sum_{j \neq i} \gamma_{ij} H v(x, j) \\ & + (\overline{\mu} - \mu_i) v(x, i), \end{aligned} \tag{9}$$

where the optimal bid b_{I} is defined in Eqs. (1), (6), and (7) for MB, SB, and DB policies, respectively. In order to facilitate the derivation and presentation of our structural results, we define the following set of functions with specific properties.

Definition 1 Let N be the set of non-negative integers and $M = \{1, 2, \ldots, K\}$. Define $\mathcal{V}$ as the set of real-valued functions defined on $N \times M$ such that if $v \in \mathcal{V}$, for all $x \in N$ and $i \in M$, the following two conditions are satisfied:
Condition 1. $v(x + 2, i) - v(x + 1, i) \le v(x + 1, i) - v(x, i)$;
Condition 2. $v(x + 1, i) - v(x, i) \ge -1$.

Condition 1 implies that $v(x, i)$ is concave with respect to x. This condition is a classical condition in the inventory control literature to prove the optimality of base-stock policies (see, e.g., Porteus 2002). Condition 2 implies that the first difference is bounded below by -1. Such a bounded difference condition is often used in the literature. For example, Gayon et al. (2009a) and Ha (1997) utilize the condition that the first difference is bounded below by the negative of the largest lost sales cost.

Proposition 1 (Preservation Property) *Suppose that $h(x)$ is convex increasing in x and satisfies $h(x + 1) - h(x) \le \alpha$. Then, $v \in \mathcal{V} \Rightarrow T_I v \in \mathcal{V}$ for $I \in \{MB, SB, DB\}$.*

The above proposition states that if a function satisfies Conditions 1 and 2, the operator T_{I} preserves these two conditions. Since the operator T_{I} is a linear combination of F and H, it suffices to show that F and H preserve these two conditions. The details of the proofs of Proposition 1 and the remaining results are presented in Appendix 1.

Based on Proposition 1, we have the following theorem, which characterizes the structure of the optimal procurement strategy associated with each bidding strategy I.

Theorem 1 (Optimal Procurement Strategy) *For bidding strategies MB, SB, and DB, the optimal procurement strategy is of base-stock type. That is, there exists a set of base-stock levels W_i, for all $i = 1, 2, \ldots, K$, such that*

(i) If the price changes to p_j and the current inventory level x is strictly less than W_j, it is optimal to procure $W_j - x$ units from the spot market. Otherwise, it is optimal not to procure.
(ii) Given the current price level p_j and the inventory level x, it is optimal to satisfy demand using on-hand inventory if $x > W_j$, and procure a unit from the spot market if $x \le W_j$.

One may conjecture that the base-stock level W_i is monotonically decreasing with respect to the price level i. However, the monotonicity may not hold in general. Intuitively, if the price is likely to increase, more inventory should be carried. On the other hand, if the price is likely to decrease, less inventory should be carried. In both cases, the price trends matter more than the current price levels when determining the base-stock levels.[6] Here, we demonstrate the non-monotonicity of the base-stock levels for the DB strategy. The parameters are estimated from historical daily prices of copper. The details for the estimation procedure are presented in Sect. 5. Suppose that the commodity price process is discretized at 10 price points.[7] The parameters of the DSCTMC are:

$$p = (0.025, 0.079, 0.141, 0.212, 0.294, 0.387, 0.493, 0.615, 0.755, 0.915),$$
$$\mu = (22.400, 21.126, 18.707, 32.256, 59.294, 32.000, 43.556, 37.882, 30.546, 24.275),$$

and

$$\gamma = \begin{pmatrix} 0 & 1.000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.214 & 0 & 0.786 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.588 & 0 & 0.412 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.375 & 0 & 0.625 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.450 & 0 & 0.550 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.625 & 0 & 0.375 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.357 & 0 & 0.643 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.348 & 0 & 0.652 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.417 & 0 & 0.583 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.000 & 0 \end{pmatrix}.$$

Example 1 Let $h(x) = 0.05x$, $\lambda = 12$, and $\alpha = 0.08$. It can be computed that the optimal DB base-stock levels are (20, 10, 0, 3, 1, 0, 0, 0, 0, 0).

Note that the base-stock levels are non-monotonic. This non-monotonicity can be attributed to the structure of the underlying price process characterized by μ_i and γ_{ij}. The inventory strategy depends not only on how long the price stays in the current price level, i.e., μ_i, but also on how likely the price is going to rise or fall, i.e., γ_{ij}. This emphasizes the importance of price dynamics in determining the optimal policy. In particular, when the current price is p_3, the price is more likely to transit to a lower price level, since $\gamma_{32} = 0.588 > \gamma_{34} = 0.412$. In contrast, when the current price is p_4, the price is more likely to transit to a higher price level, since $\gamma_{45} = 0.625 > \gamma_{43} = 0.375$. So, we have a positive base-stock level for the price level 4. In a similar vein, when the price level is p_5, we see that μ_5 is very large and $\gamma_{56} > \gamma_{54}$, leading to a positive W_5.

Recall that bid prices for MB (and ZI) are determined myopically based only on the current raw material price; hence, they do not depend on the current inventory level. In the other two strategies, SB and DB, we determine the optimal bid prices by optimizing their respective value functions. Since the bid price for SB is constant, it does not depend on the inventory level. In the next theorem, we characterize how bid prices for DB change with respect to the inventory level.

Theorem 2 (Optimal Dynamic Bidding Strategy) *The optimal bid prices for DB, $b_{DB}(x, i)$, are monotonically decreasing in the current inventory level x.*

[6] We refer the readers to Gavirneni (2004) and Yang and Xia (2009) for conditions under which W_i is monotonically decreasing with respect to p_i under slightly different modeling frameworks.

[7] The values of these 10 price points and the corresponding γ_{ij} are the same as those used in Sect. 5.

The monotonicity of bid prices for DB with respect to the inventory level is intuitive since if the current inventory level is high, it will result in high inventory holding costs. To lower such costs, the firm should submit lower bid prices to have higher winning probabilities and hence reduce its inventory.

5 Numerical study

In this section we evaluate and compare the performances of the optimal (DB) and three heuristic policies (ZI, MB, and SB) discussed in Sect. 3. Our objective is to realistically assess the scale of profit improvements that a company can generate by following the optimal integrated bidding and procurement strategy, as well as identify conditions under which it might be reasonable to adopt the simpler, heuristic policies. To this end, we use historical prices of different commodities to calibrate the parameters of our Markov price processes. This estimation procedure is explained in Sect. 5.1. Section 5.2 provides additional details about our experimental setup. Section 5.3 reports our numerical results and discusses their managerial implications. Section 5.4 conducts a robustness check where historical prices, instead of estimated price processes, are used as input to check the impact of estimation errors on policy performances.

5.1 Calibration of price processes

We test our models on real data obtained for four commodities: copper, food index, oil, and platinum. We collect daily commodity prices from http://www.datastream.com for the four commodities from January 1, 2004 to November 5, 2009. In total, we have 1526 daily prices for each commodity. Recall that we model commodity price processes as a DSCTMC with K states (we will explain shortly how to choose K). Utilizing the historical commodity price data, we estimate the average time the price stays in level i (μ_i^{-1}) and the transition probability across different price levels (γ_{ij}). The unit for μ_i is 1/year. Let q_{ij} be the instantaneous transition rate from state i to state j. From the literature of statistical inference for DSCTMCs (see, e.g., Billingsley 1968; Guttorp 1995), if the sample is taken continuously during a time period between 0 and T, the maximum likelihood estimate (MLE) for q_{ij} can be analytically derived. Specifically, let $N_T(i,j)$ be the number of transitions from state i to state j up to time T. Let $A_T(i)$ be the total time (in years) spent in state i up to time T. Then for $i \neq j$, the MLE for q_{ij} is

$$\hat{q}_{ij} = \frac{N_T(i,j)}{A_T(i)}.$$

The corresponding MLEs for μ_i and γ_{ij} are

$$\hat{\mu}_i = \sum_{j \neq i} \hat{q}_{ij} \quad \text{and} \quad \hat{\gamma}_{ij} = \frac{\hat{q}_{ij}}{\hat{\mu}_i}.$$

Because we can treat daily data as essentially continuous data (see, e.g., Bladt and Sørensen 2009), the MLEs for μ_j and γ_{ij} can be easily calculated from historical daily commodity prices.

Next, we describe how to discretize the almost continuous daily data into K states. Let d_K and d_0 be the maximum and minimum prices in the sample data. First, we find $K-1$ endpoints $d_1, d_2, \ldots, d_{K-1}$ between d_0 and d_K such that $\ln(d_1) - \ln(d_0) = \ln(d_2) - \ln(d_1) =$

$\cdots = \ln(d_K) - \ln(d_{K-1})$. Therefore, we divide the whole range into K intervals such that the natural logarithms of these $K+1$ endpoints are evenly arranged. Since it is likely that the firm would procure from the spot market when prices are relatively low, the idea of using the natural logarithm operation is to fine tune these lower prices such that we have more states for such prices. The concept of using such an operation to classify data into states is often used in the mathematical finance literature (see, e.g., Chapter 10 in Ross 1999). Because we assume that $0 < p_i < 1$ in our models, we transform $d_0, d_1, d_2, \ldots, d_{K-1}, d_K$ into $K+1$ new endpoints denoted by $d_0^{'}, d_1^{'}, \ldots, d_K^{'}$, respectively, between 0 and 1 via the formula $\frac{d-d_0}{d_K-d_0}$. If a transformed price falls between $d_{i-1}^{'}$ and $d_i^{'}$, we say that the price is at the level i. We define the corresponding price for the price level i as $\frac{d_i^{'}+d_{i-1}^{'}}{2}$.

Regarding the number of price states K, we note that there is a trade-off between the accuracy of estimation and the loss of accuracy due to discretization, which depends on K. In general, the more the number of states, the less accurate the estimates become. On the other hand, our actual price process is a continuous one that we are approximating through discretization; this approximation improves as K increases. After extensive numerical experiments with various values of K, keeping the above trade-off in mind, we decide to use 10 price levels. Note that our qualitative results are valid for a relatively large range of K. Nevertheless, we test the robustness of our choice by conducting a simulation study using historical price data in Sect. 5.4.

5.2 Design of experiments

Since the expected total discounted profit depends on the starting state and the discount factor, we consider the average reward version of the dynamic programs to illustrate our results. We note that in our implementation, time is not scaled. Given a strategy I $\in$ {MB, SB, DB}, let G_{I} be the average reward and $V_{\mathrm{I}}^{'}(x, i)$ be the relative value function in state (x, i). Then, the optimality equations are given by

$$V_{\mathrm{I}}^{'}(x, i) + G_{\mathrm{I}} = \frac{1}{\lambda + \overline{\mu}} T_{\mathrm{I}} V_{\mathrm{I}}^{'}(x, i),$$

where the operator T_{I} is defined in (9). To solve the above optimality equations, we use the relative value iteration algorithm[8] stated in Chapter 8 of Puterman (1994).

Inventory holding costs are one of the prime drivers of the performances of the strategies. In our computational study we assume that the holding cost has two components: the physical holding cost and the financial holding cost. That is, $h(x, p_i) = (h + \delta p_i)x$, where h is the physical holding cost rate, δ is the financial holding cost rate, and p_i is the current spot market price. We vary $h \in \{0.01, 0.10, 0.20\}$ and $\delta \in \{0.01, 0.05\}$. We observe that the project arrival rate λ simply scales profits and inventories, without changing the qualitative results. For this reason, we report results with a fixed $\lambda = 6$. On the other hand, we generalize the functional form for the probability of winning projects to a power-linear form, and moreover we incorporate the dependence of the probability of winning on the current spot market price. That is, $P(b, p_i) = (1-b)^{(1-\theta p_i)\beta}$, where θ is a sensitivity parameter that represents the spot market price effect and β is a sensitivity parameter that represents the competition on the demand side for winning projects. Note that the case with $\theta = 0$ and $\beta = 1$ represents a linear $P(b)$. A higher θ means a higher probability of winning when submitting the same bid price, while a higher β represents a more competitive environment since in that case the probability

[8] We terminate the algorithm once the average reward reaches four-digit accuracy.

of winning decreases rapidly in b. We vary $\theta \in \{0.1, 0.3\}$ and $\beta \in \{0.5, 1.0, 2.0\}$. So in total, for each commodity there are 36 $(\beta, h, \theta, \delta)$ combinations. We remark that the directional changes in the profits under all strategies as well as the relative performances of the heuristic strategies are similar for all commodities. We use copper as our illustrative commodity for these results (the results for the other commodities are reported in Tables 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, and 18 in Appendix 2). The magnitudes of the changes and the differences among the strategies, however, depend on the volatility of the raw material prices and hence the commodity type. We show the effect of volatility in Sect. 5.3, utilizing the results for all four commodities.

5.3 Numerical results

In this section, we compare DB with ZI, MB, and SB on average profit rates, base-stock levels, average inventory levels, and average bid prices. Tables 2, 3, and 4 report optimal average profit rates, base-stock levels, and average bid prices and inventory levels for the different strategies, respectively. Note that Q_{MB}, Q_{SB}, and Q_{DB} represent the base-stock levels for MB, SB, and DB strategies, respectively (recall that under ZI the firm does not hold inventory). In Table 3, we write D_i when the base-stock level is D for the price level i. For example, 31_1 means that the base-stock level for the price level 1 is 31. For brevity, zero base-stock levels are not listed in the table. Note that the column $\mathrm{RI_I} = \frac{G_{\mathrm{DB}} - G_{\mathrm{I}}}{G_{\mathrm{I}}} \times 100\,\%$ reports the relative profit improvements of DB against I for I $\in$ {ZI, MB, SB}, where G_{DB} and G_{I} are the average profit rates under DB and I, respectively.

We have the following observations based on Tables 2, 3, and 4.

Observation 1 (Effect of Physical Holding Cost) *If the physical holding cost is low, SB performs better than both MB and ZI. As the physical holding cost increases, the performance of SB deteriorates while those of MB and ZI improve.*

When the physical holding cost is low, the firm can purchase a significant amount of inventory at low prices to satisfy demand when prices are high. Consequently, the incremental value generated from tracking the spot market price when submitting bids is relatively less. This explains why SB can perform reasonably well when h is low. However, when the physical holding cost is high, it is costly to hold inventory, which reduces the value of procuring and holding inventory for future use. Rather, such a scenario increases the value of tracking the spot market price. Hence, the relative performances of MB and ZI improve as the physical holding cost increases. Obviously, any increase in the holding cost reduces the average inventory levels for all policies. As regards the average bid prices, under DB and SB they increase as h increases (to counterbalance the increase in costs), while for MB and ZI they are, by definition, independent of h. As for the effect of the financial holding cost δ, when δ increases, all strategies except ZI have decreasing average profit rates and hence the relative performance of ZI improves.

Observation 2 (Effect of Demand Sensitivity) *As the demand sensitivity increases, in general, the performances of all three heuristics deteriorate. Moreover, the performances of MB and ZI approach each other.*

As discussed before, a higher level of β exemplifies a more competitive market for winning projects. This accentuates the importance of the bidding decision. As expected, SB faces larger shortfalls as β increases. The same is also true for MB. When the holding cost is

Table 2 Average profit rate comparison results for all strategies: copper

β	h	θ	δ	G_{ZI}	G_{MB}	G_{SB}	G_{DB}	$\mathrm{RI}_{\mathrm{ZI}}$ (%)	$\mathrm{RI}_{\mathrm{MB}}$ (%)	$\mathrm{RI}_{\mathrm{SB}}$ (%)
0.5	0.01	0.1	0.01	0.0128	0.0227	0.0266	0.0272	112.06	19.64	2.30
0.5	0.01	0.1	0.05	0.0128	0.0202	0.0225	0.0234	82.92	16.12	4.23
0.5	0.01	0.3	0.01	0.0137	0.0250	0.0286	0.0293	114.23	16.81	2.38
0.5	0.01	0.3	0.05	0.0137	0.0222	0.0242	0.0253	85.09	13.79	4.33
0.5	0.10	0.1	0.01	0.0128	0.0160	0.0159	0.0173	34.61	8.13	8.23
0.5	0.10	0.1	0.05	0.0128	0.0155	0.0152	0.0166	29.41	7.09	9.22
0.5	0.10	0.3	0.01	0.0137	0.0174	0.0172	0.0186	36.55	7.43	8.56
0.5	0.10	0.3	0.05	0.0137	0.0168	0.0163	0.0179	30.89	6.44	9.33
0.5	0.20	0.1	0.01	0.0128	0.0139	0.0126	0.0143	11.47	3.15	13.46
0.5	0.20	0.1	0.05	0.0128	0.0137	0.0124	0.0141	10.04	3.06	13.81
0.5	0.20	0.3	0.01	0.0137	0.0149	0.0135	0.0153	12.23	2.66	13.56
0.5	0.20	0.3	0.05	0.0137	0.0147	0.0132	0.0151	10.56	2.46	14.31
1.0	0.01	0.1	0.01	0.0069	0.0117	0.0156	0.0165	139.76	40.98	5.52
1.0	0.01	0.1	0.05	0.0069	0.0103	0.0124	0.0138	100.01	32.95	10.66
1.0	0.01	0.3	0.01	0.0075	0.0134	0.0173	0.0182	144.42	36.36	5.47
1.0	0.01	0.3	0.05	0.0075	0.0117	0.0138	0.0152	104.23	29.56	10.26
1.0	0.10	0.1	0.01	0.0069	0.0082	0.0076	0.0095	37.81	16.24	25.00
1.0	0.10	0.1	0.05	0.0069	0.0079	0.0071	0.0091	31.84	14.32	28.07
1.0	0.10	0.3	0.01	0.0075	0.0091	0.0085	0.0105	40.67	14.54	23.79
1.0	0.10	0.3	0.05	0.0075	0.0088	0.0078	0.0100	34.20	13.16	27.74
1.0	0.20	0.1	0.01	0.0069	0.0072	0.0055	0.0076	10.78	5.65	38.34
1.0	0.20	0.1	0.05	0.0069	0.0071	0.0054	0.0075	9.33	5.60	38.87
1.0	0.20	0.3	0.01	0.0075	0.0079	0.0060	0.0083	11.66	4.74	37.53
1.0	0.20	0.3	0.05	0.0075	0.0079	0.0059	0.0082	10.19	4.46	40.16
2.0	0.01	0.1	0.01	0.0030	0.0047	0.0075	0.0086	184.33	82.64	15.05
2.0	0.01	0.1	0.05	0.0030	0.0041	0.0051	0.0068	124.98	66.22	32.28
2.0	0.01	0.3	0.01	0.0033	0.0055	0.0085	0.0098	196.60	78.65	14.17
2.0	0.01	0.3	0.05	0.0033	0.0047	0.0060	0.0077	135.35	63.06	29.69
2.0	0.10	0.1	0.01	0.0030	0.0033	0.0022	0.0042	39.18	29.44	87.69
2.0	0.10	0.1	0.05	0.0030	0.0032	0.0019	0.0040	31.45	23.94	105.05
2.0	0.10	0.3	0.01	0.0033	0.0037	0.0025	0.0047	44.25	28.18	86.22
2.0	0.10	0.3	0.05	0.0033	0.0036	0.0022	0.0045	36.52	26.02	100.23
2.0	0.20	0.1	0.01	0.0030	0.0030	0.0013	0.0033	8.00	8.00	142.66
2.0	0.20	0.1	0.05	0.0030	0.0030	0.0013	0.0032	6.32	6.32	148.02
2.0	0.20	0.3	0.01	0.0033	0.0033	0.0014	0.0036	10.45	9.20	155.06
2.0	0.20	0.3	0.05	0.0033	0.0033	0.0014	0.0036	8.20	8.20	158.71

high, there is less inventory, and the performance of ZI approaches that of MB. Indeed, when both β and h are high, the importance of optimally deciding how much inventory to procure/stock reduces (since the firm will then hold a low level of inventory for any policy). Consequently, ZI performs creditably well under these conditions. Nevertheless, all policies face significant profit losses compared to DB when β is high. Hence, we can identify the

Table 3 Base-stock levels comparison results for MB, SB, and DB strategies: copper

β	h	θ	δ	Q_{MB}	Q_{SB}	Q_{DB}
0.5	0.01	0.1	0.01	$21_1, 10_2, 3_4, 1_5, 1_7$	$34_1, 17_2, 6_4, 2_5, 2_7$	$33_1, 15_2, 4_4, 1_5, 1_7$
0.5	0.01	0.1	0.05	$14_1, 8_2, 3_4, 1_5, 1_7$	$21_1, 12_2, 5_4, 2_5, 1_7$	$20_1, 11_2, 4_4, 1_5, 1_7$
0.5	0.01	0.3	0.01	$24_1, 12_2, 4_4, 1_5, 1_7$	$36_1, 18_2, 6_4, 2_5, 2_7$	$34_1, 16_2, 5_4, 2_5, 1_7$
0.5	0.01	0.3	0.05	$15_1, 9_2, 3_4, 1_5, 1_7$	$22_1, 13_2, 5_4, 2_5, 1_7$	$21_1, 12_2, 4_4, 1_5, 1_7$
0.5	0.10	0.1	0.01	$7_1, 4_2, 2_4$	$9_1, 6_2, 2_4, 1_5, 1_7$	$9_1, 5_2, 2_4, 1_5, 1_7$
0.5	0.10	0.1	0.05	$6_1, 3_2, 1_4$	$8_1, 5_2, 2_4, 1_5, 1_7$	$8_1, 5_2, 2_4, 1_5$
0.5	0.10	0.3	0.01	$7_1, 4_2, 2_4, 1_5$	$10_1, 6_2, 3_4, 1_5, 1_7$	$10_1, 6_2, 2_4, 1_5, 1_7$
0.5	0.10	0.3	0.05	$6_1, 4_2, 2_4$	$8_1, 5_2, 2_4, 1_5, 1_7$	$9_1, 5_2, 2_4, 1_5, 1_7$
0.5	0.20	0.1	0.01	$3_1, 2_2, 1_4$	$4_1, 2_2, 1_4$	$5_1, 2_2, 1_4$
0.5	0.20	0.1	0.05	$3_1, 2_2, 1_4$	$4_1, 2_2, 1_4$	$4_1, 2_2, 1_4$
0.5	0.20	0.3	0.01	$4_1, 2_2, 1_4$	$5_1, 3_2, 1_4, 1_7$	$5_1, 3_2, 1_4$
0.5	0.20	0.3	0.05	$3_1, 2_2, 1_4$	$4_1, 2_2, 1_4$	$4_1, 2_2, 1_4$
1.0	0.01	0.1	0.01	$12_1, 5_2, 1_4$	$27_1, 14_2, 5_4, 2_5, 1_7$	$25_1, 11_2, 3_4, 1_5$
1.0	0.01	0.1	0.05	$7_1, 4_2, 1_4$	$16_1, 9_2, 3_4, 1_5, 1_7$	$16_1, 8_2, 2_4$
1.0	0.01	0.3	0.01	$14_1, 6_2, 2_4$	$29_1, 15_2, 5_4, 2_5, 1_7$	$27_1, 12_2, 3_4, 1_5$
1.0	0.01	0.3	0.05	$9_1, 5_2, 1_4$	$17_1, 10_2, 4_4, 1_5, 1_7$	$17_1, 9_2, 2_4, 1_5$
1.0	0.10	0.1	0.01	$3_1, 2_2$	$6_1, 4_2, 2_4, 1_5$	$7_1, 3_2, 1_4$
1.0	0.10	0.1	0.05	$3_1, 2_2$	$5_1, 3_2, 1_4$	$6_1, 3_2, 1_4$
1.0	0.10	0.3	0.01	$4_1, 2_2, 1_4$	$7_1, 4_2, 2_4, 1_5, 1_7$	$7_1, 4_2, 1_4$
1.0	0.10	0.3	0.05	$4_1, 2_2, 1_4$	$6_1, 4_2, 2_4, 1_5, 1_7$	$6_1, 3_2, 1_4$
1.0	0.20	0.1	0.01	$2_1, 1_2$	$3_1, 1_2, 1_4$	$3_1, 1_2$
1.0	0.20	0.1	0.05	$1_1, 1_2$	$2_1, 1_2$	$3_1, 1_2$
1.0	0.20	0.3	0.01	$2_1, 1_2$	$3_1, 2_2, 1_4$	$3_1, 2_2$
1.0	0.20	0.3	0.05	$2_1, 1_2$	$3_1, 1_2, 1_4$	$3_1, 1_2$
2.0	0.01	0.1	0.01	$5_1, 2_2$	$20_1, 10_2, 3_4, 1_5, 1_7$	$19_1, 7_2, 1_4$
2.0	0.01	0.1	0.05	$3_1, 2_2$	$11_1, 6_2, 2_4, 1_5, 1_7$	$11_1, 5_2, 1_4$
2.0	0.01	0.3	0.01	$6_1, 3_2$	$22_1, 11_2, 4_4, 1_5, 1_7$	$20_1, 8_2, 1_4$
2.0	0.01	0.3	0.05	$4_1, 2_2$	$12_1, 7_2, 3_4, 1_5, 1_7$	$12_1, 5_2, 1_4$
2.0	0.10	0.1	0.01	1_1	$3_1, 2_2, 1_4$	$4_1, 2_2$
2.0	0.10	0.1	0.05	1_1	$2_1, 1_2$	$4_1, 2_2$
2.0	0.10	0.3	0.01	$2_1, 1_2$	$4_1, 2_2, 1_4$	$5_1, 2_2$
2.0	0.10	0.3	0.05	$1_1, 1_2$	$3_1, 2_2, 1_4$	$4_1, 2_2$
2.0	0.20	0.1	0.01	0_1	1_1	2_1
2.0	0.20	0.1	0.05	0_1	1_1	2_1
2.0	0.20	0.3	0.01	1_1	1_1	$2_1, 1_2$
2.0	0.20	0.3	0.05	0_1	1_1	$2_1, 1_2$

intensity of competition in the market for winning projects as one of the main factors that favor the adoption of an integrated dynamic procurement and bidding strategy. Also note that a higher β forces the firm to reduce its bid price under all policies; even then the chance of winning the project is relatively low, which reduces the average inventory level. As for the

Table 4 Average bid price and average inventory comparison results for all strategies: copper

β	h	θ	δ	ZI	MB		SB		DB	
				Avg bid	Avg bid	Avg inv	Avg bid	Avg inv	Avg bid	Avg inv
0.5	0.01	0.1	0.01	0.8531	0.8531	11.58	0.7400	19.74	0.7382	18.25
0.5	0.01	0.1	0.05	0.8516	0.8516	6.95	0.7600	10.74	0.7608	9.60
0.5	0.01	0.3	0.01	0.8596	0.8596	13.58	0.7600	20.98	0.7590	18.92
0.5	0.01	0.3	0.05	0.8599	0.8599	7.36	0.7800	11.22	0.7815	10.07
0.5	0.10	0.1	0.01	0.8534	0.8534	2.48	0.8100	3.48	0.8078	3.11
0.5	0.10	0.1	0.05	0.8510	0.8510	1.87	0.8100	3.04	0.8103	2.74
0.5	0.10	0.3	0.01	0.8588	0.8588	2.51	0.8200	3.99	0.8151	3.71
0.5	0.10	0.3	0.05	0.8595	0.8595	2.09	0.8300	2.94	0.8205	3.12
0.5	0.20	0.1	0.01	0.8530	0.8530	0.83	0.8400	1.03	0.8339	1.15
0.5	0.20	0.1	0.05	0.8533	0.8533	0.83	0.8400	1.02	0.8372	0.93
0.5	0.20	0.3	0.01	0.8610	0.8610	1.01	0.8400	1.61	0.8446	1.30
0.5	0.20	0.3	0.05	0.8609	0.8609	0.81	0.8500	1.01	0.8490	0.93
1.0	0.01	0.1	0.01	0.7758	0.7758	6.46	0.6000	15.88	0.6051	13.41
1.0	0.01	0.1	0.05	0.7765	0.7765	3.19	0.6400	8.17	0.6390	7.30
1.0	0.01	0.3	0.01	0.7853	0.7853	7.85	0.6300	17.56	0.6248	15.00
1.0	0.01	0.3	0.05	0.7863	0.7863	4.26	0.6700	8.81	0.6627	7.83
1.0	0.10	0.1	0.01	0.7768	0.7768	0.93	0.7100	2.43	0.7136	2.12
1.0	0.10	0.1	0.05	0.7805	0.7805	0.92	0.7300	1.71	0.7227	1.74
1.0	0.10	0.3	0.01	0.7885	0.7885	1.28	0.7300	2.79	0.7330	2.24
1.0	0.10	0.3	0.05	0.7865	0.7865	1.31	0.7300	2.50	0.7394	1.78
1.0	0.20	0.1	0.01	0.7772	0.7772	0.46	0.7500	0.85	0.7558	0.56
1.0	0.20	0.1	0.05	0.7759	0.7759	0.29	0.7600	0.46	0.7545	0.55
1.0	0.20	0.3	0.01	0.7881	0.7881	0.41	0.7700	0.94	0.7667	0.71
1.0	0.20	0.3	0.05	0.7858	0.7858	0.43	0.7700	0.83	0.7677	0.57
2.0	0.01	0.1	0.01	0.6978	0.6978	2.53	0.4700	12.07	0.4632	9.77
2.0	0.01	0.1	0.05	0.7022	0.7022	1.40	0.5200	5.71	0.5272	4.50
2.0	0.01	0.3	0.01	0.7110	0.7110	3.13	0.4900	13.05	0.4974	10.16
2.0	0.01	0.3	0.05	0.7109	0.7109	1.73	0.5400	6.24	0.5509	4.85
2.0	0.10	0.1	0.01	0.6981	0.6981	0.19	0.6200	1.28	0.6252	1.14
2.0	0.10	0.1	0.05	0.7002	0.7002	0.18	0.6500	0.64	0.6267	1.08
2.0	0.10	0.3	0.01	0.7082	0.7082	0.67	0.6400	1.61	0.6331	1.42
2.0	0.10	0.3	0.05	0.7092	0.7092	0.40	0.6500	1.20	0.6473	1.07
2.0	0.20	0.1	0.01	0.7032	0.7032	0.00	0.6800	0.22	0.6814	0.28
2.0	0.20	0.1	0.05	0.7015	0.7015	0.00	0.6800	0.23	0.6790	0.28
2.0	0.20	0.3	0.01	0.7088	0.7088	0.17	0.7200	0.22	0.6801	0.42
2.0	0.20	0.3	0.05	0.7100	0.7100	0.00	0.7200	0.22	0.6815	0.40

effect of the spot market price sensitivity θ, when θ increases, all strategies have increasing average profit rates.

Observation 3 (Comparison of Average Bid Prices and Inventory Levels among Policies) *(i) The amount of inventory carried by MB (SB) is less (more) than the optimal level of inventory* *under DB.*[9] *(ii) For the bid prices, ZI and MB charge, on average, higher bid prices than SB and DB. Specifically, the average bid prices are ordered as follows: ZI = MB > SB ≈ DB.*

MB is based on the current price level and ignores the inventory state. This results in higher bid prices since it does not take advantage of the possibility of satisfying demand from the inventory procured at lower prices. Since ZI also does not hold inventory, its average bid price is exactly the same as that of MB. An interesting point is that even though the bid prices from DB depend on both inventory and price levels, the average bid price is comparable to the SB average bid price. The observation on average inventory can be explained as follows. Since the bid price is fixed for SB, inventory is the only lever available. Therefore, the firm tends to purchase more inventory when prices are low. On the other hand, MB passes the increasing commodity prices to customers, and therefore carries less inventory.

Next, we illustrate the effect of price volatility on the benefits of DB by comparing the results from different commodities. We use the following approach to compare the price variability of the different commodities. First, we estimate a ten-state DSCTMC model for each commodity. Then, we calculate the limiting probabilities of the ten states. The standard deviation of prices evaluated at the limiting probabilities is used as an approximate measure for the volatility of the commodity, where a higher standard deviation means higher variability. Based on this approach, the standard deviations for the raw material commodity prices are ordered as follows: $\sigma_{\text{Copper}} = 0.2895 > \sigma_{\text{Oil}} = 0.2711 > \sigma_{\text{Platinum}} = 0.2696 > \sigma_{\text{Food index}} = 0.2466$.

Observation 4 (Effect of Price Volatility) *If the price of a commodity is more volatile, the corresponding base-stock levels and average bid price are higher. Furthermore, higher volatility increases the relative improvement of DB over SB.*

Higher price volatility increases the strategic value of procurement and inventory, and naturally leads to higher base-stock levels. On the other hand, enhanced price risk prompts the firm to be more cautious about pricing, and leads to higher bid prices. In general, higher volatility increases the potential benefits of integrated procurement and bidding strategies. The reasoning is that when price risk is high, both inventory and bidding levers can be better exploited. Specifically, the relative gain of using DB over SB increases with volatility. However, for ZI and MB, the gains might be non-monotonic. For example, the order of RI_{MB} for the four commodities depends on the physical holding cost. For a physical holding cost 0.01, the order of increasing RI_{MB} is: platinum, food index, oil, and copper. But for physical holding costs 0.10 and 0.20, the order becomes platinum, oil, food index, and copper. Indeed, our numerical experiments suggest that the gains are lowest for medium levels of volatility (note that the gains for high levels of volatility are much higher than those of low levels). Figure 2 shows the relative improvements for different commodities when $\beta = 1$, $\theta = 0.1$, and $\delta = 0.01$.

5.4 Robustness check by simulating historical price data

The results reported so far use the estimated price processes as model input. This estimation procedure involves discretizing continuous prices into different levels. In order to test the robustness of our approach, we compare the performances of different strategies by simulating historical daily prices from January 1, 2004 to November 5, 2009.[10] For each parameter

[9] ZI, by definition, holds no inventory.

[10] Note that the different strategies are derived from the estimated parameters for the price processes.

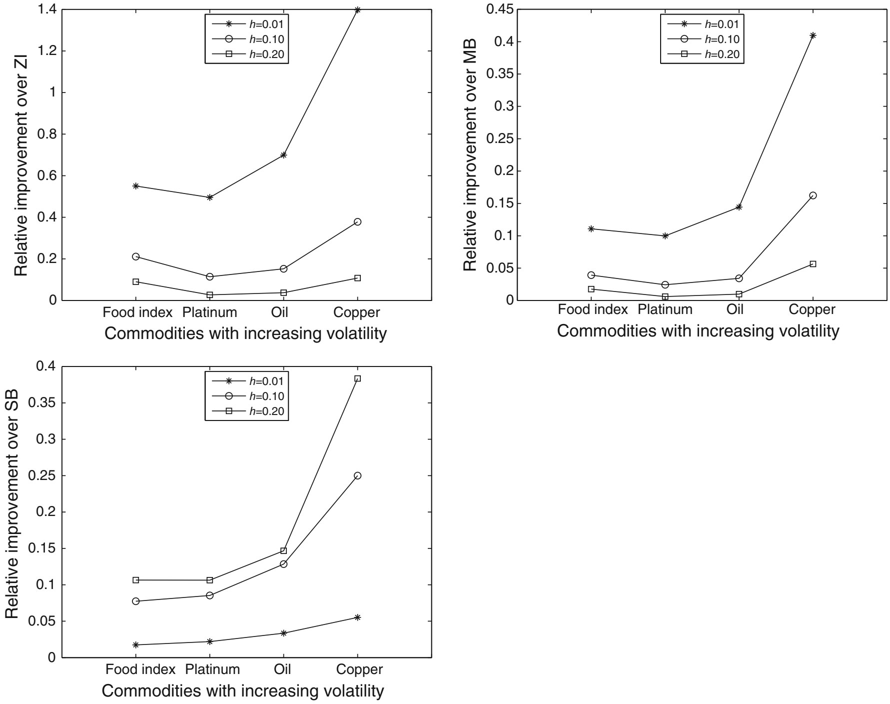

Fig. 2 $\mathrm{RI}_{\mathrm{ZI}}$, $\mathrm{RI}_{\mathrm{MB}}$, and $\mathrm{RI}_{\mathrm{SB}}$ with respect to increasing commodity volatility

combination, we simulate the demand arrival process 40 times. The profits are recorded by using the prices from historical data instead of the discrete price levels used in the estimation procedure. By conducting simulation in this fashion, we account for the potential errors introduced by the estimation procedure for the price processes. Table 5 reports the average total profits for different strategies. In the table, ATP represents average total profit, and $\mathrm{RP}_{\mathrm{ZI}}$, $\mathrm{RP}_{\mathrm{MB}}$, and $\mathrm{RP}_{\mathrm{SB}}$ are the relative improvements of DB over ZI, MB, and SB, respectively.

Observation 5 *In general, RP_{ZI}, RP_{MB}, and RP_{SB} are positive. The ranking of the strategies is similar to the one observed in Table 2.*

The highest $\mathrm{RP}_{\mathrm{ZI}}$ is 151.44 %, the highest $\mathrm{RP}_{\mathrm{MB}}$ is 88.13 %, and the highest $\mathrm{RP}_{\mathrm{SB}}$ is 210.01 %. We also observe similar results for other commodities. Therefore, DB is beneficial even accounting for potential estimation errors. Obviously, since in this section, in contrast to Sect. 5.3, we focus on one price path, in certain cases the heuristics perform better than the optimal DB strategy. Also, as noted in Sect. 5.3, MB works relatively well in high h-low β settings, SB in low h-low β settings, and ZI in high h-high β settings.

6 Concluding remarks

In this paper, we analyze an optimal joint bidding and inventory procurement strategy for a firm facing supply-side risk in terms of randomness in raw material prices (as well as demand-side risk in terms of project arrivals). We model commodity prices in a spot market as a continuous-time Markov chain, and formulate the firm's problem as a Markov decision

Table 5 Average total profit comparison results for all strategies using real data: copper

β	h	θ	δ	ATP_{ZI}	ATP_{MB}	ATP_{SB}	ATP_{DB}	RP_{ZI} (%)	RP_{MB} (%)	RP_{SB} (%)
0.5	0.01	0.1	0.01	6.0203	9.5514	10.2457	11.0498	83.54	15.69	7.85
0.5	0.01	0.1	0.05	6.2439	9.5295	10.4445	11.3120	81.17	18.70	8.31
0.5	0.01	0.3	0.01	6.9599	10.2345	12.0850	12.1060	73.94	18.29	0.17
0.5	0.01	0.3	0.05	6.9859	11.0007	12.1857	12.2168	74.88	11.05	0.26
0.5	0.10	0.1	0.01	6.5448	7.3405	7.0065	7.0858	8.27	−3.47	1.13
0.5	0.10	0.1	0.05	6.3734	7.3352	6.7049	7.1884	12.79	−2.00	7.21
0.5	0.10	0.3	0.01	6.2475	7.0291	7.0342	8.0118	28.24	13.98	13.90
0.5	0.10	0.3	0.05	6.4522	7.0389	6.9691	7.5216	16.57	6.86	7.93
0.5	0.20	0.1	0.01	6.0274	6.2405	4.9994	6.0520	0.41	−3.02	21.05
0.5	0.20	0.1	0.05	6.1520	6.1866	5.5141	5.9562	−3.18	−3.73	8.02
0.5	0.20	0.3	0.01	7.0187	6.4683	5.8662	6.7074	−4.44	3.70	14.34
0.5	0.20	0.3	0.05	6.7398	6.3417	5.6067	6.1503	−8.75	−3.02	9.70
1.0	0.01	0.1	0.01	3.1305	6.0715	7.1419	7.6996	145.95	26.81	7.81
1.0	0.01	0.1	0.05	3.3522	5.0803	5.7669	7.3259	118.54	44.20	27.03
1.0	0.01	0.3	0.01	3.7262	5.5239	7.5514	8.5277	128.86	54.38	12.93
1.0	0.01	0.3	0.05	3.8899	5.6947	6.8315	7.5442	93.94	32.48	10.43
1.0	0.10	0.1	0.01	3.7551	3.8771	3.6612	4.5254	20.51	16.72	23.60
1.0	0.10	0.1	0.05	3.5918	3.8030	3.3508	4.0611	13.07	6.79	21.20
1.0	0.10	0.3	0.01	3.7323	4.0466	3.8124	4.4705	19.78	10.48	17.26
1.0	0.10	0.3	0.05	3.8990	4.3434	3.2619	4.2652	9.39	−1.80	30.76
1.0	0.20	0.1	0.01	3.4011	3.3463	2.2766	3.4982	2.85	4.54	53.66
1.0	0.20	0.1	0.05	3.5301	3.5149	2.4327	3.3590	−4.85	−4.43	38.08
1.0	0.20	0.3	0.01	3.9199	3.4399	2.7253	3.5641	−9.08	3.61	30.78
1.0	0.20	0.3	0.05	3.7742	3.6286	2.5269	3.5357	−6.32	−2.56	39.92

Table 5 continued

β	h	θ	δ	ATP_{ZI}	ATP_{MB}	ATP_{SB}	ATP_{DB}	RP_{ZI} (%)	RP_{MB} (%)	RP_{SB} (%)
2.0	0.01	0.1	0.01	1.6178	2.5443	3.5397	4.0680	151.44	59.88	14.92
2.0	0.01	0.1	0.05	1.6257	2.1253	2.7030	3.3883	108.42	59.43	25.35
2.0	0.01	0.3	0.01	1.8688	2.3472	3.5486	4.4158	136.29	88.13	24.44
2.0	0.01	0.3	0.05	1.7467	2.3536	2.7958	3.9688	127.22	68.63	41.96
2.0	0.10	0.1	0.01	1.5901	1.6179	1.2096	1.7492	10.01	8.12	44.61
2.0	0.10	0.1	0.05	1.6231	1.6476	0.8668	1.8041	11.15	9.50	108.15
2.0	0.10	0.3	0.01	1.9337	1.8426	1.6850	2.1136	9.30	14.71	25.44
2.0	0.10	0.3	0.05	1.6235	1.8527	0.9128	1.9481	19.99	5.15	113.41
2.0	0.20	0.1	0.01	1.9223	1.7207	0.7822	1.7851	−7.14	3.74	128.22
2.0	0.20	0.1	0.05	1.5362	1.5266	0.6862	1.4180	−7.69	−7.11	106.66
2.0	0.20	0.3	0.01	1.9238	1.8588	0.7342	1.6034	−16.66	−13.74	118.37
2.0	0.20	0.3	0.05	1.9440	1.8717	0.5536	1.7162	−11.72	−8.30	210.01

Table 6 Suggested policies for different operating conditions

	Low volatility		High volatility	
	Low holding cost	High holding cost	Low holding cost	High holding cost
Low demand competition	SB	MB	DB	DB
High demand competition	DB	ZI	DB	DB

process. To the best of our knowledge, our paper makes a distinct contribution in the literature by analyzing the integrated bidding (pricing) and procurement (inventory) decisions under input price risk. From a theoretical perspective, our contribution is the characterizations of the optimal policy and the structural properties. Specifically, we prove that the optimal procurement strategy is of base-stock type, where base-stock levels depend on the commodity prices. We also illustrate that base-stock levels are not necessarily decreasing in the commodity prices by constructing a counterexample. The bid prices, however, are decreasing in the inventory level.

The optimal joint policy has significant potential for improving the firm's profits, but might require organizational changes and additional coordination costs. Consequently, from a practical perspective, it is important to identify policies that are easier to implement, but perform relatively well. To this end, we develop and analyze three heuristic strategies with varying degrees of sophistication. Comparing these strategies based on real historical data obtained for four commodities, we are able to provide a realistic assessment of their relative performances and generate managerial insights as to when it would be advisable to adopt the optimal or the heuristic policies.

The simplest heuristic policy we examine is ZI that does not hold any raw material inventory and decides on the bid prices myopically, based on the current spot market price. We show that such a policy can perform reasonably well when the financial and physical costs of holding inventory are significant, especially if the price competition in the demand market for securing projects is also intense and the raw material price volatility is low to medium. Coupling ZI with the possibility of storing inventory, even if procurement and bidding are conducted independently by different organizational units, leads to improved profits for the firm. We call this heuristic strategy MB. We find that the incremental gains of adopting MB are higher when inventory holding costs are at high/moderate levels. When the inventory cost is low, however, it is more important to integrate procurement and bidding strategies. Even a heuristic policy that uses a static (constant) bid price throughout the planning horizon (that is, SB) might perform close to optimal when holding inventory is less costly and the likelihood of winning projects is less sensitive to bid prices submitted (i.e., less competitive demand markets). In such environments, the ability to coordinate the procurement policy with the bid price is more critical than the ability to track spot market prices and utilize a more adaptive bidding policy. Nevertheless, the firm always leaves some potential profits on the table by adopting these heuristic policies. Our experiments indicate that the benefits of joint dynamic optimization of bidding and procurement decisions are most pronounced when the underlying commodity faces high price volatility and there is intense competition in terms of winning projects. We summarize the above insights in Table 6.

Table 6 also enables us to provide suggestions to the firm in the motivating example about which strategy to follow, keeping the particular operating environment it faces in mind. Recall that one main raw material of this firm is copper. The price volatility is quite high. Moreover, being relatively small, it is expensive for the firm to access operating funds and there is a shortage of storage space; consequently, the holding cost is quite high. Lastly, the industry where the firm competes is a mature one with a large number of players. Because of this, the demand market is highly competitive. Our analysis suggests that the firm should invest in getting the necessary expertise and making the required changes to implement an integrated dynamic bidding and procurement policy.

There are several potential avenues for future research. First, in this paper, the firm uses the spot market as its unique procurement channel. One possible direction is to incorporate other procurement channels, for example, forward and option contracts. Second, we assume that the winning probability of a project is given by an exogenously determined function. A game-theoretic model can be constructed to endogenize the winning probability. Clearly, the analysis in that case is more complex, and potentially involves auction models. However, we believe that at least some of the qualitative results in this study can be generalized to that case, and hope that this research spurs work on the above extensions.

Appendix 1: Proofs of Proposition 1 and Theorems 1 and 2

Proof of Proposition 1 In this proposition, we verify that the functional operators associated with MB, SB, and DB preserve Conditions 1 and 2 stated in Definition 1. Since the verification steps for MB and SB are very similar to those for DB, here, we only provide the proof for DB.

To simplify the proof, we rewrite T_{DB} as a linear combination of functional operators G and H as follows:

$$T_{\mathrm{DB}}v(x,i) = -h(x) + \lambda Gv(x,i) + \mu_i \sum_{j\neq i} \gamma_{ij} Hv(x,j) + (\overline{\mu} - \mu_i)v(x,i),$$

where

$$Gv(x,i) = \max_{b\geq 0}\Big\{(1-b)(Fv(x,i)+b) + bv(x,i)\Big\},$$

$$Fv(x,i) = \begin{cases} \max\big\{v(x,i)-p_i,\, v(x-1,i)\big\}, & \text{if } x\geq 1\\ v(x,i)-p_i, & \text{if } x=0,\end{cases}$$

and

$$Hv(x,j) = \max_{q\geq 0}\Big\{v(x+q,j) - p_j q\Big\}.$$

Now, we show that the functional operator T_{DB} preserves Conditions 1 and 2. We divide the proof into two steps, one for each condition. In each step, we consider G and H separately and show that Conditions 1 and 2 are preserved by each one of them.

Step 1. Verification of Condition 1.
Step 1.1. Preservation of Condition 1 by *G*. We first focus on the concavity of $Gv(x,i)$ with respect to x.

If $x = 0$, we want to prove $Gv(2,i) - Gv(1,i) \leq Gv(1,i) - Gv(0,i)$, that is, $\max_{b\geq 0}\big\{(1-b)(\max\{v(2,i)-p_i, v(1,i)\}+b)+bv(2,i)\big\} - \max_{b\geq 0}\big\{(1-b)(\max\{v(1,i)-p_i, v(0,i)\} + b) + bv(1,i)\big\} \leq \max_{b\geq 0}\big\{(1-b)(\max\{v(1,i)-p_i, v(0,i)\} + b) + bv(1,i)\big\} - \max_{b\geq 0}\big\{(1-b)(v(0,i)-p_i+b)+bv(0,i)\big\}$. We consider the following four cases:

Case A1. $v(2,i) - p_i \geq v(1,i)$ and $v(1,i) - p_i \geq v(0,i)$;
Case A2. $v(2,i) - p_i \geq v(1,i)$ and $v(1,i) - p_i < v(0,i)$;
Case A3. $v(2,i) - p_i < v(1,i)$ and $v(1,i) - p_i \geq v(0,i)$;
Case A4. $v(2,i) - p_i < v(1,i)$ and $v(1,i) - p_i < v(0,i)$.

Case A1. $\Leftrightarrow \max_{b\geq 0}\{(1-b)(v(2,i)+b-p_i)+bv(2,i)\} - \max_{b\geq 0}\{(1-b)(v(1,i)+b-p_i)+bv(1,i)\} \leq \max_{b\geq 0}\{(1-b)(v(1,i)+b-p_i)+bv(1,i)\} - \max_{b\geq 0}\{(1-b)(v(0,i)+b-p_i)+bv(0,i)\}$. $\Leftrightarrow \max_{b\geq 0}\{(1-b)(b-p_i)\}+v(2,i)-\max_{b\geq 0}\{(1-b)(b-p_i)\}-v(1,i) \leq \max_{b\geq 0}\{(1-b)(b-p_i)\}+v(1,i)-\max_{b\geq 0}\{(1-b)(b-p_i)\}-v(0,i)$. $\Leftrightarrow v(2,i)-v(1,i) \leq v(1,i)-v(0,i)$.

Case A2. This case is impossible since it contradicts with $v(2,i)-v(1,i) \leq v(1,i)-v(0,i)$.

Case A3. $\Leftrightarrow \max_{b\geq 0}\{(1-b)(v(1,i)+b)+bv(2,i)\} - \max_{b\geq 0}\{(1-b)(v(1,i)+b-p_i)+bv(1,i)\} \leq \max_{b\geq 0}\{(1-b)(v(1,i)+b-p_i)+bv(1,i)\} - \max_{b\geq 0}\{(1-b)(v(0,i)+b-p_i)+bv(0,i)\}$. $\Leftrightarrow \max_{b\geq 0}\{(1-b)b-b(v(1,i)-v(2,i))\} \leq \max_{b\geq 0}\{(1-b)(b-p_i)\}+v(1,i)-v(0,i)$. As for $\max_{b\geq 0}\{(1-b)b-b(v(1,i)-v(2,i))\}$, $b^* = \frac{1}{2}(1-(v(1,i)-v(2,i)))$ and $\max_{b\geq 0}\{(1-b)b-b(v(1,i)-v(2,i))\} = \frac{1}{4}(1-(v(1,i)-v(2,i)))^2$. As for $\max_{b\geq 0}\{(1-b)(b-p_i)\}$, $b^* = \frac{1}{2}(1+p_i)$ and $\max_{b\geq 0}\{(1-b)(b-p_i)\} = \frac{1}{4}(1-p_i)^2$. Hence, $\Leftrightarrow \frac{1}{4}(1-(v(1,i)-v(2,i)))^2 \leq \frac{1}{4}(1-p_i)^2 + v(1,i)-v(0,i)$. Since $v(2,i)-v(1,i) < p_i$ and $1+v(2,i)-v(1,i) \geq 1-1 = 0$, $\frac{1}{4}(1-(v(1,i)-v(2,i)))^2 = \frac{1}{4}(1+v(2,i)-v(1,i))^2 \leq \frac{1}{4}(1+p_i)^2$. Moreover, $\frac{1}{4}(1-p_i)^2 + v(1,i)-v(0,i) \geq \frac{1}{4}(1-p_i)^2 + p_i = \frac{1}{4}(1+p_i)^2$.

Case A4. $\Leftrightarrow \max_{b\geq 0}\{(1-b)(v(1,i)+b)+bv(2,i)\} - \max_{b\geq 0}\{(1-b)(v(0,i)+b)+bv(1,i)\} \leq \max_{b\geq 0}\{(1-b)(v(0,i)+b)+bv(1,i)\} - \max_{b\geq 0}\{(1-b)(v(0,i)+b-p_i)+bv(0,i)\}$. $\Leftrightarrow \max_{b\geq 0}\{(1-b)b-b(v(1,i)-v(2,i))\} - \max_{b\geq 0}\{(1-b)b-b(v(0,i)-v(1,i))\} - (v(0,i)-v(1,i)) \leq \max_{b\geq 0}\{(1-b)b-b(v(0,i)-v(1,i))\} - \max_{b\geq 0}\{(1-b)(b-p_i)\}$. $\Leftrightarrow \frac{1}{4}(1-(v(1,i)-v(2,i)))^2 - \frac{1}{4}(1-(v(0,i)-v(1,i)))^2 - (v(0,i)-v(1,i)) \leq \frac{1}{4}(1-(v(0,i)-v(1,i)))^2 - \frac{1}{4}(1-p_i)^2$. $\Leftrightarrow (1-(v(1,i)-v(2,i)))^2 - (1+v(0,i)-v(1,i))^2 \leq (1+v(1,i)-v(0,i))^2 - (1-p_i)^2$. Since $v(2,i)-v(1,i) \leq v(1,i)-v(0,i)$, $1+v(2,i)-v(1,i) \geq 1-1 = 0$, and $1+v(1,i)-v(0,i) \geq 1-1 = 0$, we have $(1-(v(1,i)-v(2,i)))^2 \leq (1+v(1,i)-v(0,i))^2$. Moreover, since $v(0,i)-v(1,i) > -p_i$, we have $1+v(0,i)-v(1,i) > 1-p_i \geq 0$. Then, we have $(1+v(0,i)-v(1,i))^2 > (1-p_i)^2$.

If $x \geq 1$, we want to prove $Gv(x+2,i) - Gv(x+1,i) \leq Gv(x+1,i) - Gv(x,i)$, that is, $\max_{b\geq 0}\{(1-b)(\max\{v(x+2,i)-p_i, v(x+1,i)\}+b)+bv(x+2,i)\} - \max_{b\geq 0}\{(1-b)(\max\{v(x+1,i)-p_i, v(x,i)\}+b)+bv(x+1,i)\} \leq \max_{b\geq 0}\{(1-b)(\max\{v(x+1,i)-p_i, v(x,i)\}+b)+bv(x+1,i)\} - \max_{b\geq 0}\{(1-b)(\max\{v(x,i)-p_i, v(x-1,i)\}+b)+bv(x,i)\}$. Since $v(x,i)$ is concave with respect to x, there exists an integer x^* such that if $x \leq x^*$, $v(x,i)-p_i \geq v(x-1,i)$ and if $x > x^*$, $v(x,i)-p_i < v(x-1,i)$. We consider the following four cases:

Case B1. $x \leq x^*-2$;
Case B2. $x = x^*-1$;
Case B3. $x = x^*$;
Case B4. $x \geq x^*+1$.

Case B1. $\max_{b\geq 0}\{(1-b)(v(x+2,i)+b-p_i)+bv(x+2,i)\} - \max_{b\geq 0}\{(1-b)(v(x+1,i)+b-p_i)+bv(x+1,i)\} \leq \max_{b\geq 0}\{(1-b)(v(x+1,i)+b-p_i)+bv(x+1,i)\} - \max_{b\geq 0}\{(1-b)(v(x,i)+b-p_i)+bv(x,i)\}$. $\Leftrightarrow \max_{b\geq 0}\{(1-b)(b-p_i)\}+v(x+2,i)-\max_{b\geq 0}\{(1-b)(b-p_i)\}-v(x+1,i) \leq \max_{b\geq 0}\{(1-b)(b-p_i)\}+v(x+1,i)-\max_{b\geq 0}\{(1-b)(b-p_i)\}-v(x,i)$. $\Leftrightarrow v(x+2,i)-v(x+1,i) \leq v(x+1,i)-v(x,i)$.

Case B2. $\max_{b\geq 0}\{(1-b)(v(x+1,i)+b)+bv(x+2,i)\}-\max_{b\geq 0}\{(1-b)(v(x+1,i)+b-p_i)+bv(x+1,i)\} \leq \max_{b\geq 0}\{(1-b)(v(x+1,i)+b-p_i)+bv(x+1,i)\}-\max_{b\geq 0}\{(1-b)(v(x,i)+b-p_i)+bv(x,i)\}$. $\Leftrightarrow \max_{b\geq 0}\{(1-b)b-b(v(x+1,i)-v(x+2,i))\}-\max_{b\geq 0}\{(1-b)(b-p_i)\} \leq v(x+1,i)-v(x,i)$. $\Leftrightarrow \frac{1}{4}(1-(v(x+1,i)-v(x+2,i)))^2-\frac{1}{4}(1-p_i)^2 \leq v(x+1,i)-v(x,i)$. Since $x+2 = x^*-1+2 = x^*+1 > x^*$, we have $v(x+2,i)-v(x+1,i) < p_i$. So, $1-(v(x+1,i)-v(x+2,i)) = 1+v(x+2,i)-v(x+1,i) < 1+p_i$. Combining with the fact that $1-(v(x+1,i)-v(x+2,i)) = 1+v(x+2,i)-v(x+1,i) \geq 1-1 = 0$, we have $\frac{1}{4}(1-(v(x+1,i)-v(x+2,i)))^2-\frac{1}{4}(1-p_i)^2 \leq \frac{1}{4}(1+p_i)^2-\frac{1}{4}(1-p_i)^2 = p_i$. Moreover, since $x = x^*-1$, that is, $x+1 = x^*$, we have $v(x+1,i)-p_i \geq v(x,i)$. Hence, $v(x+1,i)-v(x,i) \geq p_i$.

Case B3. $\max_{b\geq 0}\{(1-b)(v(x+1,i)+b)+bv(x+2,i)\}-\max_{b\geq 0}\{(1-b)(v(x,i)+b)+bv(x+1,i)\} \leq \max_{b\geq 0}\{(1-b)(v(x,i)+b)+bv(x+1,i)\}-\max_{b\geq 0}\{(1-b)(v(x,i)+b-p_i)+bv(x,i)\}$. $\Leftrightarrow \max_{b\geq 0}\{(1-b)b-b(v(x+1,i)-v(x+2,i))\}-\max_{b\geq 0}\{(1-b)b-b(v(x,i)-v(x+1,i))\}-(v(x,i)-v(x+1,i)) \leq \max_{b\geq 0}\{(1-b)b-b(v(x,i)-v(x+1,i))\}-\max_{b\geq 0}\{(1-b)(b-p_i)\}$. $\Leftrightarrow \frac{1}{4}(1-(v(x+1,i)-v(x+2,i)))^2-\frac{1}{4}(1-(v(x,i)-v(x+1,i)))^2-(v(x,i)-v(x+1,i)) \leq \frac{1}{4}(1-(v(x,i)-v(x+1,i)))^2-\frac{1}{4}(1-p_i)^2$. $\Leftrightarrow (1+v(x+2,i)-v(x+1,i))^2-(1+v(x,i)-v(x+1,i))^2 \leq (1+v(x+1,i)-v(x,i))^2-(1-p_i)^2$. Since $v(x+2,i)-v(x+1,i) \leq v(x+1,i)-v(x,i)$, $1+v(x+2,i)-v(x+1,i) \geq 1-1 = 0$, and $1+v(x+1,i)-v(x,i) \geq 1-1 = 0$, we have $(1+v(x+2,i)-v(x+1,i))^2 \leq (1+v(x+1,i)-v(x,i))^2$. Moreover, since $v(x+1,i)-v(x,i) < p_i$, we have $v(x,i)-v(x+1,i) > -p_i$. Combining with the fact that $1+v(x,i)-v(x+1,i) \geq 1-p_i \geq 0$, we have $(1+v(x,i)-v(x+1,i))^2 \geq (1-p_i)^2$.

Case B4. $\Leftrightarrow \max_{b\geq 0}\{(1-b)(v(x+1,i)+b)+bv(x+2,i)\}-\max_{b\geq 0}\{(1-b)(v(x,i)+b)+bv(x+1,i)\} \leq \max_{b\geq 0}\{(1-b)(v(x,i)+b)+bv(x+1,i)\}-\max_{b\geq 0}\{(1-b)(v(x-1,i)+b)+bv(x,i)\}$. $\Leftrightarrow \max_{b\geq 0}\{(1-b)b-b(v(x+1,i)-v(x+2,i))\}-\max_{b\geq 0}\{(1-b)b-b(v(x,i)-v(x+1,i))\}-(v(x,i)-v(x+1,i)) \leq \max_{b\geq 0}\{(1-b)b-b(v(x,i)-v(x+1,i))\}-\max_{b\geq 0}\{(1-b)b-b(v(x-1,i)-v(x,i))\}-(v(x-1,i)-v(x,i))$. $\Leftrightarrow$ $\frac{1}{4}(1-(v(x+1,i)-v(x+2,i)))^2-\frac{1}{4}(1-(v(x,i)-v(x+1,i)))^2-(v(x,i)-v(x+1,i)) \leq \frac{1}{4}(1-(v(x,i)-v(x+1,i)))^2-\frac{1}{4}(1-(v(x-1,i)-v(x,i)))^2-(v(x-1,i)-v(x,i))$. $\Leftrightarrow (1+v(x+2,i)-v(x+1,i))^2-(1+v(x,i)-v(x+1,i))^2 \leq (1+v(x+1,i)-v(x,i))^2-(1+v(x-1,i)-v(x,i))^2$. Since $v(x+2,i)-v(x+1,i) \leq v(x+1,i)-v(x,i)$, $1+v(x+2,i)-v(x+1,i) \geq 1-1 = 0$, and $1+v(x+1,i)-v(x,i) \geq 1-1 = 0$, we have $(1+v(x+2,i)-v(x+1,i))^2 \leq (1+v(x+1,i)-v(x,i))^2$. Moreover, since $v(x+1,i)-v(x,i) \leq v(x,i)-v(x-1,i)$, we have $v(x,i)-v(x+1,i) \geq v(x-1,i)-v(x,i)$. Combining with the facts that $1+v(x,i)-v(x+1,i) \geq 0$ (since $v(x+1,i)-p_i < v(x,i)$, we have $v(x,i)-v(x+1,i) > -p_i$. So, $1+v(x,i)-v(x+1,i) > 1-p_i \geq 0$) and $1+v(x-1,i)-v(x,i) \geq 0$ (since $v(x,i)-p_i < v(x-1,i)$, we have $v(x-1,i)-v(x,i) > -p_i$. So, $1+v(x-1,i)-v(x,i) > 1-p_i \geq 0$), we have $(1+v(x,i)-v(x+1,i))^2 \geq (1+v(x-1,i)-v(x,i))^2$.

Step 1.2. Preservation of Condition 1 by $\boldsymbol{H}$. Now, we focus on the concavity of $Hv(x,j)$ with respect to x. Define

$$\tilde{H}v(x,j) = \max_{y\geq x}\{v(y,j)-p_j y\}.$$

So, $Hv(x,j) = \tilde{H}v(x,j)+p_j x$. Since $p_j x$ is linear in x, we focus on the concavity of $\tilde{H}v(x,j)$ with respect to x, that is, we want to prove $\tilde{H}v(x+2,j)-\tilde{H}v(x+1,j) \leq$

$\tilde{H}v(x+1, j) - \tilde{H}v(x, j)$. Let y^* be an integer such that $y^* = \arg\max_{y=0,1,\ldots}\{v(y, j) - p_j y\}$. Then, from the concavity of $v(x, j)$ with respect to x, we have

$$\tilde{H}v(x, j) = \begin{cases} v(y^*, j) - p_j y^*, & \text{if } y^* > x \\ v(x, j) - p_j x, & \text{if } y^* \leq x. \end{cases}$$

We consider the following three cases:

Case C1. $x \leq y^* - 2$;
Case C2. $x = y^* - 1$;
Case C3. $x \geq y^*$.

Case C1. $\tilde{H}v(x+2, j) - \tilde{H}v(x+1, j) = v(y^*, j) - p_j y^* - (v(y^*, j) - p_j y^*) = 0 = \tilde{H}v(x+1, j) - \tilde{H}v(x, j)$.

Case C2. $\tilde{H}v(x+2, j) - \tilde{H}v(x+1, j) = v(y^*+1, j) - p_j(y^*+1) - (v(y^*, j) - p_j y^*) \leq 0$. The last inequality follows because of the optimality of y^*. Moreover, $\tilde{H}v(x+1, j) - \tilde{H}v(x, j) = v(y^*, j) - p_j y^* - (v(y^*, j) - p_j y^*) = 0$. Hence, $\tilde{H}v(x+2, j) - \tilde{H}v(x+1, j) \leq \tilde{H}v(x+1, j) - \tilde{H}v(x, j)$.

Case C3. $\tilde{H}v(x+2, j) - \tilde{H}v(x+1, j) = v(x+2, j) - p_j(x+2) - (v(x+1, j) - p_j(x+1)) = v(x+2, j) - v(x+1, j) - p_j$. $\tilde{H}v(x+1, j) - \tilde{H}v(x, j) = v(x+1, j) - p_j(x+1) - (v(x, j) - p_j x) = v(x+1, j) - v(x, j) - p_j$. Since $v(x+2, j) - v(x+1, j) \leq v(x+1, j) - v(x, j)$, we have $\tilde{H}v(x+2, j) - \tilde{H}v(x+1, j) \leq \tilde{H}v(x+1, j) - \tilde{H}v(x, j)$.

Step 1.3. Preservation of Condition 1 by T_{DB}. Summarizing the above results and observing that $-h(x)$ is concave, we conclude that T_{DB} preserves Condition 1, since it is the linear combination of terms that satisfy Condition 1.

Step 2. Verification of Condition 2.

Step 2.1. Preservation of Condition 2 by G. First, we focus on $Gv(x, i)$. We want to prove $Gv(x+1, i) - Gv(x, i) \geq -1$ for $x \in N$ and $i \in M$.

If $x = 0$, $Gv(1, i) - Gv(0, i) = \max_{b\geq 0}\{(1-b)(\max\{v(1, i) - p_i, v(0, i)\} + b) + bv(1, i)\} - \max_{b\geq 0}\{(1-b)(b-p_i)\} - v(0, i)$. We consider the following two cases:

Case D1. $v(1, i) - p_i \geq v(0, i)$;
Case D2. $v(1, i) - p_i < v(0, i)$.

Case D1. $Gv(1, i) - Gv(0, i) = \max_{b\geq 0}\{(1-b)(v(1, i) + b - p_i) + bv(1, i)\} - \max_{b\geq 0}\{(1-b)(b-p_i)\} - v(0, i) = v(1, i) - v(0, i) \geq -1$.

Case D2. $Gv(1, i) - Gv(0, i) = \max_{b\geq 0}\{(1-b)(v(0, i) + b) + bv(1, i)\} - \max_{b\geq 0}\{(1-b)(b-p_i)\} - v(0, i) = \frac{1}{4}(1 - (v(0, i) - v(1, i)))^2 - \frac{1}{4}(1-p_i)^2$. Since $v(1, i) - v(0, i) \geq -1$, we have $1 - (v(0, i) - v(1, i)) = 1 + v(1, i) - v(0, i) \geq 1 - 1 = 0$. Hence, $\frac{1}{4}(1 - (v(0, i) - v(1, i)))^2 - \frac{1}{4}(1-p_i)^2 \geq 0 - \frac{1}{4}(1-p_i)^2 = -\frac{1}{4}(1-p_i)^2 \geq -1$.

If $x \geq 1$, we consider the following three cases:

Case E1. $x \leq x^* - 1$;
Case E2. $x = x^*$;
Case E3. $x \geq x^* + 1$.

Case E1. $Gv(x+1, i) - Gv(x, i) = \max_{b\geq 0}\{(1-b)(v(x+1, i) + b - p_i) + bv(x+1, i)\} - \max_{b\geq 0}\{(1-b)(v(x, i) + b - p_i) + bv(x, i)\} = v(x+1, i) - v(x, i) \geq -1$.

Case E2. $Gv(x+1, i) - Gv(x, i) = \max_{b\geq 0}\{(1-b)(v(x, i) + b) + bv(x+1, i)\} - \max_{b\geq 0}\{(1-b)(v(x, i) + b - p_i) + bv(x, i)\} = \frac{1}{4}(1 - (v(x, i) - v(x+1, i)))^2 - \frac{1}{4}(1-p_i)^2$. Since $v(x+$

$1, i)-v(x,i) \geq -1$, we have $1-(v(x,i)-v(x+1,i)) = 1+v(x+1,i)-v(x,i) \geq 1-1 = 0$. Hence, $\frac{1}{4}(1-(v(x,i)-v(x+1,i)))^2 - \frac{1}{4}(1-p_i)^2 \geq 0 - \frac{1}{4}(1-p_i)^2 = -\frac{1}{4}(1-p_i)^2 \geq -1$.

Case E3. $Gv(x+1,i)-Gv(x,i) = \max_{b\geq 0}\{(1-b)(v(x,i)+b)+bv(x+1,i)\} - \max_{b\geq 0}\{(1-b)(v(x-1,i)+b)+bv(x,i)\} = \frac{1}{4}(1-(v(x,i)-v(x+1,i)))^2 - \frac{1}{4}(1-(v(x-1,i)-v(x,i)))^2 - (v(x-1,i)-v(x,i)) = \frac{1}{4}(1-(v(x,i)-v(x+1,i)))^2 - \frac{1}{4}(1+v(x-1,i)-v(x,i))^2$. Since $v(x+1,i)-v(x,i) \geq -1$, we have $1+v(x+1,i)-v(x,i) \geq 1-1 = 0$. Hence, $\frac{1}{4}(1-(v(x,i)-v(x+1,i)))^2 = \frac{1}{4}(1+v(x+1,i)-v(x,i))^2 \geq 0$. Moreover, since $v(x+1,i)-v(x,i) \geq -1$ and $v(x+1,i)-v(x,i) \leq v(x,i)-v(x-1,i)$, we have $v(x,i)-v(x-1,i) \geq -1$, that is, $v(x-1,i)-v(x,i) \leq 1$. Since $0 \leq 1-p_i \leq 1+v(x-1,i)-v(x,i) \leq 1+1 = 2$, we have $\frac{1}{4}(1+v(x-1,i)-v(x,i))^2 \leq \frac{1}{4}\times 2^2 = 1$. Hence, $Gv(x+1,i)-Gv(x,i) \geq 0-1 = -1$.

Step 2.2. Preservation of Condition 2 by H. Here, we focus on $Hv(x,j)$. We want to prove $Hv(x+1,j)-Hv(x,j) \geq -1$ for $x \in N$ and $i \in M$. We consider the following two cases:

Case F1. $x \leq y^* - 1$;
Case F2. $x \geq y^*$.

Case F1. $Hv(x+1,j)-Hv(x,j) = v(y^*,j)-p_jy^*+p_j(x+1)-(v(y^*,j)-p_jy^*+p_jx) = p_j \geq -1$.
Case F2. $Hv(x+1,j)-Hv(x,j) = v(x+1,j)-p_j(x+1)+p_j(x+1)-(v(x,j)-p_jx+p_jx) = v(x+1,j)-v(x,j) \geq -1$.

Step 2.3. Preservation of Condition 2 by T_{DB}. In conclusion,

$$\begin{aligned} T_{\mathrm{DB}}v(x+1,i) - T_{\mathrm{DB}}v(x,i) &\geq -h(x+1) - (-h(x)) + \lambda(-1) \\ &\quad + \mu_i \sum_{j\neq i} \gamma_{ij}(-1) + (\overline{\mu} - \mu_i)(-1) \\ &= -(h(x+1)-h(x)) + (\lambda + \overline{\mu})(-1) \\ &\geq -\alpha + (\lambda+\overline{\mu})(-1) = (\alpha+\lambda+\overline{\mu})(-1) = -1. \end{aligned}$$

Proof of Theorem 1 Since the results follow from the concavity of V_{I}, we only need to prove $V_{\mathrm{I}} \in \mathcal{V}$. This can be easily shown by using the facts that (i) $V_{\mathrm{I}} = \lim_{n\to\infty} T_{\mathrm{I}}^n v$ for any v in $\mathcal{V}$, where T_{I}^n is the n-fold composition of T_{I} for I $\in$ {MB, SB, DB} and (ii) V_{I} is the unique solution of $v = T_{\mathrm{I}}v$ (see Theorem 5.1 of Porteus 1982 and Theorem 6.10.4 of Puterman 1994).

Proof of Theorem 2 Recall that

$$T_{\mathrm{DB}}v(x,i) = -h(x) + \lambda Gv(x,i) + \mu_i \sum_{j\neq i} \gamma_{ij} Hv(x,j) + (\overline{\mu}-\mu_i)v(x,i),$$

where

$$Gv(x,i) = \max_{b\geq 0}\left\{(1-b)(Fv(x,i)+b) + bv(x,i)\right\},$$

$$Fv(x,i) = \begin{cases} \max\{v(x,i)-p_i, v(x-1,i)\}, & \text{if } x \geq 1 \\ v(x,i)-p_i, & \text{if } x = 0, \end{cases}$$

and

$$Hv(x,j) = \max_{q\geq 0}\left\{v(x+q,j) - p_jq\right\}.$$

Note that the optimal bid prices are determined by the maximization of $(1-b)(FV_{\mathrm{DB}}(x,i)+b)+bV_{\mathrm{DB}}(x,i)$ with respect to b. Let $Z(b,x)=(1-b)(FV_{\mathrm{DB}}(x,i)+b)+bV_{\mathrm{DB}}(x,i)$. To prove the monotonicity of the optimal bid prices with respect to the inventory level, it suffices to show $Z(b^{'},x+1)-Z(b^{'},x)\leq Z(b,x+1)-Z(b,x)$ for all $x\geq 0$ and $1\geq b^{'}\geq b\geq 0$.

If $x=0$, we want to prove $Z(b^{'},1)-Z(b^{'},0)\leq Z(b,1)-Z(b,0)$, that is, $(1-b^{'})(FV_{\mathrm{DB}}(1,i)+b^{'})+b^{'}V_{\mathrm{DB}}(1,i)-(1-b^{'})(FV_{\mathrm{DB}}(0,i)+b^{'})-b^{'}V_{\mathrm{DB}}(0,i)\leq(1-b)(FV_{\mathrm{DB}}(1,i)+b)+bV_{\mathrm{DB}}(1,i)-(1-b)(FV_{\mathrm{DB}}(0,i)+b)-bV_{\mathrm{DB}}(0,i)$. We consider the following two cases:

Case G1. $V_{\mathrm{DB}}(1,i)-p_i\geq V_{\mathrm{DB}}(0,i)$;
Case G2. $V_{\mathrm{DB}}(1,i)-p_i< V_{\mathrm{DB}}(0,i)$.

Case G1. $\Leftrightarrow (1-b^{'})(V_{\mathrm{DB}}(1,i)-p_i+b^{'})+b^{'}V_{\mathrm{DB}}(1,i)-(1-b^{'})(V_{\mathrm{DB}}(0,i)-p_i+b^{'})-b^{'}V_{\mathrm{DB}}(0,i)\leq(1-b)(V_{\mathrm{DB}}(1,i)-p_i+b)+bV_{\mathrm{DB}}(1,i)-(1-b)(V_{\mathrm{DB}}(0,i)-p_i+b)-bV_{\mathrm{DB}}(0,i)$. $\Leftrightarrow V_{\mathrm{DB}}(1,i)-V_{\mathrm{DB}}(0,i)\leq V_{\mathrm{DB}}(1,i)-V_{\mathrm{DB}}(0,i)$.
Case G2. $\Leftrightarrow (1-b^{'})(V_{\mathrm{DB}}(0,i)+b^{'})+b^{'}V_{\mathrm{DB}}(1,i)-(1-b^{'})(V_{\mathrm{DB}}(0,i)-p_i+b^{'})-b^{'}V_{\mathrm{DB}}(0,i)\leq(1-b)(V_{\mathrm{DB}}(0,i)+b)+bV_{\mathrm{DB}}(1,i)-(1-b)(V_{\mathrm{DB}}(0,i)-p_i+b)-bV_{\mathrm{DB}}(0,i)$. $\Leftrightarrow b^{'}(V_{\mathrm{DB}}(1,i)-V_{\mathrm{DB}}(0,i)-p_i)+p_i\leq b(V_{\mathrm{DB}}(1,i)-V_{\mathrm{DB}}(0,i)-p_i)+p_i$. $\Leftrightarrow b^{'}\geq b$.

If $x\geq 1$, we want to prove $Z(b^{'},x+1)-Z(b^{'},x)\leq Z(b,x+1)-Z(b,x)$, that is, $(1-b^{'})(FV_{\mathrm{DB}}(x+1,i)+b^{'})+b^{'}V_{\mathrm{DB}}(x+1,i)-(1-b^{'})(FV_{\mathrm{DB}}(x,i)+b^{'})-b^{'}V_{\mathrm{DB}}(x,i)\leq(1-b)(FV_{\mathrm{DB}}(x+1,i)+b)+bV_{\mathrm{DB}}(x+1,i)-(1-b)(FV_{\mathrm{DB}}(x,i)+b)-bV_{\mathrm{DB}}(x,i)$. From the proof of Theorem 1, $V_{\mathrm{DB}}(x,i)$ is concave with respect to x. Hence, there exists an integer x^* such that for $x\geq 1$,

$$FV_{\mathrm{DB}}(x,i)=\begin{cases} V_{\mathrm{DB}}(x,i)-p_i, & \text{if}\, x\leq x^* \\ V_{\mathrm{DB}}(x-1,i), & \text{if}\, x> x^*. \end{cases}$$

We consider the following three cases:

Case H1. $x\leq x^*-1$;
Case H2. $x=x^*$;
Case H3. $x\geq x^*+1$.

Case H1. $\Leftrightarrow (1-b^{'})(V_{\mathrm{DB}}(x+1,i)-p_i+b^{'})+b^{'}V_{\mathrm{DB}}(x+1,i)-(1-b^{'})(V_{\mathrm{DB}}(x,i)-p_i+b^{'})-b^{'}V_{\mathrm{DB}}(x,i)\leq(1-b)(V_{\mathrm{DB}}(x+1,i)-p_i+b)+bV_{\mathrm{DB}}(x+1,i)-(1-b)(V_{\mathrm{DB}}(x,i)-p_i+b)-bV_{\mathrm{DB}}(x,i)$. $\Leftrightarrow V_{\mathrm{DB}}(x+1,i)-V_{\mathrm{DB}}(x,i)\leq V_{\mathrm{DB}}(x+1,i)-V_{\mathrm{DB}}(x,i)$.
Case H2. $\Leftrightarrow (1-b^{'})(V_{\mathrm{DB}}(x,i)+b^{'})+b^{'}V_{\mathrm{DB}}(x+1,i)-(1-b^{'})(V_{\mathrm{DB}}(x,i)-p_i+b^{'})-b^{'}V_{\mathrm{DB}}(x,i)\leq(1-b)(V_{\mathrm{DB}}(x,i)+b)+bV_{\mathrm{DB}}(x+1,i)-(1-b)(V_{\mathrm{DB}}(x,i)-p_i+b)-bV_{\mathrm{DB}}(x,i)$. $\Leftrightarrow b^{'}(V_{\mathrm{DB}}(x+1,i)-V_{\mathrm{DB}}(x,i))+(1-b^{'})p_i\leq b(V_{\mathrm{DB}}(x+1,i)-V_{\mathrm{DB}}(x,i))+(1-b)p_i$. $\Leftrightarrow b^{'}(V_{\mathrm{DB}}(x+1,i)-V_{\mathrm{DB}}(x,i)-p_i)\leq b(V_{\mathrm{DB}}(x+1,i)-V_{\mathrm{DB}}(x,i)-p_i)$. Since $x+1=x^*+1>x^*$, we have $V_{\mathrm{DB}}(x+1,i)-V_{\mathrm{DB}}(x,i)-p_i<0$. Hence, $\Leftrightarrow b^{'}\geq b$.
Case H3. $\Leftrightarrow (1-b^{'})(V_{\mathrm{DB}}(x,i)+b^{'})+b^{'}V_{\mathrm{DB}}(x+1,i)-(1-b^{'})(V_{\mathrm{DB}}(x-1,i)+b^{'})-b^{'}V_{\mathrm{DB}}(x,i)\leq(1-b)(V_{\mathrm{DB}}(x,i)+b)+bV_{\mathrm{DB}}(x+1,i)-(1-b)(V_{\mathrm{DB}}(x-1,i)+b)-bV_{\mathrm{DB}}(x,i)$. $\Leftrightarrow V_{\mathrm{DB}}(x,i)-V_{\mathrm{DB}}(x-1,i)+b^{'}(V_{\mathrm{DB}}(x-1,i)-V_{\mathrm{DB}}(x,i)-V_{\mathrm{DB}}(x,i)+V_{\mathrm{DB}}(x+1,i))\leq V_{\mathrm{DB}}(x,i)-V_{\mathrm{DB}}(x-1,i)+b(V_{\mathrm{DB}}(x-1,i)-V_{\mathrm{DB}}(x,i)-V_{\mathrm{DB}}(x,i)+V_{\mathrm{DB}}(x+1,i))$. $\Leftrightarrow b^{'}(V_{\mathrm{DB}}(x-1,i)-V_{\mathrm{DB}}(x,i)-V_{\mathrm{DB}}(x,i)+V_{\mathrm{DB}}(x+1,i))\leq b(V_{\mathrm{DB}}(x-1,i)-V_{\mathrm{DB}}(x,i)-V_{\mathrm{DB}}(x,i)+V_{\mathrm{DB}}(x+1,i))$. From the proof of Theorem 1, $V_{\mathrm{DB}}(x,i)$ is concave with respect to x, that is, $V_{\mathrm{DB}}(x+1,i)-V_{\mathrm{DB}}(x,i)\leq V_{\mathrm{DB}}(x,i)-V_{\mathrm{DB}}(x-1,i)$. So, $V_{\mathrm{DB}}(x-1,i)-V_{\mathrm{DB}}(x,i)-V_{\mathrm{DB}}(x,i)+V_{\mathrm{DB}}(x+1,i)\leq 0$. Hence, $\Leftrightarrow b^{'}\geq b$.

Appendix 2. Tables for the other commodities: food index, oil, and platinum

See Tables 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, and 18

Table 7 Average profit rate comparison results for all strategies: food index

β	h	θ	δ	G_{ZI}	G_{MB}	G_{SB}	G_{DB}	RI_{ZI} (%)	RI_{MB} (%)	RI_{SB} (%)
0.5	0.01	0.1	0.01	0.0208	0.0283	0.0296	0.0298	43.20	5.30	0.76
0.5	0.01	0.1	0.05	0.0208	0.0275	0.0284	0.0287	37.94	4.55	1.26
0.5	0.01	0.3	0.01	0.0215	0.0295	0.0307	0.0309	43.70	4.80	0.96
0.5	0.01	0.3	0.05	0.0215	0.0286	0.0294	0.0298	38.33	4.06	1.49
0.5	0.10	0.1	0.01	0.0208	0.0241	0.0237	0.0245	17.72	1.72	3.44
0.5	0.10	0.1	0.05	0.0208	0.0239	0.0235	0.0243	16.64	1.85	3.57
0.5	0.10	0.3	0.01	0.0215	0.0250	0.0245	0.0254	17.77	1.39	3.65
0.5	0.10	0.3	0.05	0.0215	0.0248	0.0242	0.0251	16.74	1.49	4.03
0.5	0.20	0.1	0.01	0.0208	0.0222	0.0216	0.0225	8.01	1.18	4.30
0.5	0.20	0.1	0.05	0.0208	0.0222	0.0215	0.0224	7.41	0.86	4.33
0.5	0.20	0.3	0.01	0.0215	0.0231	0.0222	0.0233	8.18	0.97	4.76
0.5	0.20	0.3	0.05	0.0215	0.0229	0.0221	0.0232	7.60	1.01	4.80
1.0	0.01	0.1	0.01	0.0121	0.0169	0.0184	0.0187	55.04	11.08	1.74
1.0	0.01	0.1	0.05	0.0121	0.0163	0.0174	0.0179	47.95	9.63	2.97
1.0	0.01	0.3	0.01	0.0126	0.0178	0.0193	0.0197	55.84	10.32	2.13
1.0	0.01	0.3	0.05	0.0126	0.0172	0.0181	0.0188	48.67	9.07	3.37
1.0	0.10	0.1	0.01	0.0121	0.0141	0.0136	0.0146	21.12	3.92	7.74
1.0	0.10	0.1	0.05	0.0121	0.0139	0.0133	0.0145	19.77	4.06	8.58
1.0	0.10	0.3	0.01	0.0126	0.0148	0.0141	0.0153	21.67	3.63	8.73
1.0	0.10	0.3	0.05	0.0126	0.0146	0.0139	0.0151	19.86	3.28	9.07
1.0	0.20	0.1	0.01	0.0121	0.0129	0.0119	0.0132	9.04	1.76	10.65
1.0	0.20	0.1	0.05	0.0121	0.0129	0.0118	0.0131	8.58	1.86	10.72
1.0	0.20	0.3	0.01	0.0126	0.0136	0.0123	0.0138	9.09	1.39	11.76
1.0	0.20	0.3	0.05	0.0126	0.0135	0.0123	0.0137	8.66	1.47	11.84
2.0	0.01	0.1	0.01	0.0059	0.0084	0.0099	0.0104	76.97	22.81	4.27
2.0	0.01	0.1	0.05	0.0059	0.0080	0.0091	0.0097	66.30	20.96	7.32
2.0	0.01	0.3	0.01	0.0062	0.0090	0.0105	0.0110	78.61	22.26	4.93
2.0	0.01	0.3	0.05	0.0062	0.0086	0.0095	0.0103	67.70	19.84	8.27
2.0	0.10	0.1	0.01	0.0059	0.0068	0.0061	0.0074	26.99	9.62	21.10
2.0	0.10	0.1	0.05	0.0059	0.0067	0.0060	0.0073	24.27	8.77	21.93
2.0	0.10	0.3	0.01	0.0062	0.0072	0.0064	0.0079	27.98	9.42	23.79
2.0	0.10	0.3	0.05	0.0062	0.0071	0.0062	0.0077	25.43	8.57	24.62
2.0	0.20	0.1	0.01	0.0059	0.0059	0.0050	0.0064	9.67	9.67	29.32
2.0	0.20	0.1	0.05	0.0059	0.0059	0.0049	0.0064	8.60	8.60	29.94
2.0	0.20	0.3	0.01	0.0062	0.0063	0.0051	0.0068	10.39	8.32	32.68
2.0	0.20	0.3	0.05	0.0062	0.0062	0.0051	0.0067	9.39	8.87	33.27

Table 8 Average profit rate comparison results for all strategies: oil

β	h	θ	δ	G_{ZI}	G_{MB}	G_{SB}	G_{DB}	RI_{ZI} (%)	RI_{MB} (%)	RI_{SB} (%)
0.5	0.01	0.1	0.01	0.0360	0.0518	0.0546	0.0554	54.02	6.93	1.39
0.5	0.01	0.1	0.05	0.0360	0.0482	0.0497	0.0509	41.40	5.44	2.25
0.5	0.01	0.3	0.01	0.0377	0.0548	0.0573	0.0582	54.32	6.15	1.57
0.5	0.01	0.3	0.05	0.0377	0.0510	0.0521	0.0534	41.71	4.82	2.48
0.5	0.10	0.1	0.01	0.0360	0.0399	0.0387	0.0405	12.60	1.49	4.58
0.5	0.10	0.1	0.05	0.0360	0.0394	0.0380	0.0399	11.04	1.27	5.10
0.5	0.10	0.3	0.01	0.0377	0.0420	0.0406	0.0425	12.67	1.24	4.75
0.5	0.10	0.3	0.05	0.0377	0.0414	0.0398	0.0419	11.09	1.19	5.20
0.5	0.20	0.1	0.01	0.0360	0.0370	0.0351	0.0372	3.41	0.47	5.95
0.5	0.20	0.1	0.05	0.0360	0.0369	0.0350	0.0370	2.97	0.28	5.94
0.5	0.20	0.3	0.01	0.0377	0.0389	0.0367	0.0390	3.46	0.38	6.38
0.5	0.20	0.3	0.05	0.0377	0.0387	0.0365	0.0389	3.04	0.36	6.44
1.0	0.01	0.1	0.01	0.0199	0.0296	0.0328	0.0339	69.89	14.44	3.35
1.0	0.01	0.1	0.05	0.0199	0.0273	0.0288	0.0304	52.64	11.38	5.63
1.0	0.01	0.3	0.01	0.0212	0.0319	0.0349	0.0361	70.21	13.22	3.55
1.0	0.01	0.3	0.05	0.0212	0.0294	0.0307	0.0325	52.99	10.37	5.84
1.0	0.10	0.1	0.01	0.0199	0.0222	0.0204	0.0230	15.23	3.43	12.84
1.0	0.10	0.1	0.05	0.0199	0.0219	0.0199	0.0226	13.16	2.82	13.17
1.0	0.10	0.3	0.01	0.0212	0.0238	0.0216	0.0245	15.32	3.10	13.64
1.0	0.10	0.3	0.05	0.0212	0.0234	0.0211	0.0241	13.40	2.74	14.17
1.0	0.20	0.1	0.01	0.0199	0.0205	0.0180	0.0207	3.77	0.99	14.68
1.0	0.20	0.1	0.05	0.0199	0.0204	0.0180	0.0206	3.35	0.92	14.77
1.0	0.20	0.3	0.01	0.0212	0.0219	0.0191	0.0220	3.81	0.80	15.67
1.0	0.20	0.3	0.05	0.0212	0.0218	0.0190	0.0220	3.42	0.82	15.85
2.0	0.01	0.1	0.01	0.0089	0.0137	0.0164	0.0179	100.85	30.67	9.07
2.0	0.01	0.1	0.05	0.0089	0.0124	0.0133	0.0155	73.80	24.60	16.06
2.0	0.01	0.3	0.01	0.0096	0.0150	0.0178	0.0194	101.99	29.17	9.34
2.0	0.01	0.3	0.05	0.0096	0.0136	0.0145	0.0168	74.99	23.36	16.30
2.0	0.10	0.1	0.01	0.0089	0.0098	0.0078	0.0106	18.90	7.68	36.49
2.0	0.10	0.1	0.05	0.0089	0.0096	0.0075	0.0103	16.15	7.10	38.38
2.0	0.10	0.3	0.01	0.0096	0.0107	0.0083	0.0115	19.50	7.34	39.14
2.0	0.10	0.3	0.05	0.0096	0.0105	0.0079	0.0112	16.78	6.51	41.54
2.0	0.20	0.1	0.01	0.0089	0.0089	0.0064	0.0092	2.90	2.35	42.31
2.0	0.20	0.1	0.05	0.0089	0.0089	0.0064	0.0091	2.11	1.95	42.10
2.0	0.20	0.3	0.01	0.0096	0.0097	0.0068	0.0099	3.38	2.38	46.77
2.0	0.20	0.3	0.05	0.0096	0.0097	0.0067	0.0099	2.66	2.00	46.57

Table 9 Average profit rate comparison results for all strategies: platinum

β	h	θ	δ	G_{ZI}	G_{MB}	G_{SB}	G_{DB}	RI_{ZI} (%)	RI_{MB} (%)	RI_{SB} (%)
0.5	0.01	0.1	0.01	0.0277	0.0366	0.0380	0.0384	38.81	4.87	0.95
0.5	0.01	0.1	0.05	0.0277	0.0352	0.0360	0.0366	32.24	4.05	1.51
0.5	0.01	0.3	0.01	0.0286	0.0381	0.0394	0.0398	39.25	4.40	1.12
0.5	0.01	0.3	0.05	0.0286	0.0366	0.0373	0.0379	32.61	3.63	1.74
0.5	0.10	0.1	0.01	0.0277	0.0301	0.0294	0.0304	10.01	1.09	3.63
0.5	0.10	0.1	0.05	0.0277	0.0299	0.0291	0.0302	9.00	0.87	3.68
0.5	0.10	0.3	0.01	0.0286	0.0312	0.0303	0.0315	10.13	0.95	4.06
0.5	0.10	0.3	0.05	0.0286	0.0310	0.0300	0.0312	9.16	0.82	4.11
0.5	0.20	0.1	0.01	0.0277	0.0283	0.0271	0.0283	2.28	0.10	4.41
0.5	0.20	0.1	0.05	0.0277	0.0282	0.0271	0.0283	2.17	0.11	4.42
0.5	0.20	0.3	0.01	0.0286	0.0292	0.0279	0.0292	2.26	0.08	4.87
0.5	0.20	0.3	0.05	0.0286	0.0292	0.0278	0.0292	2.16	0.09	4.88
1.0	0.01	0.1	0.01	0.0160	0.0218	0.0234	0.0240	49.55	9.99	2.20
1.0	0.01	0.1	0.05	0.0160	0.0208	0.0218	0.0225	40.73	8.44	3.55
1.0	0.01	0.3	0.01	0.0167	0.0230	0.0245	0.0251	50.15	9.33	2.52
1.0	0.01	0.3	0.05	0.0167	0.0219	0.0227	0.0236	41.28	7.79	3.98
1.0	0.10	0.1	0.01	0.0160	0.0174	0.0164	0.0178	11.41	2.43	8.53
1.0	0.10	0.1	0.05	0.0160	0.0173	0.0162	0.0176	10.05	1.91	8.75
1.0	0.10	0.3	0.01	0.0167	0.0182	0.0171	0.0187	11.72	2.51	9.57
1.0	0.10	0.3	0.05	0.0167	0.0181	0.0168	0.0185	10.40	1.96	9.75
1.0	0.20	0.1	0.01	0.0160	0.0164	0.0149	0.0164	2.71	0.60	10.64
1.0	0.20	0.1	0.05	0.0160	0.0163	0.0148	0.0164	2.48	0.64	10.69
1.0	0.20	0.3	0.01	0.0167	0.0171	0.0154	0.0172	2.75	0.47	11.75
1.0	0.20	0.3	0.05	0.0167	0.0171	0.0153	0.0172	2.54	0.51	11.80
2.0	0.01	0.1	0.01	0.0077	0.0109	0.0124	0.0131	69.72	20.47	5.50
2.0	0.01	0.1	0.05	0.0077	0.0103	0.0111	0.0120	56.15	17.38	8.94
2.0	0.01	0.3	0.01	0.0081	0.0116	0.0131	0.0139	70.88	19.80	6.09
2.0	0.01	0.3	0.05	0.0081	0.0109	0.0116	0.0128	57.28	17.06	9.85
2.0	0.10	0.1	0.01	0.0077	0.0084	0.0071	0.0088	14.00	5.03	24.21
2.0	0.10	0.1	0.05	0.0077	0.0083	0.0069	0.0087	12.44	4.40	25.01
2.0	0.10	0.3	0.01	0.0081	0.0089	0.0073	0.0093	14.34	5.04	27.12
2.0	0.10	0.3	0.05	0.0081	0.0088	0.0072	0.0092	12.88	4.42	27.93
2.0	0.20	0.1	0.01	0.0077	0.0077	0.0060	0.0079	1.82	1.82	30.01
2.0	0.20	0.1	0.05	0.0077	0.0077	0.0060	0.0078	1.26	1.26	29.66
2.0	0.20	0.3	0.01	0.0081	0.0081	0.0062	0.0083	2.22	2.22	33.43
2.0	0.20	0.3	0.05	0.0081	0.0081	0.0062	0.0083	1.69	1.69	33.87

Table 10 Base-stock levels comparison results for MB, SB, and DB strategies: food index

β	h	θ	δ	Q_{MB}	Q_{SB}	Q_{DB}
0.5	0.01	0.1	0.01	8_1	12_1	11_1
0.5	0.01	0.1	0.05	6_1	9_1	8_1
0.5	0.01	0.3	0.01	9_1	12_1	11_1
0.5	0.01	0.3	0.05	7_1	9_1	9_1
0.5	0.10	0.1	0.01	3_1	4_1	3_1
0.5	0.10	0.1	0.05	3_1	3_1	3_1
0.5	0.10	0.3	0.01	3_1	4_1	3_1
0.5	0.10	0.3	0.05	3_1	3_1	3_1
0.5	0.20	0.1	0.01	1_1	2_1	2_1
0.5	0.20	0.1	0.05	1_1	2_1	2_1
0.5	0.20	0.3	0.01	2_1	2_1	2_1
0.5	0.20	0.3	0.05	1_1	2_1	2_1
1.0	0.01	0.1	0.01	5_1	10_1	8_1
1.0	0.01	0.1	0.05	4_1	7_1	6_1
1.0	0.01	0.3	0.01	6_1	11_1	9_1
1.0	0.01	0.3	0.05	4_1	8_1	7_1
1.0	0.10	0.1	0.01	2_1	3_1	3_1
1.0	0.10	0.1	0.05	2_1	3_1	2_1
1.0	0.10	0.3	0.01	2_1	3_1	3_1
1.0	0.10	0.3	0.05	2_1	3_1	2_1
1.0	0.20	0.1	0.01	1_1	1_1	1_1
1.0	0.20	0.1	0.05	1_1	1_1	1_1
1.0	0.20	0.3	0.01	1_1	1_1	1_1
1.0	0.20	0.3	0.05	1_1	1_1	1_1
2.0	0.01	0.1	0.01	3_1	8_1	6_1
2.0	0.01	0.1	0.05	2_1	6_1	5_1
2.0	0.01	0.3	0.01	3_1	9_1	7_1
2.0	0.01	0.3	0.05	3_1	7_1	5_1
2.0	0.10	0.1	0.01	1_1	2_1	2_1
2.0	0.10	0.1	0.05	1_1	2_1	2_1
2.0	0.10	0.3	0.01	1_1	2_1	2_1
2.0	0.10	0.3	0.05	1_1	2_1	2_1
2.0	0.20	0.1	0.01	0_1	1_1	1_1
2.0	0.20	0.1	0.05	0_1	1_1	1_1
2.0	0.20	0.3	0.01	1_1	1_1	1_1
2.0	0.20	0.3	0.05	1_1	1_1	1_1

Table 11 Base-stock levels comparison results for MB, SB, and DB strategies: oil

β	h	θ	δ	Q_{MB}	Q_{SB}	Q_{DB}
0.5	0.01	0.1	0.01	$23_1, 12_2, 2_3, 1_4, 1_7$	$30_1, 16_2, 3_3, 2_4, 1_7$	$29_1, 15_2, 3_3, 1_4, 1_7$
0.5	0.01	0.1	0.05	$15_1, 9_2, 2_3, 1_4, 1_7$	$20_1, 12_2, 2_3, 1_4, 1_7$	$20_1, 11_2, 2_3, 1_4, 1_7$
0.5	0.01	0.3	0.01	$24_1, 12_2, 2_3, 1_4, 1_7$	$31_1, 17_2, 3_3, 2_4, 1_7$	$30_1, 15_2, 3_3, 2_4, 1_7$
0.5	0.01	0.3	0.05	$16_1, 9_2, 2_3, 1_4, 1_7$	$21_1, 12_2, 3_3, 1_4, 1_7$	$20_1, 11_2, 2_3, 1_4, 1_7$
0.5	0.10	0.1	0.01	$6_1, 4_2, 1_3$	$7_1, 4_2, 1_3, 1_4, 1_7$	$7_1, 4_2, 1_3, 1_4$
0.5	0.10	0.1	0.05	$5_1, 3_2$	$6_1, 4_2, 1_3, 1_7$	$6_1, 4_2, 1_3$
0.5	0.10	0.3	0.01	$6_1, 4_2, 1_3, 1_4$	$7_1, 4_2, 1_3, 1_4, 1_7$	$7_1, 4_2, 1_3, 1_4$
0.5	0.10	0.3	0.05	$5_1, 3_2, 1_3$	$6_1, 4_2, 1_3, 1_7$	$6_1, 4_2, 1_3$
0.5	0.20	0.1	0.01	$2_1, 2_2$	$3_1, 2_2$	$3_1, 2_2$
0.5	0.20	0.1	0.05	$2_1, 1_2$	$2_1, 1_2$	$3_1, 2_2$
0.5	0.20	0.3	0.01	$3_1, 2_2$	$3_1, 2_2$	$3_1, 2_2$
0.5	0.20	0.3	0.05	$2_1, 1_2$	$2_1, 2_2$	$3_1, 2_2$
1.0	0.01	0.1	0.01	$15_1, 7_2, 1_3, 1_4$	$25_1, 13_2, 2_3, 1_4, 1_7$	$23_1, 11_2, 2_3, 1_4$
1.0	0.01	0.1	0.05	$10_1, 6_2, 1_3, 1_4$	$15_1, 9_2, 2_3, 1_4, 1_7$	$15_1, 8_2, 1_3, 1_4$
1.0	0.01	0.3	0.01	$16_1, 8_2, 2_3, 1_4$	$26_1, 14_2, 3_3, 1_4, 1_7$	$25_1, 12_2, 2_3, 1_4$
1.0	0.01	0.3	0.05	$11_1, 6_2, 1_3, 1_4$	$17_1, 10_2, 2_3, 1_4, 1_7$	$16_1, 9_2, 2_3, 1_4$
1.0	0.10	0.1	0.01	$4_1, 2_2$	$5_1, 3_2$	$5_1, 3_2$
1.0	0.10	0.1	0.05	$3_1, 2_2$	$4_1, 3_2$	$4_1, 3_2$
1.0	0.10	0.3	0.01	$4_1, 2_2$	$5_1, 3_2$	$5_1, 3_2$
1.0	0.10	0.3	0.05	$4_1, 2_2$	$4_1, 3_2$	$5_1, 3_2$
1.0	0.20	0.1	0.01	$1_1, 1_2$	$2_1, 1_2$	$2_1, 1_2$
1.0	0.20	0.1	0.05	$1_1, 1_2$	$1_1, 1_2$	$2_1, 1_2$
1.0	0.20	0.3	0.01	$2_1, 1_2$	$2_1, 1_2$	$2_1, 1_2$
1.0	0.20	0.3	0.05	$1_1, 1_2$	$2_1, 1_2$	$2_1, 1_2$
2.0	0.01	0.1	0.01	$8_1, 4_2, 1_3$	$19_1, 10_2, 2_3, 1_4, 1_7$	$18_1, 8_2, 1_3, 1_4$
2.0	0.01	0.1	0.05	$5_1, 3_2$	$11_1, 7_2, 1_3, 1_4, 1_7$	$11_1, 6_2, 1_3$
2.0	0.01	0.3	0.01	$9_1, 4_2, 1_3$	$20_1, 11_2, 2_3, 1_4, 1_7$	$19_1, 8_2, 1_3, 1_4$
2.0	0.01	0.3	0.05	$6_1, 3_2, 1_3$	$12_1, 7_2, 1_3, 1_4, 1_7$	$12_1, 6_2, 1_3$
2.0	0.10	0.1	0.01	$2_1, 1_2$	$3_1, 2_2$	$3_1, 2_2$
2.0	0.10	0.1	0.05	$2_1, 1_2$	$2_1, 1_2$	$3_1, 2_2$
2.0	0.10	0.3	0.01	$2_1, 1_2$	$3_1, 2_2$	$4_1, 2_2$
2.0	0.10	0.3	0.05	$2_1, 1_2$	$3_1, 2_2$	$3_1, 2_2$
2.0	0.20	0.1	0.01	1_1	1_1	$1_1, 1_2$
2.0	0.20	0.1	0.05	1_1	1_1	$1_1, 1_2$
2.0	0.20	0.3	0.01	1_1	1_1	$1_1, 1_2$
2.0	0.20	0.3	0.05	1_1	1_1	$1_1, 1_2$

Table 12 Base-stock levels comparison results for MB, SB, and DB strategies: platinum

β	h	θ	δ	Q_{MB}	Q_{SB}	Q_{DB}
0.5	0.01	0.1	0.01	$13_1, 2_3$	$17_1, 2_3, 1_4$	$16_1, 2_3$
0.5	0.01	0.1	0.05	$10_1, 2_3$	$13_1, 2_3, 1_4$	$12_1, 2_3$
0.5	0.01	0.3	0.01	$14_1, 2_3$	$18_1, 2_3, 1_4$	$17_1, 2_3$
0.5	0.01	0.3	0.05	$10_1, 2_3$	$14_1, 2_3, 1_4$	$13_1, 2_3$
0.5	0.10	0.1	0.01	$3_1, 1_3$	$4_1, 1_3$	$4_1, 1_3$
0.5	0.10	0.1	0.05	$3_1, 1_3$	$4_1, 1_3$	$4_1, 1_3$
0.5	0.10	0.3	0.01	$4_1, 1_3$	$4_1, 1_3$	$4_1, 1_3$
0.5	0.10	0.3	0.05	$3_1, 1_3$	$4_1, 1_3$	$4_1, 1_3$
0.5	0.20	0.1	0.01	1_1	1_1	1_1
0.5	0.20	0.1	0.05	1_1	1_1	1_1
0.5	0.20	0.3	0.01	1_1	1_1	1_1
0.5	0.20	0.3	0.05	1_1	1_1	1_1
1.0	0.01	0.1	0.01	$9_1, 1_3$	$15_1, 2_3$	$13_1, 1_3$
1.0	0.01	0.1	0.05	$7_1, 1_3$	$11_1, 2_3$	$10_1, 1_3$
1.0	0.01	0.3	0.01	$9_1, 1_3$	$15_1, 2_3$	$14_1, 1_3$
1.0	0.01	0.3	0.05	$7_1, 1_3$	$11_1, 2_3$	$10_1, 1_3$
1.0	0.10	0.1	0.01	2_1	$3_1, 1_3,$	$3_1, 1_3$
1.0	0.10	0.1	0.05	2_1	$3_1, 1_3$	3_1
1.0	0.10	0.3	0.01	2_1	$3_1, 1_3$	$3_1, 1_3$
1.0	0.10	0.3	0.05	2_1	$3_1, 1_3$	$3_1, 1_3$
1.0	0.20	0.1	0.01	1_1	1_1	1_1
1.0	0.20	0.1	0.05	1_1	1_1	1_1
1.0	0.20	0.3	0.01	1_1	1_1	1_1
1.0	0.20	0.3	0.05	1_1	1_1	1_1
2.0	0.01	0.1	0.01	5_1	$12_1, 2_3$	$10_1, 1_3$
2.0	0.01	0.1	0.05	4_1	$8_1, 1_3$	$7_1, 1_3$
2.0	0.01	0.3	0.01	$6_1, 1_3$	$13_1, 2_3$	$11_1, 1_3$
2.0	0.01	0.3	0.05	4_1	$9_1, 1_3$	$8_1, 1_3$
2.0	0.10	0.1	0.01	1_1	2_1	2_1
2.0	0.10	0.1	0.05	1_1	2_1	2_1
2.0	0.10	0.3	0.01	1_1	2_1	2_1
2.0	0.10	0.3	0.05	1_1	2_1	2_1
2.0	0.20	0.1	0.01	0_1	1_1	1_1
2.0	0.20	0.1	0.05	0_1	0_1	1_1
2.0	0.20	0.3	0.01	0_1	1_1	1_1
2.0	0.20	0.3	0.05	0_1	1_1	1_1

Table 13 Average bid price and average inventory comparison results for all strategies: food index

β	h	θ	δ	ZI	MB		SB		DB	
				Avg bid	Avg bid	Avg inv	Avg bid	Avg inv	Avg bid	Avg inv
0.5	0.01	0.1	0.01	0.7703	0.7703	6.49	0.7000	9.98	0.6991	8.98
0.5	0.01	0.1	0.05	0.7716	0.7716	4.56	0.7100	7.06	0.7090	6.11
0.5	0.01	0.3	0.01	0.7766	0.7766	7.40	0.7100	9.90	0.7128	8.96
0.5	0.01	0.3	0.05	0.7772	0.7772	5.50	0.7200	7.07	0.7174	7.07
0.5	0.10	0.1	0.01	0.7692	0.7692	1.94	0.7300	2.66	0.7392	1.84
0.5	0.10	0.1	0.05	0.7693	0.7693	1.94	0.7400	1.87	0.7387	1.85
0.5	0.10	0.3	0.01	0.7779	0.7779	1.90	0.7500	2.63	0.7511	1.82
0.5	0.10	0.3	0.05	0.7759	0.7759	1.92	0.7500	1.85	0.7484	1.84
0.5	0.20	0.1	0.01	0.7704	0.7704	0.49	0.7500	1.11	0.7475	1.10
0.5	0.20	0.1	0.05	0.7694	0.7694	0.49	0.7500	1.13	0.7463	1.11
0.5	0.20	0.3	0.01	0.7770	0.7770	1.15	0.7600	1.12	0.7560	1.10
0.5	0.20	0.3	0.05	0.7784	0.7784	0.48	0.7600	1.10	0.7569	1.09
1.0	0.01	0.1	0.01	0.6536	0.6536	4.01	0.5500	8.30	0.5513	6.38
1.0	0.01	0.1	0.05	0.6520	0.6520	3.08	0.5600	5.50	0.5628	4.56
1.0	0.01	0.3	0.01	0.6592	0.6592	4.98	0.5600	9.31	0.5601	7.39
1.0	0.01	0.3	0.05	0.6636	0.6636	3.01	0.5700	6.32	0.5728	5.40
1.0	0.10	0.1	0.01	0.6551	0.6551	1.29	0.6000	1.97	0.5957	1.93
1.0	0.10	0.1	0.05	0.6544	0.6544	1.29	0.6000	1.97	0.6148	1.18
1.0	0.10	0.3	0.01	0.6594	0.6594	1.29	0.6200	1.99	0.6072	1.96
1.0	0.10	0.3	0.05	0.6640	0.6640	1.28	0.6200	1.97	0.6278	1.17
1.0	0.20	0.1	0.01	0.6528	0.6528	0.56	0.6300	0.53	0.6310	0.51
1.0	0.20	0.1	0.05	0.6514	0.6514	0.55	0.6300	0.53	0.6291	0.51
1.0	0.20	0.3	0.01	0.6597	0.6597	0.55	0.6500	0.54	0.6407	0.51
1.0	0.20	0.3	0.05	0.6626	0.6626	0.55	0.6500	0.53	0.6437	0.51
2.0	0.01	0.1	0.01	0.5382	0.5382	2.47	0.4000	6.65	0.4006	4.72
2.0	0.01	0.1	0.05	0.5339	0.5339	1.52	0.4100	4.81	0.4070	3.82
2.0	0.01	0.3	0.01	0.5438	0.5438	2.41	0.4100	7.56	0.4075	5.64
2.0	0.01	0.3	0.05	0.5430	0.5430	2.41	0.4200	5.69	0.4238	3.79
2.0	0.10	0.1	0.01	0.5363	0.5363	0.65	0.4700	1.32	0.4647	1.26
2.0	0.10	0.1	0.05	0.5350	0.5350	0.65	0.4700	1.33	0.4621	1.26
2.0	0.10	0.3	0.01	0.5452	0.5452	0.63	0.4900	1.31	0.4783	1.24
2.0	0.10	0.3	0.05	0.5454	0.5454	0.64	0.4900	1.32	0.4773	1.24
2.0	0.20	0.1	0.01	0.5376	0.5376	0.00	0.5000	0.59	0.4968	0.55
2.0	0.20	0.1	0.05	0.5366	0.5366	0.00	0.5000	0.59	0.4958	0.54
2.0	0.20	0.3	0.01	0.5422	0.5422	0.64	0.5100	0.59	0.5043	0.55
2.0	0.20	0.3	0.05	0.5405	0.5405	0.64	0.5100	0.60	0.5018	0.55

Table 14 Average bid price and average inventory comparison results for all strategies: oil

β	h	θ	δ	ZI	MB		SB		DB	
				Avg bid	Avg bid	Avg inv	Avg bid	Avg inv	Avg bid	Avg inv
0.5	0.01	0.1	0.01	0.7977	0.7977	12.18	0.7200	16.36	0.7240	15.24
0.5	0.01	0.1	0.05	0.7986	0.7986	6.95	0.7400	9.71	0.7381	9.10
0.5	0.01	0.3	0.01	0.8058	0.8058	12.73	0.7400	17.38	0.7381	15.94
0.5	0.01	0.3	0.05	0.8070	0.8070	7.39	0.7500	10.30	0.7527	9.29
0.5	0.10	0.1	0.01	0.7974	0.7974	1.92	0.7700	2.41	0.7760	2.13
0.5	0.10	0.1	0.05	0.7981	0.7981	1.34	0.7700	2.09	0.7784	1.80
0.5	0.10	0.3	0.01	0.8063	0.8063	1.98	0.7900	2.42	0.7874	2.14
0.5	0.10	0.3	0.05	0.8075	0.8075	1.35	0.7900	2.07	0.7911	1.76
0.5	0.20	0.1	0.01	0.7960	0.7960	0.58	0.7900	0.72	0.7873	0.68
0.5	0.20	0.1	0.05	0.7974	0.7974	0.33	0.8000	0.34	0.7884	0.65
0.5	0.20	0.3	0.01	0.8072	0.8072	0.68	0.8000	0.69	0.7995	0.65
0.5	0.20	0.3	0.05	0.8070	0.8070	0.32	0.8000	0.57	0.7991	0.65
1.0	0.01	0.1	0.01	0.6967	0.6967	7.70	0.5800	13.89	0.5855	11.80
1.0	0.01	0.1	0.05	0.6966	0.6966	4.61	0.6100	7.30	0.6099	6.57
1.0	0.01	0.3	0.01	0.7061	0.7061	8.47	0.6000	14.80	0.5968	13.26
1.0	0.01	0.3	0.05	0.7062	0.7062	5.01	0.6200	8.56	0.6218	7.33
1.0	0.10	0.1	0.01	0.6960	0.6960	1.08	0.6700	1.59	0.6652	1.41
1.0	0.10	0.1	0.05	0.6969	0.6969	0.82	0.6700	1.31	0.6699	1.19
1.0	0.10	0.3	0.01	0.7071	0.7071	1.03	0.6800	1.54	0.6802	1.39
1.0	0.10	0.3	0.05	0.7067	0.7067	1.03	0.6900	1.32	0.6779	1.39
1.0	0.20	0.1	0.01	0.6961	0.6961	0.27	0.6900	0.42	0.6866	0.36
1.0	0.20	0.1	0.05	0.6953	0.6953	0.27	0.6900	0.28	0.6846	0.36
1.0	0.20	0.3	0.01	0.7064	0.7064	0.38	0.7000	0.41	0.6971	0.36
1.0	0.20	0.3	0.05	0.7036	0.7036	0.26	0.7000	0.42	0.6942	0.36
2.0	0.01	0.1	0.01	0.5928	0.5928	4.07	0.4400	10.70	0.4383	8.89
2.0	0.01	0.1	0.05	0.5959	0.5959	2.15	0.4700	5.52	0.4788	4.59
2.0	0.01	0.3	0.01	0.6008	0.6008	4.49	0.4600	11.52	0.4565	9.39
2.0	0.01	0.3	0.05	0.6011	0.6011	2.58	0.4900	6.05	0.4893	5.09
2.0	0.10	0.1	0.01	0.5909	0.5909	0.54	0.5500	1.06	0.5535	0.85
2.0	0.10	0.1	0.05	0.5921	0.5921	0.54	0.5700	0.56	0.5524	0.85
2.0	0.10	0.3	0.01	0.5975	0.5975	0.53	0.5700	1.07	0.5561	1.08
2.0	0.10	0.3	0.05	0.6033	0.6033	0.50	0.5700	1.03	0.5684	0.82
2.0	0.20	0.1	0.01	0.5916	0.5916	0.12	0.5900	0.15	0.5755	0.28
2.0	0.20	0.1	0.05	0.5924	0.5924	0.12	0.5900	0.15	0.5766	0.28
2.0	0.20	0.3	0.01	0.6019	0.6019	0.12	0.6100	0.15	0.5867	0.27
2.0	0.20	0.3	0.05	0.6012	0.6012	0.12	0.6000	0.15	0.5856	0.28

Table 15 Average bid price and average inventory comparison results for all strategies: platinum

β	h	θ	δ	ZI	MB		SB		DB	
				Avg bid	Avg bid	Avg inv	Avg bid	Avg inv	Avg bid	Avg inv
0.5	0.01	0.1	0.01	0.7673	0.7673	9.41	0.7100	12.48	0.7062	11.50
0.5	0.01	0.1	0.05	0.7684	0.7684	6.66	0.7100	8.82	0.7158	7.92
0.5	0.01	0.3	0.01	0.7761	0.7761	10.13	0.7200	13.17	0.7168	12.23
0.5	0.01	0.3	0.05	0.7760	0.7760	6.62	0.7300	9.65	0.7246	8.71
0.5	0.10	0.1	0.01	0.7680	0.7680	1.47	0.7500	2.01	0.7478	1.97
0.5	0.10	0.1	0.05	0.7674	0.7674	1.46	0.7500	2.00	0.7460	1.96
0.5	0.10	0.3	0.01	0.7749	0.7749	2.03	0.7600	2.00	0.7562	1.96
0.5	0.10	0.3	0.05	0.7757	0.7757	1.44	0.7600	1.97	0.7562	1.94
0.5	0.20	0.1	0.01	0.7674	0.7674	0.33	0.7700	0.33	0.7650	0.32
0.5	0.20	0.1	0.05	0.7678	0.7678	0.33	0.7700	0.33	0.7654	0.32
0.5	0.20	0.3	0.01	0.7742	0.7742	0.33	0.7700	0.33	0.7715	0.33
0.5	0.20	0.3	0.05	0.7755	0.7755	0.32	0.7700	0.32	0.7728	0.31
1.0	0.01	0.1	0.01	0.6515	0.6515	6.50	0.5500	11.10	0.5579	9.20
1.0	0.01	0.1	0.05	0.6521	0.6521	4.69	0.5700	7.62	0.5689	6.60
1.0	0.01	0.3	0.01	0.6601	0.6601	6.29	0.5700	10.94	0.5699	9.94
1.0	0.01	0.3	0.05	0.6603	0.6603	4.59	0.5900	7.57	0.5846	6.52
1.0	0.10	0.1	0.01	0.6532	0.6532	0.86	0.6200	1.55	0.6204	1.51
1.0	0.10	0.1	0.05	0.6511	0.6511	0.88	0.6200	1.57	0.6228	1.35
1.0	0.10	0.3	0.01	0.6607	0.6607	0.85	0.6400	1.54	0.6322	1.50
1.0	0.10	0.3	0.05	0.6603	0.6603	0.84	0.6400	1.55	0.6283	1.50
1.0	0.20	0.1	0.01	0.6498	0.6498	0.37	0.6400	0.37	0.6414	0.35
1.0	0.20	0.1	0.05	0.6518	0.6518	0.36	0.6400	0.36	0.6437	0.34
1.0	0.20	0.3	0.01	0.6604	0.6604	0.36	0.6600	0.36	0.6528	0.34
1.0	0.20	0.3	0.05	0.6624	0.6624	0.35	0.6600	0.35	0.6548	0.33
2.0	0.01	0.1	0.01	0.5311	0.5311	3.49	0.4100	8.99	0.4049	6.96
2.0	0.01	0.1	0.05	0.5353	0.5353	2.61	0.4300	5.42	0.4290	4.41
2.0	0.01	0.3	0.01	0.5369	0.5369	4.43	0.4200	9.90	0.4115	7.87
2.0	0.01	0.3	0.05	0.5400	0.5400	2.59	0.4400	6.25	0.4319	5.20
2.0	0.10	0.1	0.01	0.5334	0.5334	0.43	0.5000	0.96	0.4977	0.89
2.0	0.10	0.1	0.05	0.5322	0.5322	0.44	0.5000	0.98	0.4960	0.88
2.0	0.10	0.3	0.01	0.5435	0.5435	0.42	0.5200	0.95	0.5110	0.87
2.0	0.10	0.3	0.05	0.5412	0.5412	0.42	0.5200	0.96	0.5063	0.87
2.0	0.20	0.1	0.01	0.5336	0.5336	0.00	0.5100	0.41	0.5134	0.37
2.0	0.20	0.1	0.05	0.5325	0.5325	0.00	0.5400	0.00	0.5119	0.37
2.0	0.20	0.3	0.01	0.5422	0.5422	0.00	0.5300	0.41	0.5238	0.37
2.0	0.20	0.3	0.05	0.5383	0.5383	0.00	0.5300	0.42	0.5191	0.37

Table 16 Average total profit comparison results for all strategies using real data: food index

β	h	θ	δ	ATP_{ZI}	ATP_{MB}	ATP_{SB}	ATP_{DB}	RP_{ZI} (%)	RP_{MB} (%)	RP_{SB} (%)
0.5	0.01	0.1	0.01	8.3371	11.4105	11.9002	10.9954	31.88	−3.64	−7.60
0.5	0.01	0.1	0.05	8.8427	10.3895	11.0903	10.8383	22.57	4.32	−2.27
0.5	0.01	0.3	0.01	9.2379	11.6257	12.0425	11.8596	28.38	2.01	−1.52
0.5	0.01	0.3	0.05	9.1123	11.1323	11.4387	11.7194	28.61	5.27	2.45
0.5	0.10	0.1	0.01	9.7700	9.9411	9.2357	9.0610	−7.26	−8.85	−1.89
0.5	0.10	0.1	0.05	8.7971	8.6337	8.6967	8.6982	−1.12	0.75	0.02
0.5	0.10	0.3	0.01	9.3138	8.9543	8.7640	9.2312	−0.89	3.09	5.33
0.5	0.10	0.3	0.05	9.2795	9.4868	9.2777	9.4796	2.16	−0.08	2.18
0.5	0.20	0.1	0.01	8.4469	8.9469	8.0898	8.6904	2.88	−2.87	7.42
0.5	0.20	0.1	0.05	8.8621	8.5271	8.4354	8.3167	−6.15	−2.47	−1.41
0.5	0.20	0.3	0.01	9.6465	9.0220	9.1109	9.4484	−2.05	4.73	3.70
0.5	0.20	0.3	0.05	9.6641	9.0678	8.3650	9.0270	−6.59	−0.45	7.91
1.0	0.01	0.1	0.01	5.2762	6.4439	6.9209	7.1614	35.73	11.13	3.47
1.0	0.01	0.1	0.05	5.1680	5.7582	6.0276	6.2484	20.91	8.51	3.66
1.0	0.01	0.3	0.01	5.6499	7.2806	7.7154	7.7702	37.53	6.73	0.71
1.0	0.01	0.3	0.05	5.9171	6.9908	6.9129	7.6770	29.74	9.82	11.05
1.0	0.10	0.1	0.01	5.0340	4.9301	4.4535	4.8398	−3.86	−1.83	8.67
1.0	0.10	0.1	0.05	5.5858	5.3588	4.5949	4.8912	−12.43	−8.72	6.45
1.0	0.10	0.3	0.01	5.2923	5.3279	5.1666	5.2662	−0.49	−1.16	1.93
1.0	0.10	0.3	0.05	5.5969	5.3622	5.1782	5.6252	0.50	4.90	8.63
1.0	0.20	0.1	0.01	4.9256	4.9135	4.4281	5.2246	6.07	6.33	17.99
1.0	0.20	0.1	0.05	5.3197	5.1438	4.5725	5.0266	−5.51	−2.28	9.93
1.0	0.20	0.3	0.01	5.3213	5.3142	4.8058	5.2296	−1.72	−1.59	8.82
1.0	0.20	0.3	0.05	5.1525	5.0815	5.1434	5.3540	3.91	5.36	4.09

Table 16 continued

β	h	θ	δ	ATP_{ZI}	ATP_{MB}	ATP_{SB}	ATP_{DB}	RP_{ZI} (%)	RP_{MB} (%)	RP_{SB} (%)
2.0	0.01	0.1	0.01	2.6423	3.2194	3.6510	3.7626	42.40	16.87	3.06
2.0	0.01	0.1	0.05	2.6275	2.8665	3.5437	3.5720	35.95	24.61	0.80
2.0	0.01	0.3	0.01	2.6101	3.4382	3.7363	3.8621	47.97	12.33	3.37
2.0	0.01	0.3	0.05	2.7545	2.7872	3.5255	3.4831	26.45	24.97	−1.20
2.0	0.10	0.1	0.01	2.3994	2.7332	1.8498	2.4281	1.19	−11.16	31.26
2.0	0.10	0.1	0.05	2.4441	2.6013	2.0006	2.5120	2.77	−3.43	25.56
2.0	0.10	0.3	0.01	2.8914	2.5725	1.7871	2.4879	−13.96	−3.29	39.21
2.0	0.10	0.3	0.05	2.6450	2.6292	1.8795	2.7116	2.52	3.14	44.27
2.0	0.20	0.1	0.01	2.6631	2.4303	1.4434	2.2253	−16.44	−8.44	54.17
2.0	0.20	0.1	0.05	2.7169	2.6028	1.7387	2.3339	−14.10	−10.33	34.23
2.0	0.20	0.3	0.01	2.6700	2.4356	1.6906	2.3650	−11.42	−2.90	39.89
2.0	0.20	0.3	0.05	3.0045	2.5508	2.0561	2.5665	−14.58	0.62	24.83

Table 17 Average total profit comparison results for all strategies using real data: oil

β	h	θ	δ	ATP_{ZI}	ATP_{MB}	ATP_{SB}	ATP_{DB}	RP_{ZI} (%)	RP_{MB} (%)	RP_{SB} (%)
0.5	0.01	0.1	0.01	8.6892	11.8168	11.3933	11.7022	34.68	−0.97	2.71
0.5	0.01	0.1	0.05	8.1254	10.3915	10.5854	10.9212	34.41	5.10	3.17
0.5	0.01	0.3	0.01	8.5797	12.1936	12.5063	12.1386	41.48	−0.45	−2.94
0.5	0.01	0.3	0.05	8.8842	11.9509	11.4783	11.9686	34.72	0.15	4.27
0.5	0.10	0.1	0.01	8.6290	8.9816	8.4430	8.9193	3.36	−0.69	5.64
0.5	0.10	0.1	0.05	8.7959	8.9312	8.1127	8.3622	−4.93	−6.37	3.07
0.5	0.10	0.3	0.01	8.8121	8.3604	7.9526	8.6311	−2.05	3.24	8.53
0.5	0.10	0.3	0.05	8.8480	9.0756	7.8678	8.2292	−6.99	−9.33	4.59
0.5	0.20	0.1	0.01	8.8724	8.3343	8.1786	8.4031	−5.29	0.83	2.74
0.5	0.20	0.1	0.05	9.1154	7.9092	8.2969	8.4808	−6.96	7.23	2.22
0.5	0.20	0.3	0.01	9.7277	9.0769	8.4492	9.0832	−6.63	0.07	7.50
0.5	0.20	0.3	0.05	8.5475	8.5449	8.3000	8.6312	0.98	1.01	3.99
1.0	0.01	0.1	0.01	5.0406	7.2490	7.2361	7.6201	51.18	5.12	5.31
1.0	0.01	0.1	0.05	4.9376	6.1088	6.5562	6.9664	41.09	14.04	6.26
1.0	0.01	0.3	0.01	5.0054	7.2613	7.4110	7.4357	48.55	2.40	0.33
1.0	0.01	0.3	0.05	5.1162	6.5599	6.4873	6.9826	36.48	6.44	7.63
1.0	0.10	0.1	0.01	5.1369	5.8439	4.8335	5.2533	2.27	−10.11	8.69
1.0	0.10	0.1	0.05	4.7324	4.9292	4.6922	5.1630	9.10	4.74	10.03
1.0	0.10	0.3	0.01	5.0047	4.9193	4.8639	5.5403	10.70	12.62	13.91
1.0	0.10	0.3	0.05	4.8579	4.9548	4.5785	5.0113	3.16	1.14	9.45
1.0	0.20	0.1	0.01	4.8497	4.8316	4.4183	5.0097	3.30	3.69	13.38
1.0	0.20	0.1	0.05	5.3534	4.9125	4.4185	4.9239	−8.02	0.23	11.44
1.0	0.20	0.3	0.01	5.1405	4.9195	4.6990	4.9947	−2.84	1.53	6.29
1.0	0.20	0.3	0.05	5.0925	5.0538	4.2278	4.8078	−5.59	−4.87	13.72

Table 17 continued

β	h	θ	δ	ATP_{ZI}	ATP_{MB}	ATP_{SB}	ATP_{DB}	RP_{ZI} (%)	RP_{MB} (%)	RP_{SB} (%)
2.0	0.01	0.1	0.01	1.9674	3.2528	3.2039	3.9989	103.26	22.94	24.81
2.0	0.01	0.1	0.05	2.3989	3.2541	3.2498	3.7129	54.77	14.10	14.25
2.0	0.01	0.3	0.01	2.6914	3.7508	3.4306	4.6472	72.67	23.90	35.46
2.0	0.01	0.3	0.05	2.4811	3.2093	3.6953	3.9430	58.93	22.86	6.70
2.0	0.10	0.1	0.01	2.3095	2.3163	1.9458	2.6087	12.95	12.63	34.07
2.0	0.10	0.1	0.05	2.4319	2.4765	1.8009	2.4531	0.87	−0.95	36.22
2.0	0.10	0.3	0.01	2.5096	2.5249	1.9457	2.4383	−2.84	−3.43	25.32
2.0	0.10	0.3	0.05	2.4796	2.5074	2.1373	2.2991	−7.28	−8.31	7.57
2.0	0.20	0.1	0.01	2.0961	2.2518	1.6227	2.0451	−2.43	−9.18	26.03
2.0	0.20	0.1	0.05	2.4194	2.3959	1.6473	2.3935	−1.07	−0.10	45.30
2.0	0.20	0.3	0.01	2.4320	2.4136	1.8774	2.3704	−2.53	−1.79	26.26
2.0	0.20	0.3	0.05	2.3151	2.2351	1.6753	2.1128	−8.74	−5.47	26.11

Table 18 Average total profit comparison results for all strategies using real data: platinum

β	h	θ	δ	ATP_{ZI}	ATP_{MB}	ATP_{SB}	ATP_{DB}	RP_{ZI} (%)	RP_{MB} (%)	RP_{SB} (%)
0.5	0.01	0.1	0.01	9.0674	10.6505	10.4435	11.5113	26.95	8.08	10.22
0.5	0.01	0.1	0.05	9.4715	10.8376	11.1030	11.4156	20.53	5.33	2.82
0.5	0.01	0.3	0.01	10.1613	11.6000	11.9708	13.5538	33.39	16.84	13.22
0.5	0.01	0.3	0.05	9.4145	11.3428	11.5714	11.6721	23.98	2.90	0.87
0.5	0.10	0.1	0.01	8.6408	9.3442	8.5148	8.8645	2.59	−5.13	4.11
0.5	0.10	0.1	0.05	8.5828	9.1752	8.4197	8.6670	0.98	−5.54	2.94
0.5	0.10	0.3	0.01	9.3545	9.3117	9.1243	9.1646	−2.03	−1.58	0.44
0.5	0.10	0.3	0.05	9.4553	9.4853	8.8030	9.4802	0.26	−0.05	7.69
0.5	0.20	0.1	0.01	8.7389	8.9385	8.2656	8.9836	2.80	0.50	8.69
0.5	0.20	0.1	0.05	8.6946	9.1446	9.0332	9.8430	13.21	7.64	8.97
0.5	0.20	0.3	0.01	9.3556	9.0871	8.9294	8.9715	−4.11	−1.27	0.47
0.5	0.20	0.3	0.05	9.8811	9.2849	9.3523	9.2868	−6.01	0.02	−0.70
1.0	0.01	0.1	0.01	5.6476	6.7133	7.3244	6.8993	22.16	2.77	−5.80
1.0	0.01	0.1	0.05	5.2864	6.5004	7.0801	7.0950	34.21	9.15	0.21
1.0	0.01	0.3	0.01	5.8549	7.4965	7.6126	7.5682	29.26	0.96	−0.58
1.0	0.01	0.3	0.05	5.5440	6.6336	7.1609	7.1705	29.34	8.09	0.13
1.0	0.10	0.1	0.01	5.2464	5.5116	5.0497	5.2799	0.64	−4.20	4.56
1.0	0.10	0.1	0.05	5.3916	5.2715	4.8808	5.5574	3.08	5.42	13.86
1.0	0.10	0.3	0.01	5.3518	5.5053	5.0694	5.5378	3.48	0.59	9.24
1.0	0.10	0.3	0.05	5.9501	5.9349	5.2540	5.6706	−4.70	−4.45	7.93
1.0	0.20	0.1	0.01	5.6612	5.2335	4.9028	5.2912	−6.54	1.10	7.92
1.0	0.20	0.1	0.05	5.3465	5.2153	4.9600	5.1095	−4.43	−2.03	3.01
1.0	0.20	0.3	0.01	5.6498	5.3604	4.7430	5.4204	−4.06	1.12	14.28
1.0	0.20	0.3	0.05	5.6017	5.6064	4.7371	5.5152	−1.54	−1.63	16.42

Table 18 continued

β	h	θ	δ	ATP_{ZI}	ATP_{MB}	ATP_{SB}	ATP_{DB}	RP_{ZI} (%)	RP_{MB} (%)	RP_{SB} (%)
2.0	0.01	0.1	0.01	2.7775	3.6410	4.0324	4.0933	47.37	12.42	1.51
2.0	0.01	0.1	0.05	2.8342	3.1912	3.5404	3.7225	31.34	16.65	5.14
2.0	0.01	0.3	0.01	3.0156	3.3091	3.9137	4.0330	33.73	21.88	3.05
2.0	0.01	0.3	0.05	2.8128	3.3797	3.7810	3.9142	39.16	15.81	3.52
2.0	0.10	0.1	0.01	2.6310	2.8251	2.2092	2.8093	6.78	−0.56	27.16
2.0	0.10	0.1	0.05	2.5256	2.8493	2.0224	2.7294	8.07	−4.21	34.95
2.0	0.10	0.3	0.01	2.6989	2.7647	2.1917	2.5231	−6.51	−8.74	15.12
2.0	0.10	0.3	0.05	3.0151	2.8971	2.4471	2.9854	−0.98	3.05	22.00
2.0	0.20	0.1	0.01	2.7152	2.7676	2.0831	2.7375	0.82	−1.08	31.42
2.0	0.20	0.1	0.05	2.9310	2.7356	2.3016	2.6223	−10.53	−4.14	13.93
2.0	0.20	0.3	0.01	2.7573	2.7760	1.9708	2.5256	−8.40	−9.02	28.16
2.0	0.20	0.3	0.05	2.7744	2.8330	1.9876	2.5057	−9.69	−11.55	26.06

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