MANAGEMENT SCIENCE
Vol. 58, No. 9, September 2012, pp. 1694–1714
ISSN 0025-1909 (print) | ISSN 1526-5501 (online)

informs
http://dx.doi.org/10.1287/mnsc.1110.1511

Supply-Side Story: Risks, Guarantees, Competition, and Information Asymmetry

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The risk of supply disruption increases as firms seek to procure from cheaper, but unproven, suppliers. We model a supply chain consisting of a single buyer and two suppliers, both of which compete for the buyer's order and face risk of supply disruption. One supplier is comparatively more reliable but also more expensive, whereas the other one is less reliable but cheaper and faces higher risk of disruption. Moreover, the risk level of the unreliable supplier may be private information, and this lack of visibility increases the buyer's purchasing risk. In such settings, the unreliable supplier often provides a price and quantity (P&Q) guarantee to the buyer. Our objective is to study the underlying motivation for the guarantee offer and its effects on the competitive intensity and the performance of the chain partners. Our model also includes a spot market that can be utilized by any party to buy or to sell. The spot market price is random, partially depends on the available capacity of the two suppliers, and has a positive spread between buying and selling prices. We analytically characterize the equilibrium contracts for the two suppliers and the buyer's optimal procurement strategy. First, our analysis shows that P&Q guarantee allows the unreliable supplier to better compete against the more reliable one by providing supply assurance to the buyer. More importantly, when information asymmetry risk is high, use of a guarantee may enable the unreliable supplier to credibly signal her true risk, thereby improving visibility into the chain. This signal can also be used by the buyer to infer the expected spot market price. In spite of these benefits, a guarantee offer in an asymmetric setting may not always be desirable for the buyer. Rather, it can reduce competition between the suppliers, resulting in higher costs for the buyer.

Key words: supply-risk management; asymmetric information; signaling; guarantees; stochastic spot market
History: Received December 11, 2009; accepted June 20, 2011, by Martin Lariviere, operations management. Published online in *Articles in Advance* May 18, 2012.

1. Introduction

As supply chains expand to new geographies to seek lower cost solutions, the risk associated with unproven suppliers has emerged as an issue of concern for top management (Aberdeen Group 2007). Among the strategies used to deal with this concern, a popular one is performance-based contracts between chain partners (Bernstein and de Vericourt 2008). Such contracts come in many forms; we focus on supplier guarantees in terms of *price and quantity* attributes (*P&Q guarantee*). These guarantees are contractual assurances from suppliers that they will provide a certain minimum quantity/capacity at a fixed price for the buyer. We observe variants of such contracts in commodity sectors (Stevenson 2006, Creti and Fabra 2007), in the PCB (printed circuit board) industry (Heyes 2008), in fresh foods (U.S. Department of Agriculture 2001), and in spare part supply services (Alstom Transport 2009). A typical guarantee offer for commodity-type electronics items, e.g., memory, may resemble what follows (Wang 2010):

> Supplier agrees to provide (—) amount of capacity upon (—) days notice to ensure an uninterrupted flow of goods to the buyer. Supplier will be responsible for all costs incurred by the buyer due to supplier's insufficient capacity.... During the term of this agreement, the prices for the goods will be (—) and Supplier agrees to absorb all changes in costs (exceptions...).

That there is a need for effective strategies to deal with *supply risk* is undeniable. The increase in outsourcing to low-cost countries with unproven suppliers has exposed supply chains to the risk of disruptions. This can mainly be attributed to exogenous factors such as political uncertainty, natural disasters, financial breakdown, terrorism, and strikes—e.g., the 2011 earthquake in Japan (Helft and Bunkley 2011) and the financial crisis of 2008 (MacLeod and Wiseman 2008). The fragmentation of supply chains also gives rise to a lack of visibility across the chain (i.e., *information asymmetry*), which exacerbates the supply risk. Recent surveys suggest that more than 40% of buyers lack visibility into their tier-1 suppliers; this figure increases to 75% for tier-2 suppliers (*IndustryWeek* 2009).

Interestingly, so far, the literature has focussed on analyzing how buyer-led contracts are able to manage supply disruption risk and the associated information asymmetry. In contrast, we study the role of *supplier-initiated contracts*, such as P&Q guarantees, especially when there are multiple suppliers *competing* for the buyer's order. Moreover, although such guarantees are used in practice, their systemwide effects have not yet been theoretically studied. The motivation of our research is to address these gaps. Specifically, we study the following research questions.

- What motivates an unreliable supplier, competing with other more reliable suppliers, to offer a P&Q guarantee? Under what conditions will she do so?
- How effective are P&Q guarantees in dealing with information asymmetry and supply risk? In particular, when and how do they provide the buyer visibility into the supply system?
- How does provision of a P&Q guarantee affect the competitive intensity among suppliers and the cost or profit performance of the chain partners? In particular, is the buyer always better off with this type of guarantee from an unreliable supplier?

To answer these questions, we study the procurement strategy for a buyer who uses two competing suppliers to satisfy a fixed end-customer demand in a risk-neutral setting. Both suppliers face exogenous disruption risks in their supply processes. One of them (supplier S_M) is relatively "*m*ore" reliable, although she also incurs higher marginal costs.[1] Moreover, the extent of her supply risk is public knowledge. The other supplier (supplier S_L) is cheaper than S_M but is "*l*ess" reliable because of higher disruption risks. In contrast to S_M, S_L's level of risk is private information. Our framework also includes a spot market exhibiting randomness in price driven by supply risk; all three parties can access this market for buying/selling purposes. The two suppliers first compete horizontally to decide on their contract terms for the buyer. The buyer then decides on the optimal order allocation strategy. Both suppliers include their per unit prices in the contracts. However, S_L may also decide to include an additional P&Q guarantee in her contract terms.

We develop four models on the basis of whether S_L offers a P&Q guarantee and whether there is information asymmetry about the extent of risk facing S_L. We analytically characterize and compare the equilibrium strategies for the above models to answer our research questions. First, we show that when there is no information asymmetry, a P&Q guarantee from S_L *does not* affect the equilibrium order allocation strategy of the buyer or the expected costs or profits of the three chain partners. But such a guarantee has a significant impact on the expected performance when information about S_L's level of risk is not available to other parties. In that case, we establish that S_L uses the guarantee to *credibly signal* her true level of risk. Indeed, such guarantees may afford perfect *visibility* into the risk of the supply system and the expected spot market price for the buyer, especially when S_M is highly reliable or when S_M's risk is of the medium range while there is considerable uncertainty about S_L's potential risk. The visibility helps the buyer in the former case; unfortunately, it may be harmful for him in the latter scenario. Specifically, a P&Q guarantee can then *weaken competition* between the suppliers, resulting in higher contract prices and hence *higher expected costs* for the buyer. This also implies that, in an asymmetric setup, S_M may prefer to compete with a reliable (guarantee-offering) S_L. We also show that P&Q guarantees could even *increase* the expected cost incurred by the total system.

2. Related Literature

Our research falls within the general theme of managing supply risk (refer to reviews by Tang 2006 and Vakharia and Yenipazarlı 2009). However, to the best of our knowledge, ours is the first paper in the literature that studies supplier-initiated signaling contracts for a decentralized supply chain facing the risk of supply disruption. There are three streams of research directly related to our paper.

The first stream investigates guarantee contracts. Contracting has been an active area of research in the operations field (Cachon 2003). As regards the issue of supply guarantee contracts, there are papers dealing with guarantees about attributes like delivery times (Bernstein and de Vericourt 2008). However, studies related to P&Q-type guarantees in the presence of supply risk are rather sparse. The only papers on this topic are in the economics domain and study assurances regarding supply or price, but not both, in the utility sector (e.g., Creti and Fabra 2007). But their context is different from ours, and they do not model horizontal competition or information asymmetry.

The second relevant stream is related to exogenous supply risk. This stream started with the seminal paper by Karlin (1958), who investigated yield risk in the agricultural sector. Subsequently, a number

[1] Capacity and inventory are synonymous in our setting, so we use them interchangeably throughout the paper.

of papers have studied different facets of exogenous supply randomness (e.g., Ciarallo et al. 1994, Farmer 1994 and references therein). Note that all these papers use centralized decision-making frameworks. Recently, Babich et al. (2007) extend this stream by modeling exogenous supplier default risk in a decentralized context. In that paper, multiple suppliers actively compete via wholesale prices for a retailer's order, and the extent of risk faced by the suppliers is common knowledge. We add to this literature stream by incorporating information asymmetry about S_L's disruption risk in the model framework.

The most relevant stream for us is the third one, which addresses the issue of information asymmetry among decentralized channel partners. In the supply chain contracting literature, prior work has examined information asymmetry in terms of supplier cost (e.g., Corbett et al. 2004, Cachon and Zhang 2006) or retail demand (e.g., Cachon and Lariviere 2001, Özer and Wei 2006). Our research is more closely associated with papers that model information asymmetry about supplier risk or reliability (see Gurnani and Shi 2006, Chaturvedi and Martínez-de-Albéniz 2011, Tomlin 2009, Yang et al. 2009). Gurnani and Shi (2006) consider a bargaining approach where a buyer and a supplier have different estimates about supply reliability. In their model, the players do not update their beliefs and the contract terms reflect their relative beliefs about supply reliability. In contrast, Tomlin (2009) considers the case of a buyer who has forecast of a supplier's yield distribution and analyzes a Bayesian model of supply learning for the buyer to evaluate the effects of learning on sourcing and inventory strategies. In Yang et al. (2009), the buyer designs a menu contract, and private information about supplier reliability is subsequently revealed through contract choices made by the supplier. Chaturvedi and Martínez-de-Albéniz (2011) extend Yang et al. (2009) by including the supplier's cost as private information. Note that Yang et al. (2009) as well as Chaturvedi and Martínez-de-Albéniz (2011) use a screening approach to model the buyer's contracting problem. A novel feature of our paper is that, because the P&Q contract is offered by the supplier (S_L), the resulting problem is a *signaling game* rather than a screening one. Moreover, both suppliers actively compete for the buyer's order through their contract terms in our framework; such competition is less relevant in a screening contract. Note that a significant body of economics literature that examines signaling contracts has emerged since the seminal paper on job-market signaling by Spence (1973). We refer the readers to Riley (2001) for a detailed review. However, this stream of literature is not concerned with the issue of supply risk.

3. Model Framework

Our model framework involves a single-product supply chain consisting of one buyer, two suppliers (S_M and S_L), and a spot market. One supplier (S_M) is well known to the buyer as a supply source, but the other one (S_L) is new and unproven as far as the buyer is concerned. Moreover, the two suppliers and the buyer are all risk-neutral entities. Because our aim is to shed light on the effects of supply uncertainty, we assume that the end-consumer demand for the buyer is known to be Q, the selling price to end-consumers is constant, and the buyer must satisfy the entire demand.

One of the distinguishing features of our paper is that we capture the inherent difference between the two suppliers with three factors—their levels of risks (or equivalently, reliabilities), information available to the other chain partners about their risk levels, and their marginal costs for supplying the products. First, in our model setting, both suppliers face risks of disruption to their own supply systems. Because of this, although each can potentially access a maximum capacity of Q units for this particular buyer, at the time of presenting the contract terms, each is uncertain about how much capacity would actually be available to them for use (if, and when, needed). To focus on the strategic interaction between the channel partners, we assume the *supply disruption risk* facing S_M and S_L to be exogenous. Furthermore, we also assume that there is *information asymmetry* about the risk associated with S_L. Specifically, suppose that the level of risk facing S_L can be either high (h type) or low (l type); the exact type is known only to S_L (private information), whereas both the buyer and S_M only have a priori probabilistic beliefs about the type.[2] We use superscript θ to denote the type of supply risk and define ϵ_L^θ as the random variable representing risk of type $\theta \in \{h, l\}$. The supply risk of S_M, which is independent of S_L's risk, is represented by the random variable ϵ_M and the distribution of ϵ_M is common knowledge. To develop analytical managerial insights, we follow the recent supply-risk literature (e.g., Babich et al. 2007, Yang et al. 2009, and references therein) in assuming that both supply uncertainties are "all-or-nothing" types. That is,

$$\epsilon_M = \begin{cases} 0 & \text{with prob. } \alpha_M, \\ Q & \text{with prob. } 1-\alpha_M; \end{cases} \quad \text{and}$$
$$\epsilon_L^\theta = \begin{cases} 0 & \text{with prob. } \alpha_L^\theta, \\ Q & \text{with prob. } 1-\alpha_L^\theta, \end{cases} \tag{1}$$

where $\theta \in \{h, l\}$. The a priori belief for the buyer and S_M about the uncertainty level being of type θ is given

[2] However, we assume that the ex post effect of the disruption is observable by all parties.

by r^θ, where $r^h + r^l = 1$ (r^θ and distribution of ϵ_L^θ are common knowledge).

Second, as far as the comparison of the level of risk between the two suppliers is concerned, we assume that $\alpha_L^h > \alpha_L^l > \alpha_M$; i.e., the l-type S_L has a *lower* disruption risk than the h type, and S_M is known to be *more* reliable than even the l-type S_L. Last, the higher reliability of S_M comes at a premium—S_M incurs a marginal cost of c_M for only those units she actually supplies to the buyer, whereas S_L incurs c_L ($\leq c_M$) per unit only for her exact supply quantity.[3]

Another important feature of our model is a spot market that can be accessed by the buyer and the two suppliers only for buying or selling purposes.[4] The price in this spot market is stochastic, with two sources of randomness. The first comprises exogenous factors such as macroeconomic conditions. The price is also affected by an endogenous element—the actual amount of capacity available for possible use with the two suppliers, which is dependent on the realization of random variables ϵ_M and ϵ_L^θ, $\theta \in \{h, l\}$. Specifically, the greater the availability, the lower is the spot market price (refer to Kazaz and Webster 2011 and references therein for examples). We denote the spot market price by p_S, which is defined as follows:

$$p_S(\epsilon_M, \epsilon_L^\theta) = \bar{p}_S + (1-\rho)\Delta_S + \rho\Delta_E(\epsilon_M, \epsilon_L^\theta), \tag{2}$$

where $\bar{p}_S$ is the expected spot market price in the absence of any endogenous randomness, $0 < \rho < 1$ measures how strongly the spot market price is correlated with the availability at the two suppliers, and Δ_S and $\Delta_E(\epsilon_M, \epsilon_L^\theta)$ represent the randomness caused by exogenous and endogenous factors, respectively.[5] We assume that Δ_S has a distribution function F_S and, without loss of generality, $E[\Delta_S] = 0$. To capture the inverse relationship between availability and the spot market price in an analytically tractable way, we assume that $\Delta_E(\epsilon_M, \epsilon_L^\theta)$ has the following functional form:

$$\Delta_E(\epsilon_M, \epsilon_L^\theta) = \begin{cases} +\Delta & \text{if } \epsilon_M + \epsilon_L^\theta = 0 \text{ (i.e., with prob. } \alpha_M\alpha_L^\theta), \\ 0 & \text{if } \epsilon_M + \epsilon_L^\theta = Q \\ & \quad \text{(i.e., with prob. } \alpha_M(1-\alpha_L^\theta) + (1-\alpha_M)\alpha_L^\theta), \\ -\Delta & \text{if } \epsilon_M + \epsilon_L^\theta = 2Q \\ & \quad \text{(i.e., with prob. } (1-\alpha_M)(1-\alpha_L^\theta)), \end{cases} \tag{3}$$

where $\theta \in \{h, l\}$ and $\Delta > 0$ is a measure of the extent by which the spot market price changes depending on the capacity availability. We can think of $\rho\Delta$ as an overall measure of the degree of dependence of the spot market on the amount of available capacity. To deter arbitrage opportunities, we assume that there exists a positive spread ($\delta > 0$) between selling and buying prices in the spot market—the buying price is δ amount more expensive for all realizations of Δ_S and Δ_E (refer to Kazaz and Webster 2011 for more details). Last, to rule out trivial solutions, we assume that $\bar{p}_S$ is relatively high—specifically, $c_L < c_M < \bar{p}_S$. Two remarks are in order here. First, even though $\bar{p}_S$ is high, depending on the values of α_M, α_L^θ, and Δ, the expected spot market price and its specific realizations can either be higher or lower than the marginal supply costs of the suppliers. Second, because there is information asymmetry about α_L^θ, that implies that there is also information asymmetry between the chain partners about the spot market price.

Now that we have defined the characteristics of the chain partners and the spot market, we focus on the details of the contracting and allocation game. The basic contract terms for the two suppliers consist of per unit prices that they charge to the buyer. We also assume that supplier S_L may provide a further P&Q guarantee as part of her contract terms in order to make it attractive to the buyer[6] (given her characteristics, S_M does not provide such guarantees). The timing of decisions and events in our game is described as follows (also refer to Figure 1):

- Nature reveals the exact type of supply uncertainty—h or l—to supplier S_L.
- The two suppliers submit their contract terms simultaneously to the buyer. Note that S_L's terms might be a function of θ, her type.
- Based on the contracts, the buyer then (if possible) updates his a priori belief about the level of

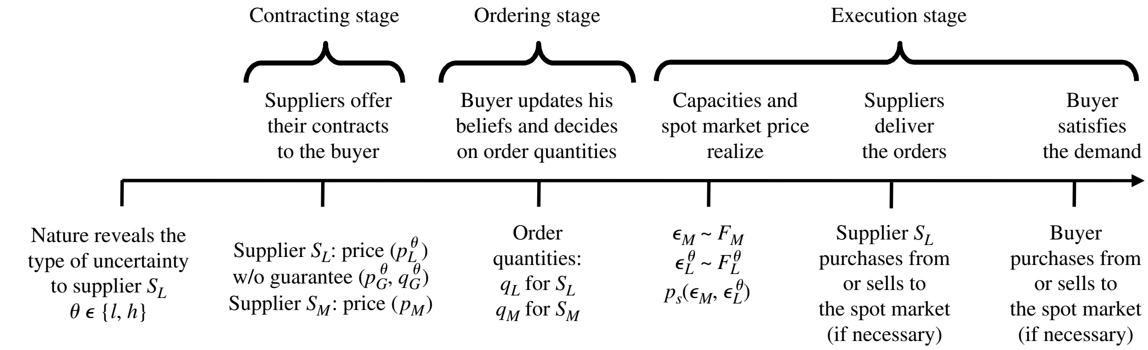
[3] We model supply uncertainty in the form of randomness in capacity availability (see Ciarallo et al. 1994). An alternative would be to model this uncertainty in the form of random production yield, in which case the marginal costs for the two suppliers (c_L and c_M) will be incurred for all units produced (rather than those delivered). We refer the readers to Wang et al. (2010) and Gurnani et al. (2012) for modeling and managerial implications of the latter framework. However, none of these papers considers the use of guarantees by the supplier to signal reliability.

[4] We do not allow trading in any type of financial derivatives in our model. Indeed, for certain products, e.g., memory, no market exists to directly deal in such derivatives contracts (see Tevelson et al. 2007). However, we believe that the analysis of a framework incorporating such contracts is a worthwhile future research direction.

[5] If the two suppliers are relatively big players in a concentrated industry, ρ would likely be large. In contrast, if all the firms in the industry are relatively small, then ρ would also be small.

[6] We assume that the rule of law ensures S_L's compliance with the guarantee contract. This is known as the "specific performance" in contract law, which is becoming increasingly common in practice (Plambeck and Taylor 2007).

Figure 1 **Timing of Events**



risk facing S_L and the spot market price. Suppose that the random variable representing the updated belief is given by $\hat{\theta}$, which is defined as follows: S_L is of h type with probability $\hat{r}^h$ and of l type with probability $\hat{r}^l$, where $\hat{r}^h + \hat{r}^l = 1$. Note that S_M cannot update her belief because she submits her contract simultaneously with S_L.

• The buyer makes the order allocation decision between the two suppliers (q_L and q_M).

• The supply disruption uncertainties facing suppliers S_L and S_M as well as the uncertainty about the spot market price resolve, i.e., ϵ_M, ϵ_L^θ, $\theta \in \{h, l\}$, and $p_S(\epsilon_M, \epsilon_L^\theta)$ realize.

• Suppliers S_L and S_M deliver to the buyer and are paid—the quantity delivered and the payment amount depend on the contract type, as discussed below. Should S_L offer a guarantee and have a shortfall, she would have to satisfy it by purchasing from the spot market at $p_S(\epsilon_M, \epsilon_L^\theta)$ per unit. If either supplier has capacity in excess of her requirement, she can sell it (if profitable) in the spot market at a price $p_S(\epsilon_M, \epsilon_L^\theta) - \delta$ per unit.[7]

• The buyer satisfies the end-customer demand Q. The buyer needs to satisfy shortfalls from the spot market at $p_S(\epsilon_M, \epsilon_L^\theta)$ per unit. Similarly, he can sell any quantity in excess of Q that he receives from the suppliers in the spot market at $p_S(\epsilon_M, \epsilon_L^\theta) - \delta$ per unit. Based on the above framework, we then develop two specific models that differ only in terms of whether supplier S_L offers P&Q guarantee. Note that both models account for asymmetric information. Their counterparts for the symmetric information setup (i.e., nature reveals S_L's type to all chain partners at the beginning of the game) will be discussed in §4.

No-Guarantee Model. The contract term for each supplier in this case only involves the per unit price—p_L^θ for S_L of type θ and p_M for S_M. Based on this, the buyer updates his belief (if possible) about the risk associated with S_L and then makes his order allocation decision. Each supplier will supply the minimum she can once the supply uncertainty resolves and the quantity is set, at price p_L^θ or p_M, for suppliers S_L and S_M, respectively. The buyer is solely responsible for satisfying any shortfall from the spot market.

Supply-Guarantee Model. Although S_M's contract still consists of only the per unit price p_M, the contract for S_L now includes a P&Q guarantee. That is, the contract for type-θ S_L now comprises three elements: a guaranteed quantity (say, $q_G^\theta \in [0, Q]$), a guaranteed price (say, p_G^θ per unit), and a per-unit price for any nonguaranteed amount (p_L^θ). To be more specific, S_L guarantees supply of up to q_G^θ units at price p_G^θ per unit to the buyer, regardless of the realization of ϵ_L^θ; however, she charges p_L^θ per unit for units ordered exceeding quantity q_G^θ (also, she does not guarantee supply in excess of q_G^θ units). Based on the two contracts, the buyer then updates his belief (if possible) about the level of supply risk facing S_L and makes the order allocation decision.

The buyer's payment to S_L will now depend on his order quantity—if q_L is less (respectively, greater) than q_G^θ, then he pays p_G^θ per unit (respectively, p_G^θ per unit for the first q_G^θ units and p_L^θ per unit for the rest). The buyer satisfies any shortfall from the spot market. Last, in contrast to the no-guarantee model, if the realized capacity available is less than $\min\{q_G^\theta, q_L\}$, then S_L procures $[\min\{q_G^\theta, q_L\} - \epsilon_L^\theta]^+$ units from the spot market to satisfy the guarantee.

Before we proceed to a detailed analysis of the asymmetric information models, it is worthwhile to understand the effectiveness of P&Q guarantees in dealing with supply risk by analyzing a symmetric information setting in the presence of supply uncertainty.

4. Role of Supply Guarantees in Symmetric Information Setting

In this section, we analyze the no-guarantee and supply-guarantee models of §3 under the assumption that when nature reveals the type of uncertainty θ

[7] We assume that both suppliers require significantly long delivery times, but the spot market can supply on a short notice. So the buyer must order from the suppliers earlier and may use the spot market as and when needed.

confronted by supplier S_L, it does so to all three parties; all other details remain the same. This means that θ can be h or l, but this information is common knowledge. Obviously, this precludes any need on the part of the buyer to update his belief about whether θ is h or l. For this information setting, we first characterize the equilibria for no-guarantee and supply-guarantee models and then compare them to show how a P&Q guarantee from S_L affects the decisions/performance of the supply chain partners.

4.1. Equilibria Characterization for No-Guarantee and Supply-Guarantee Models

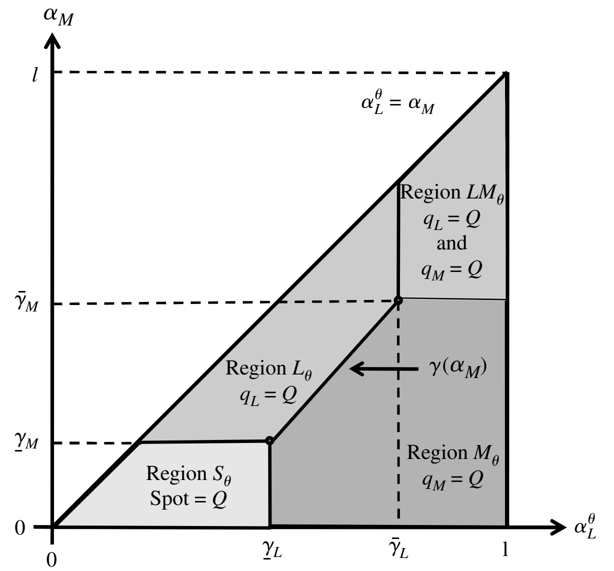
For the *no-guarantee model*, recall that the buyer bears the disruption risk and is responsible for satisfying any shortfall, relative to Q, from the spot market. We solve the problem using backward induction, starting from the buyer's optimal (cost-minimizing) allocation decision. Once the optimal allocation decision is characterized, we substitute it into S_L and S_M's profit functions and then solve the simultaneous price game between the two suppliers. The overall equilibrium characterization is provided in Proposition 1.

In the *supply-guarantee model*, for an order quantity of less than or equal to q_G^θ ($\in [0, Q]$ for type θ) to S_L, the risk for that part is borne solely by S_L; but if it is more than q_G^θ, the risk is shared by the buyer and S_L, even for the part ordered to S_L. We again solve the problem using backward induction approach and obtain the equilibrium characterization as shown in Proposition 1. Note that the detailed proofs for all propositions are provided in the appendix.

Proposition 1. *The unique equilibrium allocation, contract terms, and expected costs/profits for the channel partners in the no-guarantee and supply-guarantee scenarios under symmetric information are presented in Figure* 2 *and in Table* 1.

When both suppliers are quite reliable (i.e., Region S_θ), they can charge low contract prices for both no-guarantee and supply-guarantee contracts, and S_L can offer a full guarantee. However, this scenario also indicates to the buyer that the probability is quite high that the total capacity available in the market will be large ($= 2Q$), and hence the expected spot market price will be low. Consequently, the buyer actually opts to buy only from the spot market: we term this a *spot-sourcing* scenario. As the reliability of the two suppliers becomes more risky, the expected price in the spot market increases, and then the buyer orders from the suppliers rather than the spot market. The buyer opts for *sole sourcing* from S_M when she is relatively more reliable than S_L, but not too much more expensive (Region M_θ). In this region, for both contracts, S_M sets her price such that the expected cost for the buyer is infinitesimally less than his expected

Figure 2 Equilibrium Allocation Under Symmetric Information (Both No-Guarantee and Supply-Guarantee Scenarios)



cost of buying from S_L. Obviously, as the difference in the risk levels between the two suppliers decreases (i.e., α_M increases) and/or the cost premium for S_M increases (i.e., $c_M - c_L$ increases), the buyer opts to sole source from S_L (Region L_θ). Last, when both suppliers are quite risky (Region LM_θ), (i) S_L does not offer any guarantee, and (ii) the buyer, keeping in mind the increased supply risk, decides to *dual source* by ordering Q units from both suppliers.

In Figure 2, note that we present all the possible regions. Obviously, the presence and size of these regions depend crucially on the system parameters. For example:

- As the spot market price becomes less dependent on the available capacity and more dependent on exogenous factors (i.e., as $\rho\Delta$ decreases), then the spot-sourcing region decreases.[8] For sufficiently low $\rho\Delta$, this region totally vanishes.
- When the spread in the spot market price is high (i.e., high δ), the dual-sourcing region is small (because the buyer does not want to sell the excess Q units back to the spot market and incur extra costs), whereas the two sole-sourcing regions are relatively large, and vice versa.
- As the marginal cost for S_M (respectively, S_L) increases, it results in less dual sourcing and more spot sourcing as well as more sole sourcing from S_L (respectively, S_M).

4.2. Effects of Supply Guarantees

Table 1 clearly reveals that, although the equilibrium contract parameters might be different under

[8] Throughout the paper, we use increase/decrease and higher/lower in the weak sense, unless otherwise specified.

Table 1 Equilibrium Order Allocation, Contracts, and Costs/Profits for the No-Guarantee and Supply-Guarantee Scenarios Under Symmetric Information ($\theta \in \{h, l\}$)

Regions	$\alpha_L^\theta \le \underline{\gamma}_L$ $\alpha_M \le \underline{\gamma}_M$ Region S_θ	$\alpha_L^\theta \ge \gamma(\alpha_M)$ and $\alpha_L^\theta > \underline{\gamma}_L$ and $\alpha_M \le \bar{\gamma}_M$ Region M_θ	$\alpha_L^\theta < \gamma(\alpha_M)$ and $\alpha_L^\theta < \bar{\gamma}_L$ and $\alpha_M > \underline{\gamma}_M$ Region L_θ	$\alpha_L^\theta \ge \bar{\gamma}_L$ and $\alpha_M > \bar{\gamma}_M$ Region LM_θ
Allocation	$q_M = 0, q_L = 0$	$q_M = Q, q_L = 0$	$q_M = 0, q_L = Q$	$q_M = Q, q_L = Q$
NG				
$p_M^{\theta*}$	c_M	$\left[c_M + \dfrac{\min(k_L^\theta, k_S^\theta) - k_M^\theta}{1-\alpha_M}\right]^{(-)}$	c_M	$[\bar{p}_S - (1-\alpha_L^\theta)(\rho\Delta + \delta)]^{(-)}$
$p_L^{\theta*}$	c_L	c_L	$\left[c_L + \dfrac{\min(k_M^\theta, k_S^\theta) - k_L^\theta}{1-\alpha_L^\theta}\right]^{(-)}$	$[\bar{p}_S - (1-\alpha_M)(\rho\Delta + \delta)]^{(-)}$
G				
$p_M^{\theta*}$	c_M	$\left[c_M + \dfrac{\min(k_L^\theta, k_S^\theta) - k_M^\theta}{1-\alpha_M}\right]^{(-)}$	c_M	$[\bar{p}_S - (1-\alpha_L^\theta)(\rho\Delta + \delta)]^{(-)}$
$p_L^{\theta*}$	NA	NA	NA	$[\bar{p}_S - (1-\alpha_M)(\rho\Delta + \delta)]^{(-)}$
$p_G^{\theta*}$	k_L^θ	k_L^θ	$[\min(k_M^\theta, k_S^\theta)]^{(-)}$	NA
$q_G^{\theta*}$	Q	Q	Q	0
$\Pi_L^{\theta*}$	0	0	$(\min(k_M^\theta, k_S^\theta) - k_L^\theta)Q$	$(1-\alpha_L^\theta)(p_L^{\theta*} - c_L)Q$
$\Pi_M^{\theta*}$	0	$(\min(k_L^\theta, k_S^\theta) - k_M^\theta)Q$	0	$(1-\alpha_M)(p_M^{\theta*} - c_M)Q$
$TC_B^{\theta*}$	$k_S^\theta Q$	$\min(k_L^\theta, k_S^\theta)Q$	$\min(k_M^\theta, k_S^\theta)Q$	$k_{LM}^\theta Q$

Notes. $\gamma(\alpha_M) = (c_M - c_L)/(\bar{p}_S - c_L) + \alpha_M(\bar{p}_S - c_M)/(\bar{p}_S - c_L)$; $\bar{\gamma}_M = 1 - (\bar{p}_S - c_L)/(\rho\Delta + \delta)$; $\underline{\gamma}_M = 1 - (\bar{p}_S - c_L)/(\rho\Delta)$; $\bar{\gamma}_L = 1 - (\bar{p}_S - c_M)/(\rho\Delta + \delta)$; $\underline{\gamma}_L = 1 - (\bar{p}_S - c_M)/(\rho\Delta)$; NA, not applicable. $k_M^\theta = (1-\alpha_M)c_M + \alpha_M\bar{p}_S + \rho\Delta\alpha_M\alpha_L^\theta$; $k_L^\theta = (1-\alpha_L^\theta)c_L + \alpha_L^\theta\bar{p}_S + \rho\Delta\alpha_M\alpha_L^\theta$; $k_S^\theta = \bar{p}_S + \rho\Delta\alpha_M\alpha_L^\theta - \rho\Delta(1-\alpha_M)(1-\alpha_L^\theta)$. $k_{LM}^\theta = \bar{p}_S + \rho\Delta\alpha_M\alpha_L^\theta - (\rho\Delta + \delta)(1-\alpha_M)(1-\alpha_L^\theta)$. In all tables, "$q_M = 0, q_L = 0$" implies "spot sourcing."

supply-guarantee and no-guarantee scenarios, the equilibrium order allocation and expected costs/profits for the channel partners are exactly the same for both cases. So in that respect, when there is no information asymmetry about the disruption risks facing the suppliers, P&Q guarantees do not play a significant role in our setting. However, if we consider the uncertainties associated with supplies/costs of the buyer, then the two contracts may be different. For example, the buyer's supply as well as his costs are uncertain for all the regions under a no-guarantee contract, whereas under a guarantee contract, his costs and supply become *risk free* in the L_θ region.

5. Supply Guarantees in Asymmetric Information Setting

We now focus on analyzing the two asymmetric information models developed in §3. In this section, nature reveals whether the supply risk θ is of type h or l *only to* S_L. This private information is not known to the buyer or S_M, both of whom only have a priori probabilistic beliefs about θ, given by r^θ ($r^h + r^l = 1$). Note that, because the exact distribution representing the randomness of the spot market price depends on S_L's type, only S_L can determine the exact distribution; the other two chain members face information asymmetry about it. We analyze the no-guarantee and supply-guarantee models in §5.1 and §5.2, respectively, and compare them in §6.

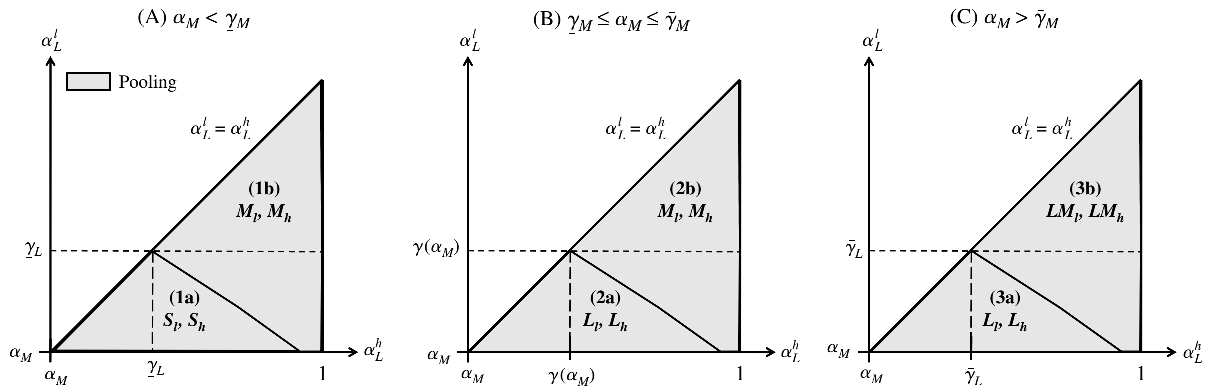
Recall that the random variable ϵ_L^θ representing supply risk for θ-type S_L is defined as follows: $\epsilon_L^\theta = 0$ with probability α_L^θ and $\epsilon_L^\theta = Q$ with probability $1-\alpha_L^\theta$, for $\theta \in \{h, l\}$ (r^θ and the distribution of ϵ_L^θ are common knowledge). Moreover, the supply risk for S_M is $\epsilon_M = 0$ with probability α_M and $\epsilon_M = Q$ with probability $1-\alpha_M$. Last, $\alpha_L^h > \alpha_L^l > \alpha_M$. For the buyer and S_M, the "expected" default risk for S_L is given by $\bar{\alpha}_L = r^h\alpha_L^h + r^l\alpha_L^l$. As discussed in §3, S_L's contract terms may help the buyer update his belief about θ. The updated beliefs about the probability of supplier S_L being h or l are $\hat{r}^h$ and $\hat{r}^l$, respectively (where $\hat{r}^h + \hat{r}^l = 1$).

5.1. No-Guarantee Model

In this case, both suppliers submit their unit contract prices, p_L^θ and p_M; based on those, the buyer makes the allocation decision. In the next proposition, we present the equilibrium contract decisions and expected costs/profits of the channel partners for this model.[9]

[9] The strategies and beliefs constitute an equilibrium in an asymmetric information setting only if (i) for each θ type, the contract terms solve the Nash game between the two suppliers, given the buyer's allocation strategy; (ii) the buyer's updated beliefs can be

Figure 3 **Equilibrium Regions for the No-Guarantee, Asymmetric Information Scenario**



Notes. The equilibrium contract, order allocation and expected costs/profits for each area can be seen from the corresponding column in Table 2. In Figures 3 and 4, a notation like L_l, M_h represents that, in that area, if S_L is of type l, then S_L gets the order, and if she is of type h, S_M gets the order (LM implies that both S_L and S_M get the order; $L|M$ implies that either S_L or S_M gets the order; S_l, S_h implies spot sourcing).

Proposition 2. *In the no-guarantee, asymmetric information scenario, the unique (pure-strategy) equilibrium is of the pooling type, where both h- and l-type supplier S_L charge the same price. That is, $p_L^{h*} = p_L^{l*}$. The unique equilibrium allocation, contract parameters, and expected costs/profits for the channel partners are shown in Figure 3 and Table 2.*

Interestingly, Proposition 2 implies that when the risk of supply disruption is private information for S_L, she *cannot credibly signal* her type (i.e., h or l) to the buyer in the no-guarantee scenario. Consequently, the buyer does not have visibility into the severity of supply risk that S_L is facing, and his allocation decision does not differentiate between h and l types. In this model setting, note that S_L does not face any risk in terms of marginal supply cost though she faces availability risk. Consequently, a separating equilibrium cannot be on the equilibrium path, because h type can increase her profit by mimicking the l type if they charge different prices.

As noted before, part of the supply risk in the no-guarantee setting is always borne by the buyer, who needs to use the spot market if S_L and/or S_M are unable to deliver the order. Because the buyer cannot separate the two S_L types, the allocation decision is driven by the expected default risk of S_L (i.e., $\bar{\alpha}$). The buyer uses $\bar{\alpha}$ to compare S_L's reliability and cost with those of S_M, as well as to calculate the expected spot market price. Assuming $\bar{\alpha}$ as the reliability of an *average*-type S_L, the equilibrium contract and allocation can be determined by comparing the buyer's expected cost of buying from the two suppliers—"average" S_L and S_M—and the spot market. Qualitatively speaking, the equilibrium allocation is quite similar to the one in §4. For example, when both suppliers are relatively more reliable, the optimal allocation strategy is spot sourcing (Region (1a)); when both are very risky, it is dual sourcing (Region (3b)), and in other sole-sourcing regions (Regions (1b), (2a), (2b), and (3a)), it depends on the relative values of the reliabilities and costs of the available options.

derived from suppliers' equilibrium strategy using Bayesian updating; and (iii) the (overall) order allocation decision minimizes the buyer's expected cost, given his updated beliefs and suppliers' contracts.

5.2. Supply-Guarantee Model

In this section, we focus on understanding how a P&Q guarantee from S_L affects the equilibrium characterization and whether it is able to deal with information asymmetry in the chain. The analysis turns out to be quite complicated because there are two issues involved—whether S_L should offer a guarantee (and if so, how much) and whether the equilibrium will be a pooling or a separating one. However, we are able to exactly characterize the equilibrium decisions and expected costs/profits of the channel partners associated with different parameter ranges, as shown below.

Proposition 3. *The type of equilibrium and the equilibrium level of the guarantee for different parameter ranges are characterized below.*

- It is possible to have either a pooling or a separating equilibrium. Specifically:

—When α_M is low ($\alpha_M < \underline{\gamma}_M$), the unique equilibrium is of the separating type.

—When α_M is medium ($\underline{\gamma}_M \le \alpha_M \le \bar{\gamma}_M$), the unique equilibrium is of the pooling type if h- and l-type suppliers' default risks—α_L^h and α_L^l, respectively—are close to each other and neither is very different from α_M; otherwise, it is of the separating type if either α_L^h and α_L^l are substantially different from each other or if they are close to each other but both are quite different from α_M.

—Last, when α_M is high ($\alpha_M \ge \bar{\gamma}_M$), the unique equilibrium is always of the pooling type.

Table 2 Equilibrium Order Allocation, Contract Terms, and Costs/Profits for the No-Guarantee, Asymmetric Information Scenario

Regions	$\bar{\alpha}_L \le \underline{\gamma}_L$ $\alpha_M \le \underline{\gamma}_M$ Region (1a)	$\bar{\alpha}_L \ge \gamma(\alpha_M)$ and $\bar{\alpha}_L > \underline{\gamma}_L$ and $\alpha_M \le \bar{\gamma}_M$ Regions (1b) and (2b)	$\bar{\alpha}_L < \gamma(\alpha_M)$ and $\bar{\alpha}_L < \bar{\gamma}_L$ and $\alpha_M > \underline{\gamma}_M$ Regions (2a) and (3a)	$\bar{\alpha}_L \ge \bar{\gamma}_L$ and $\alpha_M > \bar{\gamma}_M$ Region (3b)
Allocation	$q_M = 0, q_L = 0$	$q_M = Q, q_L = 0$	$q_M = 0, q_L = Q$	$q_M = Q, q_L = Q$
p_M^*	c_M	$\left[c_M + \dfrac{\min(\bar{k}_L, \bar{k}_S) - \bar{k}_M}{1-\alpha_M}\right]^{(-)}$	c_M	$[\bar{p}_S - (1-\bar{\alpha}_L)(\rho\Delta + \delta)]^{(-)}$
$p_L^{\theta *}$	c_L	c_L	$\left[c_L + \dfrac{\min(\bar{k}_M, \bar{k}_S) - \bar{k}_L}{1-\bar{\alpha}_L}\right]^{(-)}$	$[\bar{p}_S - (1-\alpha_M)(\rho\Delta + \delta)]^{(-)}$
$\Pi_L^{\theta *}$	0	0	$\dfrac{1-\alpha_L^\theta}{1-\bar{\alpha}_L}(\min(\bar{k}_M, \bar{k}_S) - \bar{k}_L)Q$	$(1-\alpha_L^\theta)(p_L^{\theta *} - c_L)Q$
Π_M^*	0	$(\min(\bar{k}_L, \bar{k}_S) - \bar{k}_M)Q$	0	$(1-\alpha_M)(p_M^* - c_M)Q$
TC_B^*	$\bar{k}_S Q$	$\min(\bar{k}_L, \bar{k}_S)Q$	$\min(\bar{k}_M, \bar{k}_S)Q$	$\bar{k}_{LM} Q$

Notes. $\bar{k}_M = (1-\alpha_M)c_M + \alpha_M\bar{p}_S + \rho\Delta\alpha_M\bar{\alpha}_L$; $\bar{k}_L = (1-\bar{\alpha}_L)c_L + \bar{\alpha}_L\bar{p}_S + \rho\Delta\alpha_M\bar{\alpha}_L$; $\bar{k}_S = \bar{p}_S + \rho\Delta\alpha_M\bar{\alpha}_L - \rho\Delta(1-\alpha_M)(1-\bar{\alpha}_L)$; $\bar{k}_{LM} = \bar{p}_S + \rho\Delta\alpha_M\bar{\alpha}_L - (\rho\Delta + \delta)(1-\alpha_M)(1-\bar{\alpha}_L)$. Note that Regions (3a) and (3b) are also applicable for the supply-guarantee case under asymmetric information when $\alpha_M \ge \bar{\gamma}_M$ (see Figure 4(C)).

• As for the level of guarantee, both h- and l-type suppliers offer a full guarantee (i.e., Q) on the equilibrium as long as α_M is low or medium; they offer no guarantee when α_M is high.

The unique equilibrium allocation, contract terms, and expected costs/profits for the channel partners for these three scenarios in the supply-guarantee case are shown in Figures 4(A)–4(C) *and Tables* 2–4 *depending on whether* α_M *is low, medium, or high, respectively.*[10]

The main insight of the above proposition, which reveals an interesting feature of a P&Q guarantee in an asymmetric information context, is the following:

• P&Q guarantees may enable S_L to eliminate informational asymmetry by *credibly signaling* to the buyer private information about the type of supply risk—high (h) or low (l)—she is facing. This contrasts sharply with the no-guarantee scenario of §5.1, where S_L is not able to signal her true level of risk (refer to Proposition 2).

The main reason guarantees provide signaling opportunities is that they create a cost differential between h- and l-type suppliers. By offering a guarantee, S_L internalizes the cost associated with disruption risk. Because she is responsible for satisfying the order for the guaranteed units, the expected cost per unit for l type is then *less than* that for h type. The cost difference manifests itself in the form of different guaranteed prices quoted by the two types. This enables the low-risk l type to separate herself from the high-risk h type, and the buyer to differentiate between them.

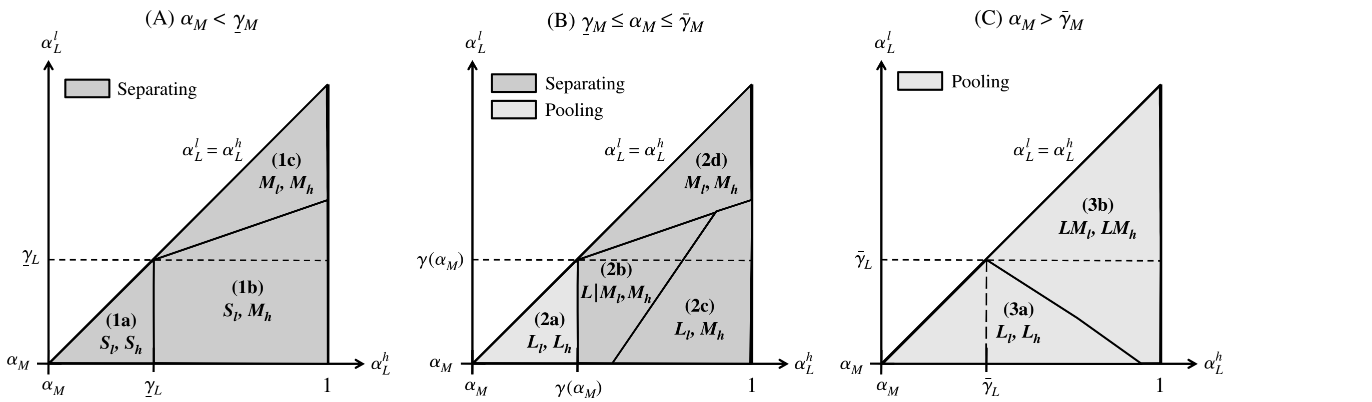
[10] In the separating case, both pure- (Regions (2c) and (2d)) and mixed-strategy (Region (2b)) equilibria are possible. In the pooling case, the equilibria are always in pure strategy.

Although S_M is not privy to S_L's price signal, she knows that the buyer might be able to use it to differentiate between h and l types. This affects S_M's contract price compared to the no-guarantee setting. In the latter scenario, S_M is competing with the buyer's cost of buying from an "average" S_L ($=\bar{k}_L$) or from a spot market based on an "average" S_L ($=\bar{k}_S$). However, because of the buyer's ability to differentiate between S_L types in the guarantee scenario, S_M needs to decide whether to compete against an l-type supplier, i.e., k_L^l, or against an h-type one, i.e., k_L^h (or against corresponding spot markets, k_S^l and k_S^h, respectively). The main trade-off is that undercutting an l-type S_L will surely win S_M the order allocation—but at the cost of a lower margin (*volume strategy*), whereas undercutting an h-type one will earn her significant margins when she wins the order, but she loses the allocation when S_L is an l type (*margin strategy*).

Based on the above discussion, we can then better understand the equilibrium *signaling, guarantee,* and *order allocation* strategies in the different regions of Proposition 3.

• *Low-risk supplier* S_M ($\alpha_M < \underline{\gamma}_M$—Figure 4(A), Table 3): Because S_M is highly reliable, S_L provides a full guarantee to compete for allocation in this region. In spite of this, the buyer never procures from S_L—the allocation goes either to S_M or to the spot market. But guarantee offers result in a separating equilibrium, and the buyer uses the price signal to determine S_L's type and the associated expected spot market price that he is going to face. When reliabilities of both S_L types are comparable to S_M (Region (1a)), the buyer opts for spot sourcing (since the spot market price would probably be low). For S_M, as discussed above, she needs to decide which strategy—volume or margin—to adopt. When she thinks that S_L

Figure 4 Equilibrium Regions for Supply-Guarantee Scenario When There Is Information Asymmetry



Notes. The equilibrium contract parameters, order allocation, and expected costs/profits for each area in (A), (B), and (C) can be seen from the corresponding column in Tables 3, 4, and 2, respectively. Note that for $\alpha_M \ge \bar{\gamma}_M$, equilibrium no-guarantee and guarantee cases are the same, so we refer to Table 2 for equilibrium values.

is most probably of h type and l type is quite reliable (high r_h and low α_L^l, Region (1b)), S_M chooses the margin strategy. But if r_l and α_L^l are high (Region (1c)), it makes more sense for S_M to use the volume strategy. The point to note in this region is that the signal from S_L primarily affects the contract price for S_M *by conveying information about the expected spot market price to the buyer*, but it never enables even l-type S_L to get any allocation (due to her higher price compared to S_M or the spot market).

• *Medium-risk supplier* S_M ($\underline{\gamma}_M \le \alpha_M \le \bar{\gamma}_M$—Figure 4(B), Table 4): S_L again provides a full guarantee to compete with S_M. When both S_L types are quite reliable (Region (2a)), in spite of their cost difference due to guarantee offers, both S_L types can charge a price such that the buyer's cost is infinitesimally lower than buying from S_M. This results in a pooling equilibrium in Region (2a), and only S_L wins the order. But in other regions, a guarantee-induced difference in supply costs results in separating equilibria.

Table 3 Equilibrium Order Allocation, Contract Terms, and Costs/Profits for the Supply-Guarantee, Asymmetric Information Scenario ($\alpha_M \le \underline{\gamma}_M$)

		Separating		
			$\alpha^h \ge \underline{\gamma}_L$	
		$\alpha^h < \underline{\gamma}_L$	$r^h \ge \dfrac{k_S^l - k_M^l}{k_S^h - k_M^h}$	$r^h < \dfrac{k_S^l - k_M^l}{k_S^h - k_M^h}$
	Regions	Region (1a)	Region (1b)	Region (1c)
Supplier S_L	(p_L^{l*}, p_L^{h*})	(NA, NA)	(NA, NA)	(NA, NA)
	(q_G^{l*}, q_G^{h*})	(Q, Q)	(Q, Q)	(Q, Q)
	(p_G^{l*}, p_G^{h*})	(k_S^l, k_S^h)	(k_S^l, k_S^h)	(k_S^l, k_S^h)
Supplier S_M	p_M	c_M	$\left[c_M + \dfrac{k_S^h - k_M^h}{1-\alpha_M}\right]^{(-)}$	$\left[c_M + \dfrac{k_S^l - k_M^l}{1-\alpha_M}\right]^{(-)}$
Beliefs	$\mu = (\hat{r}^l, \hat{r}^h)$		$\mu = \begin{cases} (1,0) & \text{if } p_G^{\theta*} = k_S^l, \\ (0,1) & \text{otherwise} \end{cases}$	
Allocation	(q_L, q_M)	$q_L = q_M = 0$	$q_L = 0,\ q_M = \begin{cases} 0 & \text{if } p_M^* > p_S^\theta, \\ Q & \text{otherwise} \end{cases}$	$q_L = 0, q_M = Q$
Profits	l-type S_L	0	0	0
	h-type S_L	0	0	0
	S_M	0	$r^h(k_S^h - k_M^h)Q$	$(k_S^l - \bar{k}_M)Q$
Cost	B	$\bar{k}_S Q$	$\bar{k}_S Q$	$(k_S^l + r^h(k_M^h - k_M^l))Q$

Table 4 Equilibrium Order Allocation, Contract Terms, and Costs/Profits for the Supply-Guarantee, Asymmetric Information Scenario ($\underline{\gamma}_M < \alpha_M \le \bar{\gamma}_M$)

			Separating		
		Pooling	$r^h > \dfrac{k_L^l - k_M^l}{k_L^h - k_M^h}$		
		$\alpha^h < \gamma(\alpha_M)$	$r^h \le \dfrac{k_S^l - k_M^l}{k_L^h - k_M^h}$	$r^h > \dfrac{k_S^l - k_M^l}{k_L^h - k_M^h}$	$r^h \le \dfrac{k_L^l - k_M^l}{k_L^h - k_M^h}$
	Regions	Region (2a)	Region (2b)	Region (2c)	Region (2d)
S_L	(p_L^{l*}, p_L^{h*})	(NA, NA)	(NA, NA)	(NA, NA)	(NA, NA)
	(q_G^{l*}, q_G^{h*})	(Q, Q)	(Q, Q)	(Q, Q)	(Q, Q)
	(p_G^{l*}, p_G^{h*})	$(\bar{k}_M^{(-)}, \bar{k}_M^{(-)})$	$p_G^\theta \in \begin{cases} [\underline{p}_L, \bar{p}_L] \text{ w.p. } F_L^l(p_G) & \text{if } \theta = l, \\ k_L^h & \text{if } \theta = h \end{cases}$	$(k_S^{l(-)}, k_L^h)$	(k_L^l, k_L^h)
S_M	p_M	c_M	$p_M \in [\underline{p}_M, \bar{p}_M)$ w.p. $F_M(p_M)$	$\left[c_M + \dfrac{k_L^h - k_M^h}{1-\alpha_M}\right]^{(-)}$	$\left(c_M + \dfrac{k_L^l - k_M^l}{1-\alpha_M}\right)^{(-)}$
Beliefs	$\mu = (\hat{r}^l, \hat{r}^h)$	$\mu = (r^l, r^h)$	$\mu = \begin{cases} (1,0) & \text{if } q_G^{\theta*} = Q \text{ and } p_G^{\theta*} < \bar{p}, \\ (0,1) & \text{otherwise} \end{cases}$		
Allocation	(q_L, q_M)	$q_L = Q, q_M = 0$	$(q_L, q_M) = \begin{cases} (Q,0) & \text{if } p_G^{\theta*} < k_M^l(p_M^*), \\ (0,Q) & \text{otherwise} \end{cases}$		$q_L = 0, q_M = Q$
Profits	l-type S_L	$(\bar{k}_M - k_L^l)Q$	$(\underline{p}_L - k_L^l)Q$	$(k_S^l - k_L^l)Q$	0
	h-type S_L	$(\bar{k}_M - k_L^h)Q$	0	0	0
	S_M	0	$r^h(k_L^h - k_M^h)Q$	$r^h(k_L^h - k_M^h)Q$	$(k_L^l - k_M^l)Q$
Cost	B	$\bar{k}_M Q$	$(E\bar{k}_M(p_M^*) - r^l E[k_M^l(p_M^*) - p_G^{l*}]^+)Q$	$r^h k_L^h + r^l k_S^l$	$(k_L^l + r^h(k_M^h - k_M^l))Q$

Notes. $F_L^l(p_G) = (1/r^l)(1 - r^h(k_L^h - k_M^h)/(p_G - k_M^l))$; $F_M(p_M) = 1 - (r^h(k_L^h - k_M^l))/(k_M^l(p_M) - k_L^l)$; $\underline{p}_L = k_M^l + r^h(k_L^h - k_M^h)$; $\bar{p}_L = \min(k_L^h - (k_M^h - k_M^l), k_S^l)$; $\underline{p}_M = h_M(\underline{p}_L)$; $\bar{p}_M = h_M(\bar{p}_L)$; $h_M(p_G) = c_M + (p_G - k_M^l)/(1-\alpha_M)$; $\bar{k}_M(p_M) = (1-\alpha_M)p_M + \alpha_M \bar{p}_S + \rho\Delta\alpha_M\bar{\alpha}_L$; $k_M^l(p_M) = (1-\alpha_M)p_M + \alpha_M\bar{p}_S + \rho\Delta\alpha_M\alpha_L^l$.

When the two S_L types are quite unreliable (Region (2d)), S_M can always win the order by undercutting l-type S_L. However, Regions (2b) and (2c) are more interesting. In contrast to the scenario in Region (2d), the lower price of the l type now enables S_L to compete with S_M for allocation. When the l type is relatively less reliable (Region (2b)), she and S_M employ a mixture of volume and margin strategies. Specifically, the prices are set to eliminate the h type from the buyer's allocation consideration. When S_L is of h type, S_M gets the allocation; otherwise, the allocation goes to whoever (l type or S_M) quotes the price that incurs lower costs for the buyer. But as the l type becomes more reliable, the buyer can use the signal about S_L's type to infer that the expected spot market price will be quite low. This reduces the maximum price that the l type and S_M can charge in the mixed strategy. In fact, when the l type is sufficiently reliable (Region (2c)), S_M may decide to no longer use the mixed strategy. Rather, she adopts a pure margin strategy by setting her price to ensure allocation when S_L is of h type and concedes allocation to S_L when S_L is of l type. So, for medium-risk supplier S_M, *the signal not only reveals the expected spot market price to the buyer, but it also allows l-type S_L to compete with S_M and get order allocation*.

- *High-risk supplier* S_M ($\alpha_M > \bar{\gamma}_M$—Figure 4(C), Table 2): Because S_M is not very reliable in this scenario, S_L does not provide any guarantee. Consequently, the effective supply costs for both h and l types are the same, and they charge the same price in equilibrium (i.e., pooling equilibrium). The buyer then makes the allocation decision by comparing an "average" S_L and S_M. Note that, because both suppliers are quite risky, the expected spot market price is high and so the buyer never uses it. The equilibrium decisions and costs/profits are then exactly the same as in the $\alpha_M > \bar{\gamma}_M$ scenario of the no-guarantee model (Figure 3(C), Table 2).

In summary, when there is information asymmetry, S_L can, under certain scenarios, effectively use P&Q guarantees to credibly signal her type (h or l) to the buyer and eliminate informational asymmetry. The larger the spread in the spot market (δ) and/or the dependence of the spot market price on the amount of supply available ($\rho\Delta$) and/or the marginal supply cost of S_L (c_L), the greater the potential regions where such signals will be provided. In contrast, as S_M becomes quite risky, such signaling becomes less likely. Moreover, how the signal is used also depends on the system parameters. When S_M is quite reliable, it is mainly used by the buyer to infer the expected spot market price. However, when S_M's risk is somewhat higher and the risk difference between the two S_L types is also quite high, the signal is also used by the l-type S_L to garner order allocation.

6. Effects of Supply Guarantee on Chain Performance

In this section, we compare the equilibrium expected profits/costs of the three chain partners and the total supply chain efficiency under the no-guarantee model in §5.1 with those of the supply-guarantee model in §5.2 in order to understand the effects of a P&Q guarantee when there is information asymmetry. For this, we need to compare the different regions in §5.1 with those in §5.2. Because some of the regions in the two do not match when we overlap them, we end up with four comparison regions for $\alpha_M < \underline{\gamma}_M$ and six regions for $\underline{\gamma}_M \le \alpha_M \le \bar{\gamma}_M$. Note that, for $\alpha_M > \bar{\gamma}_M$, the equilibrium no-guarantee and guarantee models are equivalent; thus, supply guarantees do not have any effect in that range (refer to §5). Our detailed analysis results in the following.

Proposition 4. *Effects of supply guarantee on supply chain partners' profits and costs as well as on total supply chain efficiency are fully characterized in Figure* 5.

The main takeaway from Proposition 4 is that, in contrast to the symmetric case (discussed in §4), a P&Q guarantee has a significant impact on the expected costs/profits when there is information asymmetry about S_L's risk level. Moreover, somewhat counterintuitively, we show that *the buyer and the total chain can be worse off and S_M better off* with a guarantee provision under certain conditions. To better understand the underlying reason behind this behavior, we discuss the above proposition in more detail, starting with the effects on individual parties.

- *Low-risk supplier S_M ($\alpha_M < \underline{\gamma}_M$)*: In this scenario, S_L does not get an order allocation in either the no-guarantee or the guarantee model; her profits are zero for both cases. S_M also does not obtain an allocation under either model in Region (1a, 1a). But in other regions, the two models result in different profits for S_M. Recall that in a no-guarantee setting, S_M is always competing against an "average" S_L-based spot market ($= \bar{k}_S$). However, because of the price signal, in a guarantee setting the buyer can know S_L's type exactly. This forces S_M to decide whether to adopt a margin strategy against an h-type-based spot market ($k_S^h > \bar{k}_S$) or to adopt a volume strategy against an l-type-based spot market ($k_S^l < \bar{k}_S$). In Regions (1a, 1b) and (1b, 1b), S_M opts for the former (i.e., the margin strategy) because she is very much convinced that S_L is of type h. This results in guarantee contracts generally enhancing S_M's profit because of the richer margins and/or higher chances of order allocation (compared with the no-guarantee model). But she is worse off in high α_L^l parts of Region (1b, 1b) because of loss of allocation. In contrast, in Region (1b, 1c), S_M decides to follow the volume strategy because she then strongly believes that S_L is of type l. This ensures that she always gets the allocation. But she then prefers a no-guarantee scenario because it allows her to charge a relatively high price as well as to always win the allocation.

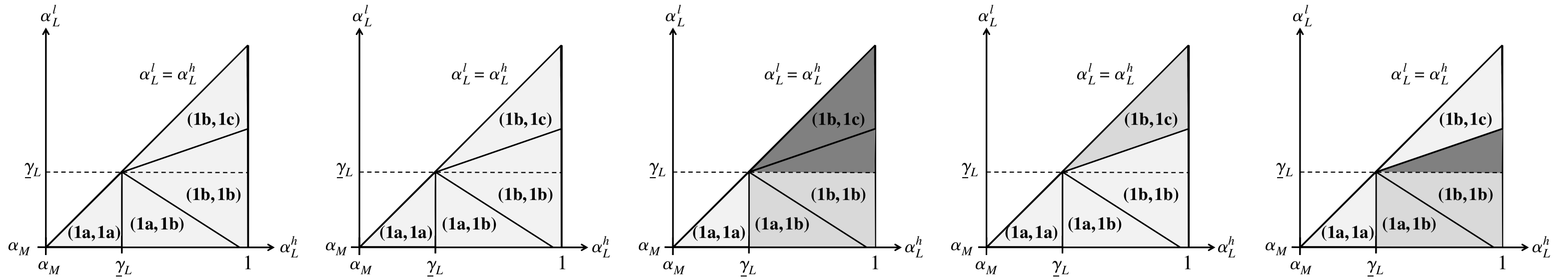
For the buyer, the expected costs remain the same under both models for all regions, except (1b, 1c). In Region (1b, 1c), the buyer actually prefers the guarantee contract because it forces S_M to compete with an l-type spot market price (k_S^l) rather than an "average"-type spot market price ($= \bar{k}_S > k_S^l$), resulting in lower prices for the buyer.

- *Medium-risk supplier S_M ($\underline{\gamma}_M \le \alpha_M \le \bar{\gamma}_M$)*: As expected, the performance of h-type S_L deteriorates with guarantee offers because she can no longer pose as an "average" supplier. By the same token, a guarantee offer normally improves the performance of an l type. In Regions (2b, 2b) and (2b, 2c), it enables her to signal the true type and get order allocations, although she did not get any in the no-guarantee scenario. As discussed in §5.2, a guarantee can also be used to deter an h type from getting any allocation. This reduces the degree of competition at the upstream level, which in turn enables an l type (and also S_M) to charge a premium price (e.g., in the upper part of Region (2a, 2b)). However, when an l type is quite reliable, the guarantee signal may indeed hurt her (e.g., in lower part of Region (2a, 2c)). In that case, the buyer correctly infers that there will probably be plenty of available capacity, and, hence, that the expected spot market price will be low. This forces the l type to charge a very low price to gain allocation, with the result that her profit under a guarantee contract is lower than in the no-guarantee model.

Interestingly, the signaling ability of a guarantee contract may actually hurt the buyer and help supplier S_M. Specifically, when the signal is used to deter the h type and allows both suppliers to charge a reliability premium, *S_M can actually benefit from a guarantee contract to the detriment of the buyer* (e.g., Regions (2a, 2b), (2a, 2c)). However, when the signal provided by a P&Q guarantee is used by the buyer to deduce the spot market price but the reliability premium is not very high (compared to the no-guarantee case) or is eliminated completely by S_M's volume strategy, *guarantees then help the buyer and decrease S_M's profit* (e.g., Region (2b, 2d)).

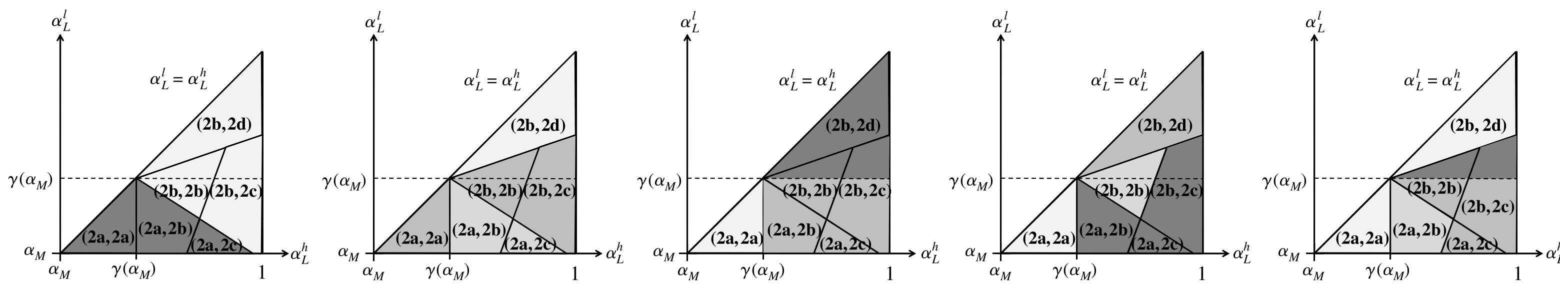
To summarize, the buyer is better off with guarantees when S_M is quite reliable and/or when both S_L

Figure 5 **Effects of Supply Guarantee on Supply Chain Partners' Profit/Costs When There Is Information Asymmetry**



(A) $\alpha_M \le \underline{\gamma}_M$

Regions	h-type S_L's profit	l-type S_L's profit	Supplier S_M's profit	Buyer's cost	Total SC efficiency
			$\alpha_M \le \underline{\gamma}_M$		
(1a, 1a)	$NG = G$	$NG = G$	$NG = G$	$NG = G$	$NG = G$
(1a, 1b)	$NG = G$	$NG = G$	$NG < G$	$NG = G$	$NG < G$
(1b, 1b)	$NG = G$	$NG = G$	$NG \le G$ iff $\alpha_L^l \le \underline{\gamma}_L$	$NG = G$	$NG \le G$ iff $\alpha_L^l \le \underline{\gamma}_L$
(1b, 1c)	$NG = G$	$NG = G$	$NG > G$	$NG > G$	$NG = G$



(B) $\underline{\gamma}_M < \alpha_M \le \bar{\gamma}_M$

Regions	h-type S_L's profit	l-type S_L's profit	Supplier S_M's profit	Buyer's cost	Total SC efficiency
			$\underline{\gamma}_M < \alpha_M \le \bar{\gamma}_M$		
(2a, 2a)	$NG > G$	$NG < G$	$NG = G$	$NG = G$	$NG = G$
(2a, 2b)	$NG > G$	$NG \le G$ iff $\Delta_1^l \ge 0$	$NG < G$	$NG < G$	$NG \le G$ iff $\Delta^{TC} \ge 0$
(2a, 2c)	$NG > G$	$NG \le G$ iff $\Delta_2^l \ge 0$	$NG < G$	$NG < G$	$NG < G$
(2b, 2b)	$NG = G$	$NG < G$	$NG \le G$ iff $\alpha_L^l \le \gamma(\alpha_M)$	$NG \le G$ iff $\Delta^B \ge 0$	$NG \le G$ iff $\alpha_L^l \le \gamma(\alpha_M)$
(2b, 2c)	$NG = G$	$NG < G$	$NG \le G$ iff $\alpha_L^l \le \gamma(\alpha_M)$	$NG < G$	$NG \le G$ iff $\alpha_L^l \le \gamma(\alpha_M)$
(2b, 2d)	$NG = G$	$NG = G$	$NG > G$	$NG > G$	$NG = G$

Notes. In (A) and (B), *NG* and *G* refer to no-guarantee and supply-guarantee models, respectively, and the first element in the "Regions" column refers to the regions in Table 2 and the second element refers to the regions in Tables 3 and 4. The expressions for Δ^B, Δ_1^l, Δ_2^l, and Δ^{TC} are provided in the appendix. Because equilibrium no-guarantee and supply-guarantee models are equivalent for $\alpha_M > \bar{\gamma}_M$, that range is not included in this figure. The dark, medium-dark, and light shaded regions in this figure represent the cases where the corresponding supply chain party is, respectively, better off with *NG* (i.e., $NG > G$), better off with *G* (i.e., $NG \le G$), and indifferent between *NG* and *G* (i.e., $NG = G$).

types are known to be quite risky. However, he favors a no-guarantee setting when S_M's risk level is medium and there is considerable uncertainty about the risk level of S_L (and vice versa for S_M). Moreover, as the spread in the spot market price (δ) and/or the dependence of the spot market price on market supply ($\rho\Delta$) increases, the area where the buyer prefers a guarantee contract also increases. Last, although guarantees may increase the buyer's expected cost by making the buyer's supply risk free, it (at least weakly) reduces his cost uncertainty.

Another issue of interest is how a P&Q guarantee affects the total supply chain efficiency (i.e., the sum of all chain partners' expected profits). Given that, in our context, the buyer's revenue from selling to end consumers is fixed irrespective of the model setting, this implies investigating the effect of guarantees on the sum of supply costs for S_L and S_M and any spot market costs for the buyer and S_L (the higher the total cost, the lower the efficiency is). As is evident from the rightmost panel of Figure 5, total supply chain efficiency *may deteriorate* with a P&Q guarantee (although most often it improves). This result is also driven by the fact that in a no-guarantee setting, S_M always competes with an "average" S_L, whereas in a guarantee model, S_M needs to decide whether to follow a volume strategy or a margin strategy. In the areas where P&Q guarantees reduce efficiency, ideally (from a supply chain efficiency viewpoint), S_M should select the volume strategy. However, S_M's own profit-maximizing strategy is to charge a relatively higher price even at the cost of allocation, i.e., to select the margin strategy. This incentive misalignment caused by decentralized decision making increases the supply chain cost, thus reducing its efficiency.

7. Conclusions and Implications

Various forms of supplier guarantees are used in procurement contracts. We focus on a particular one where the guarantee is in terms of price and quantity. Our analysis provides the insights into the causes and effects of P&Q guarantees, as shown in Table 5.

Guarantees do not play a significant role in the symmetric information case, as the allocation decision for the buyer and the expected performance of the chain partners are unaffected. The equilibrium allocation can be sole, dual, or spot sourcing, depending on the relative costs and reliabilities of S_L and S_M and the spot market price. Note that S_L does not offer a guarantee only when the default risks for both suppliers are quite high. In this case, it is likely that the spot market price would be high, which increases the costs associated with offering a guarantee. Moreover, guarantees are then not a competitive necessity for S_L. The main effect of a guarantee in the symmetric information case is its role in providing supply and cost assurances to the buyer.

On the other hand, information asymmetry regarding S_L's default risk brings to light more interesting facets of the guarantee contract. First, in the no-guarantee scenario, S_L is not able to credibly signal her default risk to the buyer. A guarantee contract may allow her to do so by differentiating between the two risk levels through the guaranteed price. The buyer can then make his allocation decision based on perfect visibility into the supply system. So P&Q contracts act both as a *supply assurance* as well as *a signaling device* (but only when S_L is able to signal her type). A signaling guarantee becomes more likely when the spread in the spot market, and/or dependence of spot market price on the supply available, and/or marginal supply cost of supplier S_L is high. Conversely, it is less likely when both suppliers have high default risks.

One interesting feature of the guarantee offer is how the resulting signal is used. When S_M is quite reliable, the signal is used to communicate the expected spot market price to the buyer. In that case, guarantees generate lower costs for the buyer and lower profits for S_M. But when S_M is somewhat risky and the two S_L types have quite different default risks, the l-type S_L uses the guarantee to get allocation from the buyer. Guarantees might then actually reduce the competitive intensity between the two suppliers. Consequently, both of them can charge the buyer a premium. So, interestingly, guarantees can then increase costs for the buyer and be beneficial for S_M. Finally, even the total supply chain efficiency may worsen under a guarantee contract.

Table 5 Summary of Results

What is the role of the guarantee?	To provide supply assurance to the buyer (symmetric and asymmetric information), and to signal the default risk of S_L (only asymmetric information)
When is the guarantee offered?	When S_M's default risk is low or medium (symmetric and asymmetric information)
How is the information conveyed through the guarantee used by the buyer?	To deduce the expected spot market price and to make allocation decision (asymmetric information)
What is the impact of the guarantee on expected costs/profits?	—No impact on the expected costs and profits (symmetric information) —In general, buyer and l type are better off but S_M and h type are worse off; but the buyer can be worse off and S_M better off because of reduced *upstream* competition (asymmetric information)

Our paper is among the first in the literature to consider supplier-led initiatives to signal their capability to buyers to gain market share. In this context, the results have a number of managerial implications. First, they show that when S_M is somewhat risky, a higher level of information asymmetry between S_L and the buyer (i.e., higher $\alpha^h - \alpha^l$) provides more incentive to S_L to provide visibility through supply guarantees. This suggests that supply guarantees can be used as an effective tool in the early stages of procurement relations when visibility into supplier reliability is likely to be low.

Our results also reveal that buyers may need to be wary about supply guarantees because of the cost associated with information rent to gain visibility via such guarantees. The amount of rent depends crucially on the level and/or likelihood of maximum risk associated with S_L (i.e., α^h and/or r^h); the higher these are, the more the buyer is likely to pay to gain visibility. This does not necessarily mean that buyers should not accept guarantee offers. Indeed, such offers can help reduce the buyer's risk as S_L picks up some of it. Hence, although guarantees in the asymmetric case can be costly for the buyer in the short term, they may pay off in the long run if they help him obtain visibility more quickly than using other means. From S_M's perspective, the above trade-off is just the opposite. Specifically, S_M normally benefits from S_L's guarantee in the short term, but loses her advantage once the buyer gains visibility over time. This suggests that, in the long run, the only option left for S_M to preserve her position is to reduce her costs.

The extra transaction costs associated with selling in the spot market (δ) and the volatility in the spot prices ($\rho\Delta$) also play critical roles in the signaling ability of S_L and the consequences on the buyer's cost. Specifically, for products whose cost is high and/or the spot price is more volatile (as has been the case in recent times for a number of commodity products), low-risk S_L can more easily differentiate from the high-risk type through guarantee offers. However, high values of δ and/or $\rho\Delta$ also create more opportunities for the two suppliers to increase their prices, which in turn increases the buyer's procurement cost.

Next, we briefly discuss the implications of relaxing some of our assumptions. First, we assume that all parties are risk-neutral players. However, as noted, P&Q guarantees can sometimes reduce the buyer's cost variability (e.g., Region (2a) in Table 4). This suggests that if we consider implications on both the mean and the variability of buyer's costs, it becomes more likely that a risk-averse buyer would benefit from a guarantee contract. Second, we assume that S_L incurs marginal costs only for the units that are actually delivered to the buyer. If she incurs marginal costs for all units produced, this would increase the cost differential between l and h types, which in turn would increase the likelihood of separating equilibria. Finally, we assume that the capacity uncertainty is of the "all-or-nothing" type. Adding intermediate levels into the capacity uncertainty structure may allow l-type S_L to offer partial guarantees, which again can be used to differentiate from h type. It is our expectation that this research spurs work on the above extensions to provide guidance to procurement managers regarding how supply-guarantee contracts can address two of their main concerns—improving visibility and dealing with disruption risk.

Acknowledgments
The authors thank department editor Martin Lariviere, an anonymous associate editor, and three anonymous referees for constructive comments that led to significant improvements in this paper. The authors are also grateful to seminar attendees at the University of British Columbia, University of Massachusetts Amherst, GERAD, HEC Montreal, Syracuse University, University of California at Irvine, University of California at Riverside, the Hong Kong University of Science and Technology, Southern Methodist University, Georgetown University, University of Pittsburgh, University of Miami, and Wilfrid Laurier University for many helpful comments and suggestions. The works of Mehmet Gümüş and Saibal Ray were supported in part by research grants from the Natural Sciences and Engineering Research Council of Canada [NSERC RGPIN 355570-09, NSERC 202889]; the Social Sciences and Humanities Research Council of Canada [SSHRC 214832]; and Fonds de recherche sur la Société et la culture of Québec [FQRSC NP-132114]. The work of Haresh Gurnani was supported in part by a research grant from the University of Miami.

Appendix. Proofs of Propositions

Proof of Proposition 1. Let us first start by characterizing the equilibrium under the no-guarantee (NG) contract. We solve the game using backward induction starting from the buyer's order allocation decision. Let TC_B^θ be the buyer's expected total cost under symmetric information scenario. Because it is linear in q_L and q_M, depending on the sign of the coefficients for q_L and q_M, optimal q_L and q_M can only be the end points; i.e., $q_L^* \in \{0, Q\}$ and $q_M^* \in \{0, Q\}$. The buyer's expected total costs under symmetric information scenario for the four possible combinations are as follows: $TC_B^\theta(q_L, q_M) = k_L^\theta(p_L) = (1-\alpha_L^\theta)p_L + \alpha_L^\theta \bar{p}_S + \rho\Delta\alpha_M\alpha_L^\theta$ if $q_L = Q$ and $q_M = 0$; $TC_B^\theta(q_L, q_M) = k_M^\theta(p_M) = (1-\alpha_M)p_M + \alpha_M\bar{p}_S + \rho\Delta\alpha_M\alpha_L^\theta$ if $q_L = 0$ and $q_M = Q$; $TC_B^\theta(q_L, q_M) = k_{LM}^\theta(p_L, p_M) = (1-\alpha_L^\theta)p_L + (1-\alpha_M)p_M + \alpha_L^\theta\alpha_M(\bar{p}_S + \rho\Delta) - (1-\alpha_M)(1-\alpha_L^\theta)(\bar{p}_S - \rho\Delta - \delta)$ if $q_L = Q$ and $q_M = Q$; and, finally, $TC_B^\theta(q_L, q_M) = k_S^\theta = \bar{p}_S + \rho\Delta\alpha_M\alpha_L^\theta - \rho\Delta(1-\alpha_M)(1-\alpha_L^\theta)$ if $q_L = 0$ and $q_M = 0$. Minimizing TC_B^θ with respect to $q_L \in \{0, Q\}$ and $q_M \in \{0, Q\}$, we obtain the following allocation rule: $q_L(p_M, p_L) = Q$ if and only if $\min(k_L^\theta(p_L), k_{LM}^\theta(p_L, p_M)) < \min(k_M^\theta(p_M), k_S^\theta)$. Similarly, $q_M(p_M, p_L) = Q$ if and only if $\min(k_M^\theta(p_M), k_{LM}^\theta(p_L, p_M)) < \min(k_L^\theta(p_L), k_S^\theta)$. After substituting q_L and q_M into supplier S_L and supplier S_M's profit functions, respectively, we can then solve the (Nash) price competition game between the two suppliers. We show that the price competition between the two suppliers leads to three sourcing equilibria: (i) no-sourcing, (ii) sole-sourcing, and (iii) dual-sourcing equilibria.

No-Sourcing and Sole-Sourcing Equilibria. In the no-sourcing and sole-sourcing equilibria, the competition between S_L and S_M can be analyzed as a pure Bertrand price competition with different marginal costs. We know that in Bertrand competition, each supplier will undercut her competitor's price as long as that price is strictly greater than her marginal cost. Let k_M^θ and k_L^θ be the expected cost for the buyer when supplier S_M and S_L offer their break-even prices, respectively, c_M and c_L, and let k_S^θ be the expected cost for the buyer if she buys in the spot market. So the undercutting argument ensures that in equilibrium there will always be incentive for the party whose expected cost to the buyer is the lowest to undercut the other parties if these parties charge more than their break-even price. This argument implies that the only equilibrium that is sustainable against all deviations is the one in which all the losing parties charge their break-even prices and the winning party charges a price that makes the expected cost for the buyer infinitesimal lower than the expected cost from the best losing party. Throughout the appendix, we use $[x]^{(-)}$ to denote a number that is infinitesimally less than x. So, from buyer's perspective, if his marginal cost of buying from the spot market is less than buying from S_L and S_M when both suppliers charge their break-even prices—i.e., $k_S^\theta < k_M^\theta$ and $k_S^\theta < k_L^\theta$, or equivalently, $\alpha_L^\theta \le \underline{\gamma}_L$ and $\alpha_M \le \underline{\gamma}_M$, where $\underline{\gamma}_M = 1-(\bar{p}_S - c_L)/(\rho\Delta)$ and $\underline{\gamma}_L = 1-(\bar{p}_S - c_M)/(\rho\Delta)$—then, in equilibrium, both suppliers charge their marginal costs, i.e., $p_M^* = c_M$, and $p_L^* = c_L$. On the other hand, if buying from supplier S_M is less than buying from supplier S_L and the spot market—i.e., $k_M^\theta < k_L^\theta$ and $k_M^\theta < k_S^\theta$, or equivalently, $\alpha_L^\theta \ge \gamma(\alpha_M)$ and $\alpha_L^\theta \ge \underline{\gamma}_L$ where $\gamma(\alpha_M) = (c_M - c_L)/(\bar{p}_S - c_L) + \alpha_M(\bar{p}_S - c_M)/(\bar{p}_S - c_L)$—then, in equilibrium, S_L can at best offer $p_L^{\theta*} = c_L$, and S_M undercuts S_L by setting her price such that the buyer's marginal cost of buying from S_M is equal to $[k_L^\theta]^{(-)}$. By equating $k_M^\theta(p_M)$ to $[k_L^\theta]^{(-)}$ and solving for p_M, we obtain equilibrium $p_M^* = [c_M + (k_L^\theta - k_M^\theta)/(1-\alpha_M)]^{(-)}$. Otherwise, if buying from supplier S_L is less than buying from supplier S_M and the spot market, i.e., if $\alpha_L^\theta < \gamma(\alpha_M)$, and $k_L^\theta < k_S^\theta$, S_M sets $p_M^* = c_M$ and S_L sets her price such that the buyer's marginal cost of buying from S_L is equal to $[k_M^\theta]^{(-)}$. By equating $k_L^\theta(p_L)$ to $[k_M^\theta]^{(-)}$ and solving for p_L, we obtain equilibrium $p_L^{\theta*} = [c_L + (k_M^\theta - k_L^\theta)/(1-\alpha_L^\theta)]^{(-)}$.

Dual-Sourcing Equilibrium. Finally, when both supplier S_M and supplier S_L become quite unreliable, then it is optimal for the buyer to procure from both to reduce the total expected cost. For dual-sourcing equilibrium prices to be sustainable, they should satisfy the following conditions: $k_{LM}^\theta(p_L, p_M) < \min(k_L^\theta(p_L), k_M^\theta(p_M), k_S^\theta)$. Because all the three functions ($k_{LM}^\theta(p_L, p_M)$, $k_L^\theta(p_L)$, and $k_M^\theta(p_M)$) are increasing in p_L and p_M, for suppliers' prices to satisfy the above condition in equilibrium, we need to find p_L^* and p_M^* that solve $k_{LM}^\theta(p_L^*, p_M^*) = [k_L^\theta(p_L^*)]^{(-)} = [k_M^\theta(p_M^*)]^{(-)}$ and check that $k_{LM}^\theta(p_L^*, p_M^*) < k_S^\theta$. The solution of the above equation gives us the following equilibrium prices: $p_L^{\theta*} = [\bar{p}_S - (1-\alpha_M)(\rho\Delta+\delta)]^{(-)}$ and $p_M^{\theta*} = [\bar{p}_S - (1-\alpha_L^\theta)\cdot(\rho\Delta+\delta)]^{(-)}$. Evaluating $k_{LM}^\theta(p_L, p_M)$ at the equilibrium prices $p_L = p_L^*$, and $p_M = p_M^*$, we can show that $k_{LM}^\theta(p_L^*, p_M^*) = \bar{p}_S + \rho\Delta\alpha_M\alpha_L^\theta - (\rho\Delta+\delta)(1-\alpha_M)(1-\alpha_L^\theta) < k_S^\theta = \bar{p}_S + \rho\Delta\alpha_M\alpha_L^\theta - \rho\Delta(1-\alpha_M)(1-\alpha_L^\theta)$, because $\delta > 0$. Now we verify that there is no profitable deviation from p_L^* and p_M^* for supplier S_L and supplier S_M, respectively. Suppose that, for a given p_M^*, supplier S_L increases her price from p_L^*. This increases both $k_L^\theta(p_L^*)$ and $k_{LM}^\theta(p_L^*, p_M^*)$ and makes both greater than $k_M^\theta(p_M^*)$, which implies that supplier S_L loses the order allocation from the buyer. Obviously, supplier S_L's profit decreases as she lowers her price from p_L^*; hence, there is no profitable deviation for S_L in either direction. The same argument also works for supplier S_M. As a final check, we need to make sure that both $p_L^{\theta*} \ge c_L$ and $p_M^{\theta*} \ge c_M$. This results in the following conditions: $p_L^{\theta*} \ge c_L$ and $p_M^{\theta*} \ge c_M$ if and only if $\alpha_L^\theta \ge \bar{\gamma}_L$ and $\alpha_M \ge \bar{\gamma}_M$, where $\bar{\gamma}_M = 1-(\bar{p}_S - c_L)/(\rho\Delta+\delta)$ and $\bar{\gamma}_L = 1-(\bar{p}_S - c_M)/(\rho\Delta+\delta)$.

To summarize, the resulting equilibrium allocation can be characterized in two stages. In the first stage, either none of the suppliers gets the order allocation if $\alpha_L^\theta \le \underline{\gamma}_L$ and $\alpha_M \le \underline{\gamma}_M$ or both of them get the order allocation if $\alpha_L^\theta \ge \bar{\gamma}_L$ and $\alpha_M \ge \bar{\gamma}_M$. In the second stage, if neither of the above conditions is satisfied, then the equilibrium is of sole-sourcing type; i.e., either supplier S_L obtains the full allocation if $\alpha_L^\theta \le \gamma(\alpha_M)$ or supplier S_M obtains the full allocation if $\alpha_L^\theta > \gamma(\alpha_M)$. The resulting equilibrium profits and costs can then be derived from $p_L^{\theta*}$ and p_M^*. Finally, we need to check whether supplier S_L and S_M benefit from selling directly to the spot market or not. Suppose that S_L decides to sell directly to the spot market. S_L's expected revenue conditional on his realized capacity being equal to Q is $\alpha_M(\bar{p}_S - \delta) + (1-\alpha_M)(\bar{p}_S - \rho\Delta - \delta) = \bar{p}_S - \alpha_M\delta - (1-\alpha_M)\cdot(\rho\Delta+\delta) \le \bar{p}_S - (1-\alpha_M)(\rho\Delta+\delta)$. Recall that $\bar{p}_S - (1-\alpha_M)\cdot(\rho\Delta+\delta) \le c_L$ if and only if $\alpha_M \le \bar{\gamma}_M$. But because S_L earns at minimum an expected revenue of $\bar{p}_S - (1-\alpha_M)(\rho\Delta+\delta)$ when $\alpha_M > \bar{\gamma}_M$, this implies that on the equilibrium, S_L never chooses to sell her capacity directly to the spot market. The same argument applies for supplier S_M, showing that S_M is worse off by selling directly to the spot market.

Regarding the supply-guarantee (G) contract setting, note that there are two extra decision variables for S_L: guaranteed quantity (q_G) and price (p_G). We can show that except for when both suppliers are very risky (i.e., region LM_θ), $q_G^* = Q$. The equilibrium price for S_M is the same as the NG case. The guaranteed price p_G^* can be obtained by using Bertrand price competition argument, as in the NG case above. The detailed expressions for equilibrium decisions and profits/costs are provided in Table 1. □

Proof of Proposition 2. We prove this proposition for the NG case in two steps:

1. We first show that in equilibrium both h- and l-type S_L charge the same price. Note that θ-type supplier L's profit function in the NG case is as follows: $\Pi_L^\theta = (1-\alpha_L^\theta)\cdot(p_L^\theta - c_L)q_L$. Based on the above, it is trivial to show that in equilibrium if $p_L^{l*} \ne p_L^{h*}$, either l or h type can increase her profit by mimicking the other type. Hence, it implies that $p_L^{l*} \ne p_L^{h*}$ cannot be on the equilibrium path. This suggests that the buyer cannot identify the correct type of supply uncertainty and hence uses a priori beliefs (i.e., $\bar{\alpha}_L = r^h\alpha_L^h + r^l\alpha_L^l$) to decide on quantity allocations.

2. Replacing $\bar{\alpha}_L$ with α_L^θ in the proof of Proposition 1, we can obtain equilibrium characterization in terms of $\bar{\alpha}_L$ and α_M. Also, equilibrium profits and costs can be derived from p_L^* and p_M^*. □

Proof of Proposition 3. For the G scenario, we will characterize the equilibrium strategies for low, medium, and high α_M cases separately.

Case (i) $\alpha_M \le \underline{\gamma}_M$. Since $\alpha_M \le \underline{\gamma}_M$, $k_S^\theta \le k_L^\theta$ for all $\alpha_L^\theta \ge 0$. This implies that the buyer never procures from supplier S_L. Both h and l types offer a full guarantee and signal their types by offering different per unit guaranteed prices; i.e., $p_G^{h*} = k_L^h$ and $p_G^{l*} = k_L^l$. So, depending on p_M, the buyer makes an allocation decision between S_M and the spot market as follows: $q_M = Q$ if $k_M^\theta(p_M) \le k_S^\theta$; otherwise, $q_M = 0$. As shown in Figure 4(A) and Table 3, we have three regions depending on α_L^h and α_L^l. Below, we analyze each region separately:

—Region (1a): Note that if $\alpha_L^h \le \underline{\gamma}_L$, $k_M^\theta(p_M) \le k_S^\theta$ for all $\theta \in \{l, h\}$; hence, the buyer procures only from the spot market. Therefore, on the equilibrium S_M offers $p_M^* = c_M$ and gets zero allocation; i.e., $q_M^* = 0$.

—Regions (1b) and (1c): Otherwise, if $\alpha_L^h > \underline{\gamma}_L$, to get allocation, supplier S_M needs to offer a p_M that makes $k_M^\theta(p_M)$ epsilon below k_S^θ. This implies that $p_M^\theta = [\bar{p}_S - \rho\Delta(1-\alpha_L^\theta)]^{(-)}$. Note that since $\alpha_L^h > \alpha_L^l$, it implies that $p_M^l < p_M^h$. Therefore, S_M can choose between two options: (i) She charges $p_M^h = [\bar{p}_S - \rho\Delta(1-\alpha_L^h)]^{(-)}$, gets full allocation only when $\theta = h$ and, hence, earns an expected profit of $r^h(k_S^h - k_L^h)$; or (ii) she charges $p_M^l = [\bar{p}_S - \rho\Delta(1-\alpha_L^l)]^{(-)}$, gets full allocation under both $\theta = h$ and $\theta = l$, and earns an expected profit of $r^l(k_S^l - k_L^l) + r^h(k_S^l - k_L^h)$. The equilibrium pricing decision for S_M depends on her a priori beliefs as follows: $p_M^* = p_M^h = [\bar{p}_S - \rho\Delta(1-\alpha_L^h)]^{(-)}$ if $r^h > (k_S^l - k_L^l)/(k_S^h - k_L^l)$; otherwise, $p_M^* = p_M^l = [\bar{p}_S - \rho\Delta(1-\alpha_L^l)]^{(-)}$.

Case (ii) $\underline{\gamma}_M < \alpha_M \le \bar{\gamma}_M$. In this case, we have four regions as shown in Figure 4(B) and Table 4. Below, we analyze each region separately:

—Region (2a): When $\alpha_L^h < \gamma(\alpha_M)$, there exists only pooling equilibrium, where both l and h types receive the full order. In this case, both l and h types can undercut S_M; hence, $p_G^l \neq p_G^h$ cannot be on the equilibrium path. Given that $p_M = c_M$, both l- and h-type supplier S_L can get the full allocation as long as $p_G^\theta \le \bar{k}_M$. Because the profit function of supplier S_L increases in p_G^θ, on the equilibrium, both types of supplier S_L charge infinitesimally less than $\bar{k}_M$. Given that p_G^θ is infinitesimally less than $\bar{k}_M$, supplier S_M's profit will be zero if S_M charges more than c_M and negative if she charges less than c_M. Hence, it implies that $p_M = c_M$, $q_G^\theta = Q$ and p_G^θ that is infinitesimally less than $\bar{k}_M$ for $\theta = \{h, l\}$ form a Nash equilibrium. We can also establish that this is a unique Nash equilibrium by noting that any contract terms other than the one specified above would give an incentive to either S_M or S_L to undercut her opponent, and hence are not sustainable on the equilibrium.

—Regions (2b) and (2c): In these regions, $\gamma(\alpha_M) < \alpha_L^h$ and $r^h > (k_L^l - k_M^l)/(k_L^h - k_M^l)$. The first condition implies that h-type S_L cannot make a profit against supplier S_M, even if she offers a full guarantee. This is because she can set p_G^h at minimum k_L^h, but S_M can still undercut her offer by charging $[c_M + (k_L^h - k_M^h)/(1-\alpha_M)]^{(-)}$. However, l-type S_L can still compete with S_M by offering a price p_G^l, which is less than $k_M^l(p_M)$. On the equilibrium, we can show that under certain conditions, there is no pure strategy equilibrium. First, we construct a mixed equilibrium and derive the conditions in order for it to be sustainable for all the parties. Note that S_L offers guaranteed price p_G^l, whereas S_M offers a nonguaranteed price p_M. Therefore, their prices are defined over different price ranges. Let $\mathcal{P}_L = [\underline{p}_L, \bar{p}_L]$ and $\mathcal{P}_M = [\underline{p}_M, \bar{p}_M]$ be the ranges of mixing equilibria for supplier S_L and S_M, respectively. Any pair of prices in $p_G \in \mathcal{P}_L$ and $p_M \in \mathcal{P}_M$ can be matched with each other by using the following transformations: $p_G = k_M^l(p_M)$ and $p_M = h_M(p_G) = c_M + (p_G - k_M^l)/(1-\alpha_M)$. Based on the above transformations, S_M (respectively, S_L) gets full allocation if and only if S_M (respectively, S_L) charges less than $h_M(p_G) = c_M + (p_G - k_M^l)/(1-\alpha_M)$ (respectively, $k_M^l(p_M)$). Let F_L^l and F_M be the pricing distributions for S_L and S_M, respectively, that are defined over the above ranges. The upper bound for the mixed strategy interval for l-type supplier S_L must be set just at either the marginal cost of h-type S_L or l-type spot market price, whichever is smaller; i.e., $\bar{p}_L = \min(k_L^h - (k_M^h - k_M^l), k_S^l)$. The upper bound for supplier S_M can be found by mapping $\bar{p}_L$ to $\mathcal{P}_M$ as follows: $\bar{p}_M = c_M + (\bar{p}_L - k_M^l)/(1-\alpha_M) \le c_M + (k_L^h - k_M^h)/(1-\alpha_M)$. Note that $\bar{p}_M$ is less than $c_M + (k_L^h - k_M^h)/(1-\alpha_M)$ because S_M always undercuts h-type supplier S_L. Mixed strategy probability distribution for S_M can be derived by the condition that if S_M mixes continuously with F_M over the interval $[\underline{p}_M, \bar{p}_M]$, the l type is indifferent between charging any price over the interval $[\underline{p}_L, \bar{p}_L]$. Similarly, the l type also mixes continuously over the interval $[\underline{p}_L, \bar{p}_L]$ to make sure that S_M is indifferent between undercutting both l and h types and undercutting only the h type. To express these conditions, we need to write down each firm's expected payoff first: $\Pi_L^l(p_G) = [1 - F_M(c_M + (p_G - k_M^l)/(1-\alpha_M))](p_G - k_L^l)$ and $\Pi_M(p_M) = r^h(1-\alpha_M)(p_M - c_M) + r^l(1-\alpha_M)(p_M - c_M)\cdot(1 - F_L^l(k_M^l(p_M)))$. Recall that supplier S_M's profit needs to be equal to $r^h(1-\alpha_M)(\bar{p}_M - c_M)$ for all p in the support; i.e., $\Pi_M(p_M) = (1 - r^l F_L^l(k_M^l(p_M)))(p_M - c_M) = r^h(\bar{p}_M - c_M)$. Note that the right-hand side of this equation represents supplier S_M's expected profit at the upper limit of her range ($\bar{p}_M$), i.e., $r^h(\bar{p}_M - c_M)$. Using the transformation between supplier S_M's price range and supplier S_L's price range, we can rewrite S_M's expected profit at $\bar{p}_M$ as $r^h(k_L^h - k_M^h)$. So, inverting the above profit equation for supplier S_M and applying variable transformation between p_G and p_M, we obtain mixing distribution function for l-type supplier S_L: $F_L^l(p_G) = (1/r^l)(1 - r^h(k_L^h - k_M^h)/(p_G - k_M^l))$. Also note that $\lim_{p_G \to \bar{p}_L} F_L^l(p_G) = 1$ if $\bar{p}_L < k_S^l$ but $\lim_{p_G \to \bar{p}_L} F_L^l(p_G) = 1 - (1/r^l)(1 - r^h(k_L^h - k_M^h)/(p_G - k_M^l)) < 1$ if $\bar{p}_L = k_S^l$. Hence, there is a probability mass at $\bar{p}_L$ if $\bar{p}_L = k_S^l$.

From $F_L^l(p_G)$, we obtain the lower bound for the support, $\underline{p}_L = k_M^l + r^h(k_L^h - k_M^h)$. Also we need to make sure that $\underline{p}_L$ must be greater than k_L^l. Otherwise, by charging infinitesimally below k_L^l, S_M can always undercut both l and h types and make a higher profit. This condition implies that $\underline{p}_L = k_M^l + r^h(k_L^h - k_M^h) > k_L^l$, but this is automatically satisfied because of the second condition of the Regions (2b) and (2c); i.e., $r^h > (k_L^l - k_M^l)/(k_L^h - k_M^h)$. Because the l type cannot charge more than k_S^l, we also need to make sure that $\underline{p}_L$

must be greater than k_S^l. Otherwise, by charging infinitesimally below k_L^h, S_M can undercut only the h type and makes more profit. This condition implies that $\underline{p}_L = k_M^l + r^h(k_L^h - k_M^h) < k_S^l$ if and only if $r^h < (k_S^l - k_M^l)/(k_L^h - k_M^h)$, but this is automatically satisfied in Region (2b), whereas the opposite is satisfied in Region (2c).

First, we assume that $r^h < (k_S^l - k_M^l)/(k_L^h - k_M^h)$, i.e., $\underline{p}_L < k_S^l$. Note that in this case, $\lim_{p\to \underline{p}_L} F_L^l(p) = 0$. If $k_L^h \le k_S^l$, then $\bar{p}_L = k_L^h$, which implies that $\lim_{p\to\bar{p}_L} F_L^l(p) = 1$. Hence, there is no mass between $\underline{p}_L$ and $\bar{p}_L$. However, if $k_L^h > k_S^l$, then $\bar{p}_L = k_S^l$, which implies that $\lim_{p\to\bar{p}_L} F_L^l(p) < 1$. Hence, there is a mass $1 - F_L^l(k_S^l)$ at $\bar{p}_L = k_S^l$.

We derive the mixing distribution for S_M by using the fact that l type's profit needs to be equal to $\underline{p}_L - k_L^l$ for all p in the support; i.e., $\Pi_L^l(p_G) = [1 - F_M(c_M + (p_G - k_M^l)/(1-\alpha_M))](p_G - k_L^l) = \underline{p}_L - k_L^l$. Inverting profit equation for l-type supplier S_L and applying the variable transformation between p_M and p_G, we obtain mixing distribution function for supplier S_M, F_M, as a function of p_M: $F_M(p_M) = 1 - (\underline{p}_L - k_L^l)/(k_M^l(p_M) - k_L^l)$. Note that $\lim_{p_M\to\underline{p}_M} F_M(p_M) = 0$ but $\lim_{p_M\to\bar{p}_M} F_M(p) = 1 - (\underline{p}_L - k_L^l)/(k_M^l(p_M) - k_L^l) < 1$. Hence, there is a mass at $\bar{p}_M$ given by $(\underline{p}_L - k_L^l)/(k_M^l(\bar{p}_M) - k_L^l)$. Also, in this region, we can establish that this mixed Nash equilibrium is unique. First, consider h-type supplier S_L. Suppose that she sets p_G^h more than k_L^h. In this case, we can construct a mixed strategy equilibrium for both the l type and S_M. But the h type can increase her profit by decreasing p_G^h. Hence, $p_G^h > k_L^h$ is not sustainable on the equilibrium. Similarly, $p_G^h < k_L^h$ would imply a negative profit for the h type, hence it cannot be on the equilibrium path. Therefore, it must be that $p_G^h = k_L^h$ on the equilibrium. Finally, given that $p_G^h = k_L^h$, by construction, the only mixed strategy for l type and S_M that is sustainable on the equilibrium path is the one specified above.

Next, we assume that $r^h \ge (k_S^l - k_M^l)/(k_L^h - k_M^h)$; i.e., $\underline{p}_L \ge k_S^l$. In this case, l-type supplier S_L offers epsilon below k_S^l, supplier S_M offers epsilon below $c_M - (k_L^h - k_M^h)/(1-\alpha_M)$, and h type supplier S_L offers k_L^h. On the equilibrium, the buyer always procures from supplier S_L if $p_G^* = [k_S^l]^{(-)}$; otherwise, he procures from supplier S_M.

—Region (2d): In this region, $\gamma(\alpha_M) < \alpha_L^h$ and $r^h > (k_L^l - k_M^l)/(k_L^h - k_M^h)$. These two conditions imply that supplier S_M has an incentive to undercut both l and h types. Given that $p_G^l = k_L^l$, supplier S_M can get the full allocation as long as $p_M < c_M - (k_L^l - k_M^l)/(1-\alpha_M)$. Because the profit function of supplier S_M increases in p_M, on the equilibrium she charges infinitesimally less than $c_M - (k_L^l - k_M^l)/(1-\alpha_M)$. Given that p_M is infinitesimally less than $c_M - (k_L^l - k_M^l)/(1-\alpha_M)$, l-type supplier S_L's profit will be zero if she charges more than k_L^l and negative if she charges less than k_L^l. This implies that $p_G^l = k_L^l$ and p_M that is infinitesimally less than $c_M - (k_L^l - k_M^l)/(1-\alpha_M)$ form a Nash equilibrium. Finally, the uniqueness of this equilibrium comes from the same fact that any strategy other than the above one would lead to a profitable deviation for S_M, l-type S_L, or h-type S_L.

Case (iii) $\alpha_M > \bar{\gamma}_M$. We can analyze this case in two regions. First, when $\alpha_L^h < \bar{\gamma}_L$, there exists only pooling equilibrium, where both l and h types receive the full order, because in this case, both l and h types can undercut S_M. Next, we can show that if $\alpha_L^h \ge \bar{\gamma}_L$, there is always a profitable deviation for the l type if equilibrium is of a separating type. This is because when $\alpha_L^h \ge \bar{\gamma}_L$ and true type is h, both supplier S_L and S_M can increase their prices and charge $p_L^{h*} = [\bar{p}_S - (1-\alpha_M)(\rho\Delta + \delta)]^{(-)}$ and $p_M^* = [\bar{p}_S - (1-\alpha_L^h)(\rho\Delta+\delta)]^{(-)}$, respectively. However, when true type is l, the highest price the l type can charge is k_M^l, which leads to a profitable deviation for l-type supplier S_L and makes her mimic h type. Therefore, separating equilibrium cannot be sustainable on the equilibrium. This implies that the buyer cannot update his belief on the type of supplier S_L; hence, he uses $\bar{\alpha}$ to decide whether only supplier S_L gets the full allocation (as in Region (3a) of Figure 4(C)) or both S_L and S_M get the allocation (as in Region (3b) of Figure 4(C)).

Finally, profit and cost expressions can be easily derived by evaluating Π_L^θ, Π_M, and TC_B at the equilibrium strategies defined in Tables 2–4. Because of space constraints, we only derive profit and cost expressions for Region (2b) here. The other derivations are available from the authors. In Region (2b), the h type always loses the order to supplier S_M. Therefore, h's profit is equal to zero. Either supplier S_M or the l type wins the full allocation, depending on who offers the lowest price (from the buyer's viewpoint). From the above analysis, we know that both S_M and the l type mix their prices in such a way that on the equilibrium their profits are the same on any point within their respective supports $[\underline{p}_M, \bar{p}_M]$, and $[\underline{p}_L, \bar{p}_L]$. So, by construction, S_M's profit in $[\underline{p}_M, \bar{p}_M]$ is equal to $r^h(k_L^h - k_M^l)Q$, and l-type S_L's profit is equal to $(\underline{p}_L - k_L^l)$. Finally, the buyer's cost is equal to $QE_{p_M^*, p_G^{\theta*}} \min(k_L(p_M^*), p_G^{l*})$, where the expectation is taken with respect to F_M and F_L^l. $\square$

Proof of Proposition 4. In this proposition, we compare the buyer's cost and suppliers' profits under NG and G by using the results of Propositions 2 and 3, respectively. Because we show that when $\alpha_M \ge \bar{\gamma}_M$, supply chain partners' profits/costs are unaffected by the guarantee contract, we skip $\alpha_M \ge \bar{\gamma}_M$ and compare profits and costs under NG and G only when (i) $\alpha_M \le \underline{\gamma}_M$ and (ii) $\underline{\gamma}_M < \alpha_M \le \bar{\gamma}_M$:

Case (i) $\alpha_M \le \underline{\gamma}_M$. Both l-type and h-type S_L do not receive an order allocation when supplier S_M's default risk is low. Therefore, a guarantee does not have any impact on their profits. Therefore, we consider only S_M and the buyer.

—$\alpha^h < \underline{\gamma}_L$, i.e., Region (1a, 1a): Note that S_M's profit is zero and the buyer's cost is $\bar{k}_S$ under both NG and G when $\alpha^h < \bar{\gamma}_L$. So the guarantee contract has no impact on S_M's profit and buyer's cost.

—$\alpha^h \ge \underline{\gamma}_L$: In this case, the equilibrium cost and profit under G depend on S_M's a priori belief.

- $r^h \ge (k_S^l - k_M^l)/(k_S^h - k_M^h)$, i.e., Regions (1a, 1b) and (1b, 1b): In these regions, under G contract, S_M would only undercut the h-type spot market and generate an expected profit of $r^h(k_S^h - k_M^h)Q$. Under the NG contract, her profit is equal to $\bar{k}_S = r^h(k_S^h - k_M^h)Q + r^l(k_S^l - k_M^l)Q$. Comparing these two expressions, we can show that S_M would prefer G over NG if and only if $r^h(k_S^h - k_M^h)Q \ge r^h(k_S^h - k_M^h)Q + r^l(k_S^l - k_M^l)Q \Leftrightarrow k_S^l \ge k_M^l \Leftrightarrow \alpha^l \ge \underline{\gamma}_L$. From the buyer's perspective, his expected cost is the same (equal to $\bar{k}_S$) under both G and NG.

• $r^h < (k_S^l - k_M^l)/(k_S^h - k_M^h)$, i.e., Region (1b, 1c): In this region, under G contract, S_M would undercut the l-type (hence also the h type) spot market and generate an expected profit of $(k_S^l - (r^h k_M^h + r^l k_M^l))Q$. However, S_M's profit under NG in this region is $(r^h k_S^h + r^l k_S^l - (r^h k_M^h + r^l k_M^l))Q$. Note that because $k_S^h \geq k_S^l$, it implies that S_M is always worse off with the G contract in Region (1b,1c). From the buyer's perspective, his cost under G is $r^l k_S^l + r^h(k_S^l + k_M^h - k_M^l)$, whereas his cost under NG is $r^h k_S^h + r^l k_S^l$. Taking the difference between these two costs, we obtain $r^h(k_S^h - k_S^l) - r^h(k_M^h - k_M^l)$. This implies that if $k_S^h - k_M^h \geq k_S^l - k_M^l$, the buyer loses under NG. Rewriting $k_S^\theta - k_M^\theta = (1-\alpha_M)(\bar{p}_S - c_M - \rho\Delta + \rho\Delta\alpha_L^\theta)$, we can show that $k_S^\theta - k_M^\theta$ increases in α_L^θ. Because $\alpha_L^h \geq \alpha_L^l$, we can show that $k_S^h - k_M^h \geq k_S^l - k_M^l$; i.e., the buyer prefers G.

Case (ii) $\underline{\gamma}_M < \alpha_M \leq \bar{\gamma}_M$. As we did in Case (i) above, we consider each region separately from the perspectives of l-type S_L, h-type S_L, S_M and the buyer.

—Region (2a, 2a): Recall that in this region, under both G and NG, the equilibrium is of the pooling type and, therefore, both the l type and h type undercuts an average S_M or spot market, whichever is least. Comparing θ-type S_L's profits under NG and G and arranging terms, we can show that she prefers NG if and only if the following holds true: $((1-\alpha_L^\theta)/(1-\bar{\alpha}))(\min(\bar{k}_S, \bar{k}_M) - \bar{k}_L) \geq (\min(\bar{k}_S, \bar{k}_M) - k_L^\theta) \Leftrightarrow (1-\alpha_L^\theta)/(1-\bar{\alpha}) \geq (\min(\bar{k}_S, \bar{k}_M) - k_L^\theta)/(\min(\bar{k}_S, \bar{k}_M) - \bar{k}_L)$. By multiplying both the denominator and numerator by $\bar{p}_S + \rho\Delta\alpha_M - c_L$, we can rewrite $(1-\alpha_L^\theta)/(1-\bar{\alpha})$ as follows: $(\bar{p}_S + \rho\Delta\alpha_M - c_L)/(\bar{p}_S + \rho\Delta\alpha_M - c_L) \times (1-\alpha_L^\theta)/(1-\bar{\alpha}) = (\bar{p}_S + \rho\Delta\alpha_M - k_L^\theta)/(\bar{p}_S + \rho\Delta\alpha_M - \bar{k}_L)$. Because $\bar{p}_S + \rho\Delta\alpha_M \geq \min(\bar{k}_S, \bar{k}_M)$, and $k_L^h \geq \bar{k}_L \geq k_L^l$, we can show that $(1-\alpha_L^h)/(1-\bar{\alpha}) = (\bar{p}_S + \rho\Delta\alpha_M - k_L^h)/(\bar{p}_S + \rho\Delta\alpha_M - \bar{k}_L) \geq (\min(\bar{k}_S, \bar{k}_M) - k_L^h)/(\min(\bar{k}_S, \bar{k}_M) - \bar{k}_L)$ and $(\min(\bar{k}_S, \bar{k}_M) - k_L^l)/(\min(\bar{k}_S, \bar{k}_M) - \bar{k}_L) \geq (\bar{p}_S + \rho\Delta\alpha_M - k_L^l)/(\bar{p}_S + \rho\Delta\alpha_M - \bar{k}_L) = (1-\alpha_L^l)/(1-\bar{\alpha})$, which implies that the h type and l type prefer NG and G, respectively. For S_M and the buyer, their profits and costs are same under G and NG. Hence, G is equal to the NG contract in Region (2a, 2a).

—Region (2a, 2b): In this region, h type's profit is zero under G, whereas her profit is positive under NG because h-type S_L can hide behind l-type S_L because of the pooling equilibrium and get allocation. Similarly, S_M gets nothing under NG but may receive nonzero allocation under G. From the buyer's perspective, the buyer's expected cost under NG is equal to $\min(\bar{k}_M, \bar{k}_S)$. However, under G, supplier S_M charges a price in the range of $[\underline{p}_M, \bar{p}_M]$, where $\underline{p}_M \geq c_M$. This implies that the buyer would incur a cost more than k_M^h (with probability r^h) and more than k_M^l (with probability r^l), both of which are greater than what he would incur under NG. Therefore, the buyer is worse off with the guarantee contract in this region. Finally, to evaluate the impact of a guarantee from the l type's perspective, we need to compare her profits under NG and G. Under NG and G, l-type S_L's profits are $((1-\alpha_L^l)/(1-\bar{\alpha})) \cdot (\min(\bar{k}_S, \bar{k}_M) - \bar{k}_L)$ and $\underline{p}_L - k_L^l$, respectively. However, depending on the actual values of the parameters, the comparison can be go either way. Because we have the closed-form profit expressions, we can state the necessary and sufficient conditions (denoted by Δ_1^l) under which l-type S_L benefits from the guarantee contract as follows: $\Delta_1^l = (\underline{p}_L - k_L^l)/(1-\alpha_L^l) - (\min(\bar{k}_S, \bar{k}_M) - \bar{k}_L)/(1-\bar{\alpha}) \geq 0$ where $\underline{p}_L = k_M^l + r^h(k_L^h - k_M^h)$.

—Region (2a, 2c): Similar to Region (2a, 2b), in this region also, a guarantee enables l-type supplier S_L to differentiate herself from h-type S_L. Therefore, the h type does not get any allocation from the buyer under G. However, under NG, h-type S_L can hide behind l-type S_L because of the pooling equilibrium and get allocation. So h-type S_L loses under a G contract. Similarly, S_M gets nothing under NG but may get nonzero allocation under G, which implies that she benefits from a G contract. From the buyer's perspective, using Proposition 2, his expected cost is $\min(\bar{k}_M, \bar{k}_S)$. This implies that his cost is not more than $\bar{k}_M = r^h k_M^h + r^l k_M^l$. Under a G contract, in this region, his expected cost is $r^h k_L^h + r^l k_S^l$. Using the conditions that define Region (2a, 2c), we can show both that $k_L^h \geq k_M^h$ and $k_S^l \geq k_M^l$. This implies that $\min(\bar{k}_M, \bar{k}_S) \leq r^h k_L^h + r^l k_S^l$. Therefore, similar to Region (2a, 2b), the buyer loses from the guarantee contract. Finally, to evaluate the impact of G on l-type supplier S_L, we need to compare his profits under NG and G. Under NG and G, l-type S_L's profits are $(1-\alpha_L^l)/(1-\bar{\alpha})(\min(\bar{k}_S, \bar{k}_M) - \bar{k}_L)$ and $k_S^l - k_L^l$, respectively. However, depending on the actual values of the parameters, the comparison can go either way. Because we have the closed-form profit expressions, we can state the necessary and sufficient conditions (denoted by Δ_2^l) under which l-type S_L benefits from the guarantee contract as follows: $\Delta_2^l = (k_S^l - k_L^l)/(1-\alpha_L^l) - (\min(\bar{k}_S, \bar{k}_M) - \bar{k}_L)/(1-\bar{\alpha}) \geq 0$. A sufficient condition for Δ_2^l to be negative can be derived by comparing $\bar{k}_S$ to $\bar{k}_M$. If the former is less than the latter, we can show that $\Delta_2^l \leq 0$ because $\alpha_L^l \leq \bar{\alpha}_L$, and $k_S^l - k_L^l \leq k_S^h - k_L^h$.

—Region (2b, 2b): Note that h-type S_L's profit is zero under both NG and G, whereas l-type S_L's profit is zero and strictly positive under NG, and G, respectively. From Proposition 2, S_M's profit under NG is equal to $\min(\bar{k}_S, \bar{k}_L) - \bar{k}_M$ ($= \bar{k}_L - \bar{k}_M$ because $\alpha_M \geq \underline{\gamma}_M$). From Proposition 3, her profit under G is equal to $r^h(k_L^h - k_M^h)$. Comparing $\bar{k}_L - \bar{k}_M$ with $r^h(k_L^h - k_M^h)$, we can show that S_M is better off with the guarantee contract if and only $\alpha_L^l \leq \gamma(\alpha_M)$ as follows: $\bar{k}_L - \bar{k}_M = r^h(k_L^h - k_M^h) + r^l(k_L^l - k_M^l) \leq r^h(k_L^h - k_M^h)$ if and only if $k_L^l \leq k_M^l \Leftrightarrow \alpha_L^l \leq \gamma(\alpha_M)$. Finally, from Proposition 2, the buyer's cost is equal to $\min(\bar{k}_L, \bar{k}_S)$. However, from Proposition 3, the calculation of the buyer's expected cost under G requires double integration (expectation) with respect to $F_M(p_M)$ and $F_L(p_L)$ in the ranges of $[\underline{p}_M, \bar{p}_M]$, and $[\underline{p}_L, \bar{p}_L]$, respectively. Therefore, we only provide the condition (denoted by Δ^B) under which the buyer is better off with the guarantee as follows: $\Delta^B = \min(\bar{k}_L, \bar{k}_S) - (E\bar{k}_M(p_M^*) - r^l E[k_M^l(p_M^*) - p_G^{l*}]^+) \geq 0$.

—Region (2b, 2c): Similar to Region (2b, 2b), h-type S_L does not get any order allocation under either NG or G. Also, l-type S_L does not get any allocation under NG; however, a guarantee contract enables her to signal her type and get order allocation from the buyer. This implies that the l type strictly increases her profit with a guarantee. From Proposition 2, S_M's profit under NG is equal to $\min(\bar{k}_S, \bar{k}_L) - \bar{k}_M$ ($= \bar{k}_L - \bar{k}_M$ because $\alpha_M \geq \underline{\gamma}_M$). From Proposition 3, her profit under G is equal to $r^h(k_L^h - k_M^h)$. Comparing $\bar{k}_L - \bar{k}_M$ with $r^h(k_L^h - k_M^h)$, we can show that S_M is better off with the guarantee contract if and only $\alpha_L^l \leq \gamma(\alpha_M)$ as follows: $\bar{k}_L - \bar{k}_M = r^h(k_L^h - k_M^h) + r^l(k_L^l - k_M^l) \leq r^h(k_L^h - k_M^h)$ if and only if $k_L^l \leq k_M^l \Leftrightarrow \alpha_L^l \leq \gamma(\alpha_M)$. This implies that in the lower portion of Region (2b, 2c) (i.e., where $\alpha_L^l \leq \gamma(\alpha_M)$), S_M is better off with the guarantee contract, whereas in the upper portion of Region (2b, 2c) (i.e., where $\alpha_L^l \geq \gamma(\alpha_M)$), she is worse off. Finally, from Proposition 2, the buyer's cost is equal to $\min(\bar{k}_L, \bar{k}_S)$ $(\leq \bar{k}_L = r^h k_L^h + r^l k_L^l)$. However, from Proposition 3, the buyer's cost under G is equal to $r^h k_L^h + r^l k_S^l$. Note that because $\alpha_M \geq \underline{\gamma}_M$, $k_L^l \leq k_S^l$, which implies that the buyer's cost increases under guarantee contract; i.e., the buyer prefers *NG*.

—Region (2b, 2d): Under both *NG* and *G*, both types of supplier S_L do not get any allocation, implying that their profits are unaffected by the guarantee contract. Supplier S_M's profits under *NG* and *G* are $\bar{k}_L - \bar{k}_M = r^l(k_L^l - k_M^l) + r^h(k_L^h - k_M^h)$, and $k_L^l - k_M^l$, respectively. Note that because $k_L^l - k_M^l \geq k_L^h - k_M^h$, we can show that $\bar{k}_L - \bar{k}_M \geq k_L^l - k_M^l$ (i.e., S_M prefers *NG*). Finally, from the buyer's perspective, the above argument can be used in the opposite direction to show that the buyer benefits from the guarantee contract.

Finally, we briefly analyze the impact of P&Q guarantees on total supply chain cost. Let us first define the total supply chain cost: $E_{\epsilon_L^\theta, \epsilon_M}\{c_L \min(q_L, \epsilon_L^\theta) + c_M \min(q_M, \epsilon_M) + p_S(\epsilon_L^\theta, \epsilon_M)[Q - \min(q_L, \epsilon_L^\theta) - \min(q_M, \epsilon_M)]\}$, where the first and second terms account for the marginal costs incurred by supplier S_L and supplier S_M, respectively, and the third term is for the spot market cost. Note that the total supply chain cost is only determined by the order allocation decision of the buyer. This implies that total supply chain cost decreases with G if and only if the allocation strategy associated with G assigns the order to a less costly source than the one associated with *NG*. Because of space constraints, we only analyze Region (2a, 2b) when $\underline{\gamma}_M < \alpha_M \leq \bar{\gamma}_M$. Under *NG*, the order is always assigned to $\bar{S}_L$, whereas under G, it is assigned to supplier S_M if θ is h type; otherwise, it is assigned to S_M or S_L, depending on whether $k_M^l(p_M^*) \leq p_G^{l*}$ or not. Even though the guarantee contract decreases the total cost when $\theta = h$ by switching the order from h-type S_L to S_M, it also increases the total cost when $\theta = l$ by assigning the order to S_M if $k_M^l(p_M^*) \leq p_G^{l*}$. Therefore, the net impact of the guarantee contract on the total cost depends on the comparison between the two. This leads to the following condition. The total supply chain cost is reduced by the guarantee contract if and only of $\Delta^{TC} = r^h(k_L^h - k_M^h) - r^l E_{p_M^*, p_G^{l*}}[I(k_M^l(p_M^*) \geq p_G^{l*})(k_M^l - k_L^l)] \geq 0$. Note that the analysis for other regions can be done in a similar fashion (details are available from the authors upon request). ☐

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