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Shipping Fees or Shipping Free? A Tale of Two Price Partitioning Strategies in Online Retailing

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In this paper, we study the price partitioning decisions of online retailers regarding shipping and handling (S&H) fees. Specifically, we analyze two partitioning formats used by retailers in this context. In the first scenario, retailers present customers with a price that is partitioned into a product price and a separate S&H surcharge (the *PS strategy*); in the second, customers are offered *free shipping* through a non-partitioned format where the product price already includes the shipping cost (the *ZS strategy*). We first develop a stylized *game-theoretic model* that captures the competitive dynamics between (and within) these two formats. Analysis of the model provides insights into how both firm and product level characteristics drive a retailer's strategic choice regarding which partitioning format to adopt, and, hence, determines the equilibrium market structure in terms of proportion of ZS and PS retailers. Subsequently, we conduct *empirical analyses*, based on product and S&H prices data for two different product categories (digital cameras and printers) collected from online retailers, to validate all the results of our theoretical model. We establish that PS retailers charge lower product prices than ZS ones, but the total price (product + S&H) charged is higher for the first group. The S&H charge for PS retailers can be significant - it is, on average, 5.4% (printers) and 3.0% (digital cameras) for our two product categories. Furthermore, retailers which are popular and/or face risky cost environment are more likely to opt for the ZS strategy, while retailers whose portfolio mostly includes large or heavy products with high cost(S&H)-to-price ratios usually choose the PS strategy. Lastly, our empirical study also illustrates that the price adjustment behavior of retailers is affected by their shipping-fee policies - for example, ZS retailers change their product prices almost 1.5 times more frequently than PS ones.

Key words: Retail, E-commerce pricing strategy, Price partitioning, Free shipping, Shipping & handling costs.

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1. Motivation

Nowadays, electronic transactions constitute a substantial portion of retail activities. Online retail sales in the US reached about $35 billion during the second quarter of 2008. This is an increase of almost 9.5% compared to the second quarter of 2007; during the same period, total US retail sales only increased 2.5% (US Census Bureau News 2008). One of the salient characteristics that differentiates *online* retailers from their *offline* counterparts (for physical products) is that, with the former, products are delivered to customers by the retailer, while, in the latter, customers usually visit the retailer to buy the product. This means that the *shipping and handling (S&H) cost*[1] is an important consideration for online retailers. This additional responsibility compels online retailers to tackle a new set of decisions. They need to choose whether to partition the total price charged to customers into a product price and a S&H surcharge (the PS strategy) or to offer them *free shipping* by charging only the product price and include delivery-related costs - either partly or wholly - in that price (the non-partitioned ZS strategy).[2] Obviously, retailers need to decide how much to charge for the product and, if applicable, for S&H. The primary goal of this paper is to study how the firm and the product characteristics drive the *strategic* price partitioning decisions of online retailers. We also investigate how the price adjustment behaviors of such retailers differ based on their strategic choices.

In order to address the above issues, it is important to understand that E-commerce related S&H *costs* can indeed be substantial. Such expenses can account for more than 30% of the total cost in a number of sectors such as groceries and toys (Barsh et al. 2000). While it is true that efficient order fulfillment through better supply chain network design and inventory management has reduced S&H costs for online retailers (refer to Johnson and Whang 2002; Netessine and Rudi 2006; and references therein), such costs can still be quite high. In fact, Amazon.com's delivery-related operations incurred losses of $630M in 2008 and $849M in 2009 because of the costs associated with free shipping offers (Amazon.com Annual Report 2009, page 27).[3] In terms of absolute value, per order costs for S&H are about $15 for prescription drugs and $26 for toys (Lewis et al. 2006). There are practitioner experts from the online retail industry who suggest that the lost income due to not charging for S&H costs might be as large as 35-40% of the operating contribution per order for some retailers (Brown 2008, Bolotsky 2008 and references therein). So, in order to recoup such costs, retailers might need to add a surcharge for S&H on top of the product price.

[1] Handling includes activities such as picking, packing and customer service. Also, note that we use the terms S&H and delivery interchangeably throughout the paper.

[2] Note that any reference to the product price in this paper excludes applicable taxes.

[3] Amazon.com's gross profits for its overall operations were $5531M and $4270M in 2009 and 2008, respectively (Amazon.com Annual Report 2009, page 26).

However, any S&H *surcharge* could have a significant *negative* effect on the purchase decisions of consumers. Studies have repeatedly shown S&H fees to be one of the consumers' main complaints about online retailing. In annual surveys by the e-tailing group, consumers consistently respond that the number one reason for them not buying more products online is the high cost of S&H, and this trend is increasing over the years (Freedman 2008). In a comscore survey from 2008, 72% of those surveyed responded that if an e-commerce site starts charging S&H fees, they would use another site that offered free shipping (Kawamoto 2008). Similar sentiments have been expressed in other surveys by Paypal and Forrester.[4] There is a segment of customers who view S&H surcharges as sneaky and unfair strategy used by a retailer to make additional profits. This group is referred to as *shipping-charge skeptics* (Schindler et al. 2005). Consequently, these consumers are very reticent when it comes to paying any such charges. Indeed, research suggests that online consumers are more sensitive to S&H surcharges than other additional charges such as taxes (Smith and Brynjolfsson 2001). The choice between PS and ZS strategies is especially relevant in this context. The literature has established that an end consumer's mental cost of moving from zero to a positive price for any service/product is more than an "equal" change in the positive price range. For example, a decrease in price from a non-zero value to zero, say from \$1 to 0, increases demand more than the same decrease in the positive price range, say from \$2 to \$1. This is termed the *zero price effect* (Shampanier et al. 2007)[5]. We would also like to refer the readers to Anderson (2009) for real-life examples which show that free offers generate much more demand than even a very low price.

When deciding on their optimal pricing strategies, retailers, then, must carefully trade-off the negative impact of delivery-related expenses on their total costs against the positive effect of not charging for S&H on demand (and hence, revenue). This results in online retailers resorting to the two distinct pricing formats explained above - the "partitioned pricing" format or the *PS strategy* and the "non-partitioned pricing" format or the *ZS strategy.*

1.1 Research Questions and Main Results

Motivated by the issues discussed above, this paper addresses the following research questions:

1. How do online retailers decide whether to add an extra surcharge for S&H services or offer free shipping (i.e., adopt PS or ZS strategy, respectively) in a competitive environment? What equilibrium market structure, in terms of proportion of PS and ZS retailers, such strategic interactions will result in for a particular setting?

[4] Paypal's July 2009 survey showed that 46% of online consumers abandoned their purchases due to high S&H costs (http://blog.searchenginewatch.com/090706-160859). According to Forrester Research, 75% of the consumers prefer free shipping retailers and 58% claim that S&H prices deter them from shopping online (Mulpuru 2008).

[5] According to Shampanier at al. (2007), when Amazon introduced free shipping in some European countries, the price in France mistakenly was reduced not to zero but to one French franc (about 10 cents). Whereas the number of orders increased dramatically in the countries with free shipping, not much change occurred in France. When Amazon.fr rectified the mistake and offered free shipping, orders in France also jumped significantly.

2. How do the particular characteristics of a retailer, or the characteristics of the products being sold by the retailer, affect the retailer's strategic decision on price partitioning?

3. How do the average retail price and the price adjustment behavior (over time) for PS firms compare to those of ZS firms?

In spite of the significant importance of S&H in online retailing, the extant literature is silent about the above issues. The primary motivation of this research is to address this gap. We use *theoretical and empirical* approaches to answer the research questions. We first construct a demand model for an oligopoly framework that incorporates how consumer purchases are affected by partitioning decisions as well as actual prices. The resulting profit functions for ZS and PS retailers capture the basic trade-off that retailers need to consider when deciding on optimal price-partitioning strategies. Subsequently, we utilize a two-stage game-theoretic approach, where retailers first decide whether or not to offer free shipping, and then decide what prices to charge. This helps us to determine the competitive *equilibrium partitioning strategy* that will be adopted by each retailer, and, hence, the equilibrium proportion of ZS and PS retailers in the online market.

We then theoretically analyze how both product and retailer characteristics shape the optimal partitioning strategy. We first assume all retailers to be symmetric to delineate the role of product characteristics, and then study a duopoly setting with asymmetric-retailers to examine the role of the firms' characteristics. We show that retailers adopting the ZS strategy charge more in terms of unit price, but less in total, than those adopting the PS strategy. Moreover, as the number of shipping-charge-skeptics in the market decreases, the product and/or its S&H gets more costly or when the future cost environment is expected to be stable, it is advantageous for most retailers in the market to implement the PS strategy. On the other hand, as an online retailer becomes more popular or if it is mostly selling relatively light/small products, adopting the ZS strategy is its optimal choice.

Finally, we conduct an empirical study to validate our theoretical findings. The data for the empirical analysis consists of actual product and S&H prices collected from a large number of online retailers for two product categories - digital cameras and printers. The results suggest that a unit price (i.e., price without S&H fees) charged by a ZS retailer is indeed higher than a unit price charged by a PS retailer. The reverse is true when total prices (i.e., prices with S&H fees) are considered. Furthermore, our results indicate that the volume of a product as well as the total cost of a product is negatively associated with the proportion of ZS retailers in the market. On the other hand, we note that the variance in cost at the retailer-product level, and the extent of a retailer's popularity both have a significant positive association with a retailer's propensity to offer free shipping. Our empirical study also demonstrates that the ZS retailers change their prices more frequently than PS ones, albeit the magnitude of the changes is not significantly different between the two groups of retailers. In summary, our analysis sheds light on the status of different S&H fee-related partitioning strategies adopted by online retailers, as well as the underlying reasons behind the differences in adoption level of such strategies.

2. Review of Related Literature

Although the specific research questions of this paper have not been addressed before, there are certain streams of existing literature that are relevant to our research. The first one deals with the effects of partitioning strategies on purchasing decisions of individual customers. To a rational customer, partitioning should not matter, because their purchase decisions ought to be based on total price, irrespective of how it is apportioned. However, behaviorial marketing literature shows that customers do not weigh product price and surcharges equally. Several studies (e.g., Morwitz et al. 1998, Cheema 2008, and references therein) suggest that with partitioned pricing customers tend to more heavily anchor on the product price and overlook the small surcharge. Consequently, customers are likely to underestimate the total price, resulting in a relatively more positive effect on demand. However, there are also studies that contrast this viewpoint (e.g., Thaler 1985; Schindler et al. 2005). They argue that partitioning sometimes induces customers to pay attention, in addition to the product price, to the surcharged secondary attributes which they would have otherwise ignored. Consequently, partitioned pricing can result in reduced demand compared to a non-partitioned format. We use the insights from this literature stream in developing our demand framework. However, while prior research focuses on the effects of partitioning on individual customer behavior, we investigate the equilibrium partitioning strategy adopted at the firm level in a competitive market setting.

The second stream of related literature consists of papers dealing with supply chain issues in the E-commerce world. Its focus has been on using the differing level of price and time sensitivities of online customers to optimally differentiate the available price and delivery choice options (e.g., Zhao et al. 2008), or using customized delivery strategies for online customers to improve inventory management (e.g., Cattani and Souza 2002), or better order fulfillment through effective inventory management and network design (e.g., Netessine and Rudi 2006; Johnson and Meller 2002; Pyke et al. 2001). Johnson and Whang (2002) provides a detailed review of this stream. While S&H fees are discussed in some of the above papers, none of them deals with the issue of price-partitioning. The paper from this stream most relevant to us is Leng and Parlar (2005) that analyzes the issue of free shipping - specifically, what should be the optimal threshold for (total) purchase order value above which free shipping should be offered by an online retailer. Subsequently, Becerril-Arreola et al. (2009) extends Leng and Parlar (2005) to also include retail prices and inventories as decision variables. Clearly, our objective, to determine the equilibrium proportion of free-shipping (ZS) and non-free-shipping (PS) retailers in an oligopolistic framework, is quite distinct. Moreover, most of the above papers are analytical ones, and do not empirically validate their results (only Becerril-Arreola et al. (2009) validate some of their results using data from Lewis et al. (2006)).

There are also a few empirical papers concerning S&H charges that are related to our research. Lewis et al. (2006) uses data from an online retailer specializing in grocery and drugstore items to show that S&H charges, and especially free shipping offers, significantly affect frequency and sizes of customer orders. On the other hand, Lewis (2004) compares the effects of loyalty programs with other marketing instruments, including S&H fees, on customer retention. This paper suggests that loyalty programs might be more effective than other marketing schemes. Clay et al. (2002) studied pricing strategies of online book retailers and showed that some retailers offer low product prices with high S&H charges while others offer high prices and low shipping fees. Note that the above papers focus on retail level strategy (and how it impacts customer behavior) and not on the equilibrium market structure, as we do in this paper. Moreover, in contrast to us, they do not use analytical modeling to develop insights which are then validated by an empirical study.

Lastly, there is a mature information system literature stream pertaining to the temporal pricing behavior of online retailers. The existing literature along this line has empirically examined a variety of issues, including the degree of price dispersion between online and offline retailers (Bakos et al. 2005) as well as among online retailers (Clemons et al. 2002) and the frequency and magnitude of price adjustments (Oh and Lucas 2006). Despite the divergence in terms of their scope and context, the consensus among these studies is that electronic markets allow sellers to promptly adjust their pricing tactics according to fluctuating market conditions. Although the present study also explores pricing behavior of online retailers, our focus is quite unique. We study how such behavior of online retailers is driven by their price partitioning policies, based on the differences in price change patterns between PS and ZS retailers.

The rest of the paper is organized as follows. In §3, we develop our game-theoretic model framework, and present both product- and retailer-based analyses. In §4, we empirically validate our theoretical results, and also discuss the temporal pricing behavior of ZS and PS retailers. §5 focuses on the managerial implications of our results. In §6, we present our concluding remarks.

3. Theoretical Model Formulation and Analysis

In this section we develop an analytical model to understand how online retailers decide whether to offer free shipping or not (i.e., opt for the ZS or the PS strategy) in a competitive environment, and what factors drive their choices.

The basic model framework involves $N(\geq 2)$ retailers selling a particular product online, and engaging in a (Nash) price competition among themselves for the potential consumer base. In our model, each retailer makes two decisions. In the first stage, each of them simultaneously decides on whether it should adopt the PS strategy or the ZS strategy. Subsequently, in the second stage, each retailer, given its partitioning decision, simultaneously determines the exact values of the product price and S&H fee (if applicable) that consumers need to pay. We assume that ZS retailers offer free shipping regardless of the end consumer's purchase value. In reality, such offers sometimes require that the value exceed a threshold (e.g., for Amazon.com it is $25). Our framework is an approximation of reality - the PS strategy is equivalent to the threshold value being very high, and the ZS strategy is equivalent to the threshold being zero. Moreover, for analytical purposes, we assume that the model horizon is relatively short so that any transient effects (e.g., customer loyalty dynamics, customer learning) do not significantly impact the retail decisions of our interest.

Before analyzing the above game, we need to develop the demand and profit functions for the retailers. Let N_{zs} and N_{ps} be the number of retailers that adopt ZS and PS partitioning strategies, respectively, in the first stage ($N_{zs} + N_{ps} = N$). We denote this decision for retailer i by $f_i \in \{ZS, PS\}$. Moreover, suppose p_i and s_i are the product and S&H prices charged by retailer i in the second stage. If retailer i opts for ZS, i.e., $f_i = ZS$, then it does not charge anything for delivery, i.e., $s_i = 0$. On the other hand, if retailer i opts for PS, i.e. $f_i = PS$, then we assume that $s_i = s * p_i$, where s ($s < 1$) is the (exogenous) percentage of the unit product price levied as S&H surcharge.

Demand and Profit Functions: Let $0 < \gamma_i < 1$ be the relative popularity of retailer i. A higher value of γ_i compared to γ_j signifies that, ceteris paribus, more customers will prefer retailer i over retailer j. One of the primary reasons behind this asymmetry in attractiveness is brand equity.[6] For example, Amazon.com has a higher brand equity, and hence a higher γ_i, compared to most other online retailers. The retailers procure the product by paying a price $c_p(\gamma_i)$ per unit, and incurs a cost $c_s(\gamma_i)$ per unit for S&H (total cost $c(\gamma_i) = c_p(\gamma_i) + c_s(\gamma_i)$).[7] We assume that a retailer with higher popularity pays less for buying the product and/or as S&H cost (due to economies of scale and/or scope). Specifically, suppose $c(\gamma_i)$ decreases in γ_i.

In order to capture the effects of both partitioning and pricing decisions in the demand model, we use results from the behaviorial marketing literature. As far as the *effect of partitioning decision* is concerned, Schindler et al. (2005) and Lewis et al. (2006) have established that that there is a group of consumers who are very sensitive to any S&H charges (*shipping-charge skeptics*),

[6] Ideally, attractiveness factor γ_i should be a function of product and shipping prices (as well as other tangibles and intangibles) in the long run. Since we focus on the short term, we assume γ_i to be exogenous.

[7] Suppose that there is only one option available for delivering to end consumers (e.g., regular delivery).

while others are willing to pay a "reasonable" amount for delivery services (*non-shipping-charge skeptics*).[8] Moreover, they suggest that the size of this skeptic group depends on the physical characteristics of the product. Specifically, ceteris paribus, customers would be more willing to pay S&H charges for larger and/or heavier products, implying that for such products the skeptic group would be relatively small. Suppose w represents the physical characteristics of a product where higher w implies heavier and/or larger products. We assume that online customers consist of two groups: $0 < \alpha(w) < 1$ proportion of them are shipping-charge-skeptics and the remaining (i.e., $1 - \alpha(w)$) portion is non-shipping-charge-skeptics. Obviously, based on our above discussion, $\alpha(w)$ is decreasing in w.

As far as the aggregate demand functions from the above two groups are concerned[9], they depend on the total prices (i.e., product price + S&H charge, if any) of all retailers in the market. We model the *price effect* via an exponential functional form (e.g., like in Jeuland and Shugan 1988), where the demand for a ZS retailer (resp., PS retailer) is decreasing in its own total price, and increasing in the total prices of its competitors, both ZS and PS. Because of the heterogeneous characteristics inherent in the two groups, we assume that their demand sensitivities towards any change in S&H charges are different - specifically, skeptics are more sensitive than the other group. We represent this in our model by parameters τ_{nsc} ($0 < \tau_{nsc} \leq 1$) and τ_{sc} (≥ 1), which represent the relative weight that non-skeptics and skeptics place on any change in S&H charges, respectively. We also assume that customers in the skeptics group are not willing to buy the product from any PS retailer; rather, they only check free-shipping ZS retailers and decide whether or not to buy from a retailer in that set. The remaining group, who is willing to pay S&H surcharges, considers all retailers as potential purchase sources. Moreover, the customer segment who considers only ZS retailers (resp., both ZS and PS retailers) is divided among N_{zs} retailers (resp., $N_{zs} + N_{ps} = N$ retailers) based on the relative popularity levels of the retailers, i.e., γ_i. Note that our model captures the fact that if the total price charged by a ZS retailer (resp., PS retailer) is too high, then not only do some of the customers move to competing ZS retailers (resp., PS retailers), but some others might indeed defect to PS retailers (resp., ZS retailers) also. Obviously, some others might decide not to buy the product at all.[10] Therefore, in our framework, the overall demand function for an arbitrary retailer

[8] Indeed, as we indicated earlier, Kawamoto (2008) refers to a comscore survey where a large segment of customers indicated that if an online retailer starts charging S&H fee, then they would opt for another "free shipping" retailer.

[9] Our demand functions are aggregate ones. Although they capture consumer preferences, they are not based directly on individual consumer choice models.

[10] An alternative formulation of the model might be to assume that both groups of customers consider all firms, ZS and PS, and the two groups are differentiated only by their sensitivities to S&H charge, i.e., τ_{nsc} and τ_{sc}. We discuss such a model in our concluding remarks (§6) and analyze it in details in (online) Appendix A.2. It turns out that almost all our qualitative theoretical insights hold true even for that model, although not all them can then be proved analytically.

i is as shown below (recall that f_i is the strategic choice for retailer i):[11]

$$d^i_{N_{zs},N_{ps}}(p_i,p_{-i}|f_i=\text{ZS}) = \overbrace{\left(\frac{\gamma_i}{\sum_{i\in N_{zs}}\gamma_i}\alpha(w)\exp(-up_i+\sum_{j\in N_{zs}\backslash\{i\}}vp_j+\sum_{j\in N_{ps}}vp_j(1+\tau_{sc}s))\right)}^{\text{Demand from shipping-charge-skeptics}}$$
$$+\overbrace{\left(\frac{\gamma_i}{\sum_{i\in N}\gamma_i}(1-\alpha(w))\exp(-up_i+\sum_{j\in N_{zs}\backslash\{i\}}vp_j+\sum_{j\in N_{ps}}vp_j(1+\tau_{nsc}s))\right)}^{\text{Demand from non-shipping-charge-skeptics}} \quad (1)$$

$$d^i_{N_{zs},N_{ps}}(p_i,p_{-i}|f_i=\text{PS}) = \overbrace{\frac{\gamma_i}{\sum_{i\in N}\gamma_i}(1-\alpha(w))\exp(-up_i(1+\tau_{nsc}s)+\sum_{j\in N_{ps}\backslash\{i\}}vp_j(1+\tau_{nsc}s)+\sum_{j\in N_{zs}}vp_j)}^{\text{Demand from non-shipping-charge-skeptics}}, \quad (2)$$

where u and v represent the direct and cross effects of prices on demands for each type of retailer ($u>v$). We can then develop the profit functions for the two types of retailers. The profit for retailer i in the ZS set is given by (recall that ZS ones offer free shipping):

$$\pi^i_{N_{zs},N_{ps}}(p_i,p_{-i}|f_i=\text{ZS}) = d^i_{N_{zs},N_{ps}}(p_i,p_{-i}|f_i=\text{ZS})(p_i-c(\gamma_i)), \quad (3)$$

while that for each PS retailer can be represented by

$$\pi^i_{N_{zs},N_{ps}}(p_i,p_{-i}|f_i=\text{PS}) = d^i_{N_{zs},N_{ps}}(p_i,p_{-i}|f_i=\text{PS})((1+s)p_i-c(\gamma_i)). \quad (4)$$

The above profit functions capture the basic trade-off of our interest indicated in the previous section. Free shipping by ZS retailers results in their loss of S&H surcharge revenues, but gain in demand since they are able to attract both shipping-charge skeptics and non-skeptics. In contrast, PS retailers receive extra S&H revenues, but lose out on demand from shipping-charge-skeptics.

3.1 Competitive (Market) Equilibrium Partitioning Strategy

The analysis in this section will provide insights as to how product characteristics affect the equilibrium proportion of PS and ZS retailers in the market. Without loss of generality, for the rest of the paper we assume $\sum_{i\in N}\gamma_i=1$. In order to focus on the product level analysis, for the time being, suppose that the retailers are symmetric in terms of their popularity, i.e. $\gamma_i=\frac{1}{N}\forall i$ (refer to §3.2 for asymmetric analysis). This means that their (unit) product, S&H and total costs are also equal (i.e., $c_p(\gamma_i)=c_p$, $c_s(\gamma_i)=c_s$ and $c(\gamma_i)=c$, $\forall i$). Recall that the retailers compete by first

[11] In order to see how a price change affects demands in our framework, note the following example. If PS retailer i increases its product price (and hence its S&H charge), then such an action decreases i's demand from the non-skeptic group. Some of these non-skeptics move to other PS retailers and also to ZS retailers. The price increase also results in more shipping-charge-skeptic customers who consider only ZS retailers ($d^i_{N_{zs},N_{ps}}(p_i,p_{-i}|f_i)$ is affected by a price increase in a PS retailer through both skeptics and non-skeptics). Evidently, the effects of the price increase on the skeptic and non-skeptic groups are different because of how they weigh any S&H charge.

choosing their partitioning strategies, i.e., f_i, and then by setting their prices, i.e., p_i. We solve the game by backward induction starting from the last stage. The following proposition characterizes the equilibrium strategy for each retailer. Proofs for all propositions and corollaries are provided in §A.1 of the (online) Appendix.

PROPOSITION 1. *There is a unique pure strategy Nash equilibrium to the game. Specifically:*

- *Suppose that the equilibrium proportion of retailers who offer free shipping to customers, i.e., adopt the ZS strategy, is given by β^* (so $1-\beta^*$ proportion adopts the PS strategy). Then, $\beta^* = \frac{N^*_{zs}}{N} = \min(\hat{\beta}, 1)$, where $\hat{\beta}$ is the unique solution to the following equation:*

$$\hat{\beta}\exp\left(\hat{\beta}vNs(\frac{c}{1+s}+\frac{u}{1+\tau_{nsc}})(\tau_{sc}-\tau_{nsc})\right)=\frac{\alpha(w)}{1-\alpha(w)}\frac{1+\tau_{nsc}s}{1+s}\frac{\exp(vNs(\frac{c}{1+s}+\frac{u}{1+\tau_{nsc}})(\tau_{sc}-\tau_{nsc}))}{\exp\left((u+v)\left[\frac{cs(1-\tau_{nsc})}{1+s}\right]\right)-\frac{1+\tau_{nsc}s}{1+s}} \tag{5}$$

- *Let p^*_{zs} be the equilibrium price charged by a retailer of the set who opts for the ZS strategy in the first stage, i.e. $f^*_i = ZS$. Then, $p^*_{zs} = c + \frac{1}{u}$ per unit ($s^*_{zs} = 0$).*
- *Let p^*_{ps} and s^*_{ps} be the equilibrium product and S&H prices, respectively, charged by a retailer of the set who opts for the PS strategy in the first stage, i.e., $f^*_i = PS$. Then, $p^*_{ps} = \frac{c}{1+s} + \frac{1}{u(1+\tau_{nsc}s)}$ per unit and $s^*_{ps} = s * p^*_{ps} = \frac{cs}{1+s} + \frac{s}{u(1+\tau_{nsc}s)}$ per unit.*

The above proposition shows the equilibrium proportion of PS and ZS retailers, and the corresponding equilibrium prices, for online markets. Moreover, we can show that $\beta^* > 0$, i.e., there will always be some ZS retailers in the market. Note that Harrington and Leahey (2007) analyze an oligopoly setting similar to ours in the presence of search cost for S&H charges, and show that in equilibrium *all* retailers will offer free shipping ($\beta^* = 1$). However, this is not the case in reality, where PS and ZS strategies co-exist. In their model, effectively, all customers are assumed to be shipping-charge-skeptics (i.e., $\alpha = 1$). Evidently, as α approaches 1, the above proposition suggests that $\beta^* = 1$, i.e., all retailers would opt for the ZS strategy. Our framework is able to reconcile the disparity between theory and practice - when α is not very high, indeed both ZS and PS retailers would exist in a competitive online market.

Before proceeding further, it is worthwhile to compare the equilibrium prices that customers will end up paying under the two strategies.

COROLLARY 1. *The following are true:*

a) The unit product price is higher for ZS retailers than PS ones, i.e., $p^*_{ps} < p^*_{zs}$.

b) The unit total price is lower for ZS retailers than PS ones, i.e., $p^*_{ps} + s^*_{ps} \geq p^*_{zs}$.

The above corollary demonstrates that, while customers generally assume that they are saving on delivery charges due to free shipping by ZS retailers, they are indeed paying higher product prices when purchasing from such retailers (although total price is still lower). Consequently, ZS retailers are "padding" their product prices to counterbalance, at least in part, the loss of revenues from free shipping offers. Their primary motivation for doing so is to attract the customer segment who is averse to paying any S&H surcharge.

As is evident from Proposition 1, the equilibrium proportion of PS and ZS retailers depends on factors such as the proportion of shipping-charge-skeptics in the market (α), the physical characteristics of the particular product (w) and total cost c. We next study how changes in these parameters affect the equilibrium proportion.

PROPOSITION 2. *The following are true:*

a) β^* increases in α and decreases in w, i.e., the equilibrium proportion of ZS retailers increases (resp., decreases) in the proportion of shipping-charge-skeptics in the market (resp., in the weight and/or volume of the products).

b) Suppose that N is sufficiently large.[12] *Then, the equilibrium proportion of ZS retailers, i.e., β^*, decreases in the total cost c.*

The first part of the above proposition implies that if the product being sold by the retailer is small and/or light (i.e., low w), then a sufficiently large number of customers in the market would be skeptical of S&H surcharges (i.e., high $\alpha(w)$). So, it is optimal for most retailers to offer free shipping to customers (charging for S&H will lead to a substantial demand loss in this case) and vice versa if the product being sold is large and/or heavy. The second part suggests that the higher the product and/or the S&H cost, the more likely it is that most of the retailers would add a S&H surcharge to recoup some of those costs. We can then expect more premium product categories (higher product costs) to have a larger proportion of PS retailers.

What if the product cost is random? While the above proposition demonstrated the effects of product cost c on the optimal partitioning strategy, the assumption there is that online retailers have precise knowledge about the value of c. However, in reality, this is rarely the case. So, one issue of managerial interest is to understand the effects of cost uncertainty facing retailers on their optimal partitioning strategies. For example, uncertainty in S&H costs may arise due to fluctuation in prices charged by delivery companies, while uncertainty of product costs might be due to fluctuating raw material costs.

In order to address the above issue we analyze a framework similar to the one before (so N symmetric retailers). The only difference is that now we assume the total selling horizon is divided into T periods, and the total cost faced by retailer i at the beginning of each period is $c_t^i, t =$

[12] Indeed, large N is a sufficient condition. The result might be true even for small N as shown in the Appendix.

$1,2,\ldots,T$ per unit, where c_t^i are i.i.d. random variables (represented by c with mean μ_c and standard deviation σ_c). At the beginning of the selling horizon (i.e., $t=0$), all retailers simultaneously decide whether each of them would adopt the PS or the ZS strategy. Once they decide on a strategy they cannot change it during the selling horizon. Subsequently, the retailers observe the realized total cost per unit at the beginning of each period, and set their unit product and S&H prices accordingly. All other assumptions remain the same as before. Analysis of this game leads to the following conclusion:[13]

PROPOSITION 3. *The equilibrium proportion of retailers who decide to adopt the ZS strategy is given by* $\beta^*_{ran}=\min(\hat{\beta},1)$, *where* $\hat{\beta}$ *is defined as follows:*

$$\hat{\beta}=\frac{\alpha(w)}{1-\alpha(w)}\frac{E_c\left[\exp((1-\hat{\beta})vN(\frac{c}{1+s}+\frac{1}{u(1+\tau_{nsc})})(\tau_{sc}-\tau_{nsc})s)\right]}{\left[\frac{1+s}{1+\tau_{nsc}s}\frac{E_c\left[\exp(-\frac{uc(1+\tau_{nsc}s)}{1+s})\right]}{E_c[\exp(-uc)]}\frac{E_c\left[\exp(\frac{-vc(1+\tau_{nsc}s)}{1+s})\right]}{E_c[\exp(-vc)]}-1\right]}. \tag{6}$$

The equilibrium product price to be charged by each retailer of ZS and PS sets in period $t, t=1,2,\ldots,T$, *is the same as in Proposition 1 with* c *replaced by* c_t^i, *the actual realization of total cost for retailer* i *in period* t.

In what follows we provide a more detailed understanding of the effects of cost variability on equilibrium proportion by assuming that the total product cost is uniformly distributed between $\mu_c-\sqrt{3}\sigma_c$ and $\mu_c+\sqrt{3}\sigma_c$, i.e., $c\sim U(\mu_c-\sqrt{3}\sigma_c,\mu_c+\sqrt{3}\sigma_c)$, implying that the mean product cost is μ_c and the standard deviation is σ_c. This yields the following result:

PROPOSITION 4. *If* $c\sim U(\mu_c-\sqrt{3}\sigma_c,\mu_c+\sqrt{3}\sigma_c)$, *then the following are true:*

a) The equilibrium proportion of retailers who opt for the ZS strategy is given by $\beta^*_{ran}=\min(\hat{\beta},1)$, *where* $\hat{\beta}$ *is defined as follows:*

$$\hat{\beta}=\frac{\alpha(w)}{1-\alpha(w)}\exp\left(\frac{(1-\hat{\beta})vN(\tau_{sc}-\tau_{nsc})s}{u(1+\tau_{nsc})}\right)\frac{f_1}{\frac{1+s}{1+\tau_{nsc}s}f_2-1}$$

where $f_1=E_c\left[\exp\left(\frac{(1-\hat{\beta})vN(\tau_{sc}-\tau_{nsc})s}{1+s}c\right)\right]$ *and* $f_2=\frac{E_c\left[\exp(-\frac{uc(1+\tau_{nsc}s)}{1+s})\right]}{E_c[\exp(-uc)]}\frac{E_c\left[\exp(\frac{-vc(1+\tau_{nsc}s)}{1+s})\right]}{E_c[\exp(-vc)]}$.

b) Moreover, for a constant μ_c, β^*_{ran} *is increasing in* σ_c. *So, the proportion of ZS retailers increases as the coefficient of variation* ($cov_c=\frac{\sigma_c}{\mu_c}$) *of the total cost becomes larger.*

This proposition proves that the degree of cost uncertainty, as represented by coefficient of variation, affects partitioning strategies of online retailers. The intuition behind the above result is as follows. In our model, firms first decide on whether to adopt ZS or PS strategy and then adjust their prices (unit and S&H, if applicable) by competitively responding to random changes in (unit)

[13] $E_x(A)$ represents expected value of expression A, where the expectation operator is over the random variable x.

product and S&H costs. Note that the uncertainty in cost implies that the equilibrium ZS and PS prices are also uncertain, which, in turn, implies uncertain demand streams for ZS and PS firms. In general, the higher is the degree of cost uncertainty, the higher is the demand risk for online retailers. In that scenario, the extra demand lift provided by a free shipping offer is very desirable since this enables a ZS firm to clear out its stock faster compared to a PS firm (thus reducing the risk), even though the ZS firm loses out on S&H revenue. Obviously, if the uncertainty is relatively low, then the benefit of demand lift in terms of reduction of risk is not significant. In that case, PS strategy becomes more desirable because the surcharge revenue can help firms recoup their S&H costs. Therefore, expectation of a relatively stable cost environment results in higher proportion of PS retailers, and as the possibility of cost uncertainty increases, more and more firms start adopting the ZS strategy to attract more demand.

3.2 Retailer-based Analysis of Partitioning Strategy

In order to focus on the effects of product characteristics on the equilibrium segmentation in the market among PS and ZS firms, the last section assumed that all retailers are symmetric. In this sub-section our goal is to study how asymmetry among the retailers, in terms of their relative popularity, i.e., γ_i, affect their partitioning strategies. For expository reasons, we assume in this sub-section that the market consists of two retailers (duopoly), who are different only in terms of their popularity levels (i.e., they are selling the same product). Specifically, the popularity of retailer $i = 1, 2$ is denoted by $\gamma_i \in [0, 1]$, and $\gamma_1 + \gamma_2 = 1$. This also implies that the product and S&H costs for the two retailers are now different. The unit total cost for retailer i is $c(\gamma_i)$, and $c(\gamma_i)$ decreases in γ_i (the decrease in cost might be due to product and/or shipping costs).[14] All other model details remain the same as before. That is, both retailers first decide on whether to adopt the ZS or the PS strategy, and subsequently compete on prices. We can then develop the demand and profit functions for the two retailers as shown in the (online) Appendix. Based on the profit functions, we then analyze the two-stage game, again using backward induction. Keeping in mind the space constraints, we focus only on the more interesting case where at least one retailer's popularity is large enough to exhibit significant cost benefits. The equilibrium strategy for the two retailers is given in the following proposition:

PROPOSITION 5. *If at least one retailer's popularity is above a threshold popularity level $\bar{\bar{\gamma}}$, i.e., when $\max(\gamma_i, \gamma_j) \geq \bar{\bar{\gamma}}$, where $\bar{\bar{\gamma}}$ is the unique solution to the following equation:*

$$\exp\left(\frac{us(1-\tau_{nsc})}{1+s}c(\bar{\bar{\gamma}})\right) = \frac{1+\tau_{nsc}s}{(1-\alpha(s))(1+s)}$$

[14] As regards product cost, many retailers get price discount for high volumes from suppliers. Since S&H for many retailers are handled by third parties, higher volumes also provide cost benefits in that case (Bolotsky 2008).

then the equilibrium partitioning strategies for the two retailers are as follows:[15]

1. *If $\gamma_i \leq \bar{\bar{\gamma}} \leq \gamma_j$ or $\gamma_j \leq \bar{\bar{\gamma}} \leq \gamma_i$, then the retailer with lower popularity should adopt the PS strategy, and the one with higher popularity should opt for the ZS strategy.*

2. *If both firms' popularities are greater than $\bar{\bar{\gamma}}$, i.e., $\min(\gamma_i, \gamma_j) \geq \bar{\bar{\gamma}}$, then both retailers should adopt the ZS strategy.*

The optimal prices under each strategy are the same as in Proposition 1 (with c replaced by $c(\gamma_i)$).

The above proposition suggests that popular retailers should take advantage of their low S&H costs (due to economies of scale/scope) and offer free shipping, while retailers who are less popular should add S&H surcharges to recoup their (relatively high) delivery costs. Offering free shipping will allow popular retailers to get a further demand lift by attracting shipping-charge-skeptics. This, in turn, will allow them to reduce their costs even further, making free shipping offers even more attractive for them. The above proposition also helps us to understand how the physical characteristics of the product, i.e., w, affects the partitioning strategies of the two asymmetric retailers (the result below is similar in spirit to Proposition 2(a)).

Corollary 2. *For given γ_i and γ_j, as w decreases, it becomes more likely that both retailer i and retailer j would adopt the ZS strategy.*

In summary, our theoretical analysis in this section identifies the strategic rationale as to why some retailers opt for partitioned pricing with S&H surcharges (the PS strategy), while others prefer non-partitioned strategy with free shipping offers (the ZS strategy). Clearly, differences in retailer as well as product characteristics shape the optimal partitioning strategy that results in the varying level of PS-ZS segmentation observed in online retail markets.

4. Empirical Validation

In this section, we empirically validate the insights obtained from game-theoretic perspective in §3. Our empirical analysis also seeks to study whether an online retailer's strategic partitioning decision related to S&H surcharge is associated with its temporal pricing behavior.

4.1 List of Hypotheses

Before proceeding further, in Table 1, we summarize our analytical results of §3 in the form of hypotheses, which will subsequently be validated empirically .

[15] As we show in the Appendix, the condition $\max(\gamma_i, \gamma_j) \geq \bar{\bar{\gamma}}$ is not very restrictive. It will be satisfied as long as w or u or s is relatively low and/or τ_{nsc} is relatively high.

Table 1 Summary of analytical results and hypotheses.

Hypothesis	Description
Hypothesis 1a	The average product price per unit charged by a PS retailer is less than the average product price per unit charged by a ZS retailer. (Corollary 1a)
Hypothesis 1b	The average total price per unit of a PS retailer is higher than the average total price per unit of a ZS retailer. (Corollary 1b)
Hypothesis 2a	The proportion of S&H charge skeptics is positively associated with the proportion of ZS retailers in the market. (Proposition 2)
Hypothesis 2b	The cost of products is negatively associated with the proportion of ZS retailers in the market. (Proposition 2)
Hypothesis 3a	The degree of uncertainty in unit costs is positively associated with the propensity of a retailer to adopt the ZS strategy. (Proposition 4b)
Hypothesis 3b	The propensity of a retailer to adopt the ZS strategy increases as the popularity of the retailer grows. (Proposition 5)

4.2 Description of Empirical Methods Data

Following previous studies (e.g., Smith 2002; Baye et al. 2004), we chose a price comparison site BizRate.com (http://www.bizrate.com/) to gather price and non-price data over a four-month period, from December 1st, 2006 to March 31st, 2007. BizRate is one of the most popular online central exchanges, on which many sellers post their up-to-date prices for numerous product categories. To facilitate data collection, we developed a proprietary software program, that automatically downloads the target HTML pages and parses their content down to extract prices and other relevant information on a daily basis. Automated proprietary software agents such as ours, often called "intelligent bots" or "spiders", are widely used as the main data collection mechanisms in the literature (Baye et al. 2004, Oh and Lucas 2006, Pan et al. 2002). To ensure the accuracy of this program, we tested it for two weeks prior to the actual data collection and manually inspected its reliability. This validation process affirmed that the software is 100% accurate.

Because S&H costs are an important aspect of our study, we chose two product categories (digital cameras and printers) that differ substantially in terms of weights and sizes and, therefore, S&H costs. For each product category, we selected 12 different products randomly from CNET's list (http://www.cnet.com), which contains comprehensive categories of computer-related products. A detailed list of all the 24 products, along with their shipping weights and volumes, is provided later on in Tables 12 and 13. As one can imagine, the printers in the sample are, on average, substantially heavier and larger than the digital cameras. The number of retailers posting their prices varied from product to product; some specialist vendors do not offer an entire range of products, but limit their sales to particular products (e.g., digital cameras). Consistent with Oh and Lucas (2006), New York City (zip code 10012) was used as the default destination city and regular delivery as the default delivery option, to compute the S&H charges. During the two-week pre-sampling period, we used another zip code (78712; Austin, Texas) to determine whether the choice of zip code significantly affects S&H fees. The results show that the calculated S&H charges were almost similar between the two cities and, therefore, only New York City was employed as the target destination during the data-collection period.

Regarding the characteristics of the retailers in our sample, 71% are "pure" online retailers who operate only in online markets and 29% are hybrid retailers who offer products in both online and offline channels. As for the types of products sold, 60% of the retailers in the sample sold a wide range of products, including digital cameras and printers, whereas the remaining 40% are specialized in only one specific product category. Finally, many retailers in our sample are popular retailers with established name brands (e.g., Amazon.com, Dell.com, Buy.com), but many others can be considered small-or medium-sized retailers who have operated for a relatively short period of time.

Variables and Measurements

Tables 2, 3 and 4 describe the definition and operationalization of the variables used in the analysis. First of all, to validate Hypotheses 1a and 1b, UNIT_PRICE and TOTAL_PRICE were used as the dependent variables, respectively, while SHIP_POLICY represented the independent variable (retailer-product-time level analysis; refer to Table 2). We used linear mixed effect models to control for the heterogeneities that could arise from both product and time characteristics. A detailed description of model specifications for all of the empirical analyses will be provided later. Note from Table 2 that, for a given product, TOTAL_PRICE can be computed as the sum of UNIT_PRICE and S&H charges. SHIP_POLICY is a binary variable indicating whether the product is offered by ZS retailers (coded as 1) or PS retailers (coded as 0).

Table 2 **Variables, Descriptions and Measurements (Retailer-Product-Time Level)**

Variable Name	Description / Measurement
UNIT_PRICE (i,j,t)	The price excluding S&H fees of a product j offered by retailer i in time t.
TOTAL_PRICE (i,j,t)	The total price (i.e., price with S&H fees) of a product j offered by retailer i in time t
SHIP_POLICY (i,j,t)	A binary variable indicating whether retailer i offers free shipping (i.e., zero S&H fee) for a product j in time t. Zero S&H charge is coded as 1 and positive S&H charge is coded as 0

To test Hypotheses 2a and 2b, ZS_PROP was employed as the dependent variable, which represents the proportion of retailers in the market who offer free shipping for product j in time t, i.e., β^* (retailer-time level analysis; refer to Table 3). The reason we used a product's average total price as a proxy for cost (PROD_COST) in the analysis is that, at the aggregate level, cost is one of the primary determinants of the price levels set by retailers, especially in a competitive setting like ours, and therefore they are highly correlated (Blinder et al. 1998). Moreover, we used products' volume in cubic inches (PROD_VOL) as proxies to operationalize the proportion of shipping-charge-skeptics in the market. Although further validation is necessary, it is reasonable to assume that there will be more consumers willing to pay S&H fees when they purchase larger products; consequently, products with larger volumes would have less proportion of shipping-charge-skeptics.[16] That is, PROD_VOL represents w, and like in §3, the proportion of skeptics $\alpha(w)$ decreases in w. Note that PROD_COST and PROD_VOL represent the main variables to be tested in the analysis. In addition, we also included a dummy variable (DEC_EFCT) that captures the possible seasonality effect. This variable was included to reflect the fact that many online retailers rely heavily on December sales, and subsequently use different S&H policies during that period.

Table 3 Variables, Descriptions and Measurements (Retailer-Time Level)

Variable Name	Description / Measurement
ZS_PROP (j,t)	The proportion of retailers who offer free shipping for product j in time t. For each product and each day we calculated the proportion of retailers offering free S&H to the total number of retailers offering the product. A total of 2,711 out of 2,904 samples (24 products x 121 days) were collected.
PROD_COST (j,t)	Product j's average total price *across all retailers* selling the product in time t. As explained later on, we use price as a proxy for cost.
PROD_VOL (j)	Product j's actual volume in cubic inches used as a proxy to operationalize consumers' skepticism to pay S&H fees.
DEC_EFCT (t)	A measurement of seasonal effect. December is coded as 1 and the other months are coded as 0.

Hypotheses 3a and 3b were tested at the retailer-product level (refer to Table 4). The dependent variable, FREE_PROPENSITY, measures retailer i's propensity to offer free shipping for a product j. Because some retailers differentiate their S&H strategies over time, we used a continuous scale with 100% representing free shipping for product j during the entire four month period. An index of 75% (resp., 0%) indicates that a product was available with free shipping for 3 (resp., 4) months, but for 1 month the retailer charged S&H fees for that product. Among the independent variables, COST_VAR, indicating the co-efficient of variation of price for a given product sold by a particular retailer, was calculated as indicated in Table 4. The degree of a retailer's popularity, POPULA, was measured as follows. On the first day of each month during the data collection period, we assessed the popularity of each retailer in our sample based on the mechanism used by Linkpopularity.com. This website measures a website's "popularity" by counting the total number of websites that link to it. More specifically, assuming that more popular retailers or websites appear more frequently in search results, Linkpopularity.com queries all three major search engines (Google, Yahoo and MSN) and returns each URL's total link counts. We computed the monthly average index based

[16] Note that our results remain the same even if we use products' weight, rather than volume, as the proxy.

on the average counts reported by the three major search engines. We then log-transformed the average index and used it to gauge the popularity of each retailer in our sample. Lastly, we added two control variables (PROD_VOL and CHANNEL) to the analysis as controls. PROD_VOL is likely to affect a retailer's actual costs associated with S&H, while a retailer's shipping- related decisions are likely to be influenced by its channel strategy (CHANNEL).

Table 4 Variables, Descriptions and Measurements (Retailer-Product Level)

Variable Name	**Description / Measurement**
FREE_PROPENSITY (i, j)	Retailer i's propensity to offer free shipping for a product j across time. It is a continuous variable with the range between 0 and 1; 0 and 1 indicate that retailer i offers free shipping for product j none and all of the time, respectively. A value of 0.5 means that the retailer offers free shipping for half of the time of the product being sold.
POPULA (i)	Retailer i's popularity. Linkpopularity.com (http://www.linkpopularity.com) rates websites' popularity based on search results. We calculated the average value of the search results from three major search engines (Google, Yahoo and MSN) to reflect each retailer's popularity and then performed log-transformation.
COST_VAR (i, j)	Co-efficient of variation of price (across time) for each retailer i-product j combination. It was computed for each such combination by dividing the standard deviation of total price across time by the average price. For example, for retailer 1 - product 1 combination, we computed the standard deviation and average of total prices charged by retailer 1 for product 1 over the data collection period and divided the standard deviation by the average.
PROD_VOL (j)	As defined in Table 1. The retailers' actual costs for shipping would be related to this product characteristics.
CHANNEL (i)	Whether or not retailer i offers only online sales: pure online retailer and hybrid retailer are coded as 0 and 1, respectively.

4.3 Results

Because our data are based on cross-sectional and longitudinal observations, we employed panel data analytics suited to exploring empirical regularities associated with shipping-charge strategies.

Retailer-Product-Time Level Analysis (Hypotheses 1a and 1b)

We used multivariate analyses to validate Hypotheses 1a and 1b (refer to Table 1) regarding the impact of shipping policies on product price. We employed a mixed-effects linear regression to test these hypotheses because both fixed (e.g., product) and random (e.g., time) effects are present in our model. A Durbin-Watson test suggests that an ordinary least squares (OLS) approach is inappropriate in our case since doing so will violate the assumption of independence (Greene 2008). Since product characteristics (e.g., volume or weight) are likely to impact a retailer's shipping strategy, we used fixed effects to control for the product effect. Moreover, the variation across time is assumed to be random and uncorrelated with the independent variable. In addition, prices of digital cameras and printers tend to be time-sensitive as they vary over time. Therefore, we used random effects to control for the time factor. By controlling for these fixed and random effects, we can assess the predictor (shipping strategy)'s net effect on prices. For model estimations, we employed restricted maximum likelihood (REML), which generates unbiased estimates of variance and covariance parameters (Cressie and Lahiri 1993). These statistical procedures address the potential problems of heteroskedasticity and autocorrelation that might arise when analyzing cross-sectional and longitudinal data. We chose linear mixed models instead of general linear models because the former has superior capabilities than the latter for handling correlated data and unequal variances (McCulloch and Searle 2001). STATA/SE 9.2 was used to perform the analyses. We present the linear effect models based on the form recommended by Laird and Ware (1982).

$$\text{UNIT\_PRICE}_{i,j,t} = \beta_0 + \beta_1 SHIP\_POLICY_{i,j,t} + V_t + \varepsilon_{i,j,t} + U_j \tag{7}$$

$$\text{TOTAL\_PRICE}_{i,j,t} = \beta_0 + \beta_1 SHIP\_POLICY_{i,j,t} + V_t + \varepsilon_{i,j,t} + U_j \tag{8}$$

where i, j and t are retailer, product and time indices, respectively, β_0 is the constant, β_1 is the fixed-effect co-efficient, $\text{UNIT\_PRICE}_{i,j,t}$ and $\text{TOTAL\_PRICE}_{i,j,t}$ are the dependent variables, U_j is the product-specific error (unobserved heterogeneity due to products), $SHIP\_POLICY_{i,j,t}$ is the regressor, $V_t \sim N(0, \psi_k^2)$ is the random-effect variable for time t assumed to be multivariately normally distributed, ψ_k^2 are the variances among the random effects assumed to be constant across times, $\varepsilon_{i,j,t} \sim N(0, \sigma^2\lambda_{i,j,t})$ and $\sigma^2\lambda_{i,j,t}$ are the covariances between errors in time t.

Tables 5 and 6 present the results of equations (7) and (8), respectively. The Wald's chi-squared statistic indicates that the model fit is strong ((Prob > chi2) <0.01) and the coefficients in the model are significantly different from zero. The results show that free shipping is *positively* associated with UNIT_PRICE (i.e., price without S&H fees) at the 99% confidence level ($z = 8.8, p < 0.01$), indicating that for a given product a unit price charged by ZS retailers is higher than a unit price charged by PS retailers (i.e., $p^*_{ps} < p^*_{zs}$). Consequently, Hypothesis 1a is supported by our data. We also found strong support for Hypothesis 1b. The analysis reveals that free shipping is *negatively* related to TOTAL_PRICE (i.e., price with S&H fees) ($z = -24.84, p < 0.01$). This result, therefore, validates Hypothesis 1b that for a given product the total price charged by a PS retailer is higher than the total price of a ZS retailer (i.e., $p^*_{ps} + s^*_{ps} \geq p^*_{zs}$).

Retailer-Time Level Analysis (Hypotheses 2a and 2b)

As shown in Table 1, Hypotheses 2a and 2b state that the proportion of ZS retailers in the market is associated with several factors, including proportion of shipping-charge-skeptics (Hypothesis 2a), and the total cost of a product (Hypothesis 2b). Regarding our data structure, we included 24 products (12 digital cameras and 12 printers), each of which resides within a time series (121 days). Therefore, the maximum sample size is 2,904 ($= 24 \times 121$), 193 of which were omitted (6.6% of the total) due to product discontinuation. Consequently, 2,711 samples remained in the analysis. The

Table 5 Mixed-effects REML regression (Retailer-Product-Time Level)

Number of obs = 84091, Number of groups = 24, Group variable: product
Wald chi2(1) = 78.02, Log restricted-likelihood = -496975.99, Prob > chi2 = 0.0000

UNIT_PRICE	Coef.	Std. Err.	z	$P > z$	95% Conf. Interval	
SHIP_POLICY	5.508448	.6236456	8.83	0.000	4.286125	6.730771
CONSTANT	603.8902	83.09398	7.27	0.000	441.029	766.7514

Random-effects Parameters		Estimate	Std. Err.	95% Conf. Interval	
product:Independent	sd(time)	.2470781	.0367285	.1846299	.3306485
	sd(_cons)	407.0624	60.01901	304.8991	543.4577
	sd(Residual)	89.0359	.2171707	88.61127	89.46256

LR test vs. linear regression: chi2(2) = 2.6e+05 , Prob > chi2 = 0.0000

Dependent variable: UNIT_PRICE
Significance Levels: $^{*}p < 0.05$; $^{**}p < 0.01$.

Table 6 Mixed-effects REML regression (Retailer-Product-Time Level)

Number of obs = 84091, Number of groups = 24, Group variable: product
Wald chi2(1) = 617.02, Log restricted-likelihood = -497064.4 , Prob > chi2 = 0.0000

TOTAL_PRICE	Coef.	Std. Err.	z	$P > z$	95% Conf. Interval	
SHIP_POLICY	-15.50747	.6242954	-24.84	0.000	-16.73107	-14.28387
CONSTANT	625.3421	84.27606	7.42	0.000	460.1641	790.5202

Random-effects Parameters		Estimate	Std. Err.	95% Conf. Interval	
product:Independent	sd(time)	.2542482	.0377416	.190065	.3401053
	sd(_cons)	412.8535	60.87275	309.237	551.1889
	sd(Residual)	89.12857	.2173967	88.7035	89.55568

LR test vs. linear regression: chi2(2) = 2.6e+05 , Prob > chi2 = 0.0000

Dependent variable: TOTAL_PRICE
Significance Levels: $^{*}p < 0.05$; $^{**}p < 0.01$.

dependent variable for the analysis indicates the proportion of ZS retailers in the market, for a given product and time. Table 7 shows the descriptive statistics and the correlation coefficients of the variables included in this analysis. The mean of ZS PROP was 0.48 - that is, for a given product and in a given day, approximately 48% of products were offered with free shipping. The average price of the products was $615.67, while the average of PROD_VOL and DEC_EFCT was 5.50 and 0.25, respectively. As for the bi-variate correlation coefficients, ZS_PROP has a significant negative association with PROD_COST and PROD_VOL, while PROD_COST was positively correlated to PROD_VOL.

Similar to the procedures used to test Hypotheses 1a and 1b, we developed a mixed effect model in which the proportion of ZS (ZS_PROP), the dependent variable, is thought to be affected by

Table 7 Summary statistics and correlation coefficients (Retailer-Time Level) ($N = 2,711$)

	Mean	STDEV	1	2	3	4
1. ZS_PROP	.4781003	.1051404	1			
2. PROD_COST	615.6742	411.1025	-0.3060**	1		
3. PROD_VOL	5.496703	2.764333	-0.6851**	0.4024**	1	
4. DEC_EFCT	.2559941	.436499	-0.0470	0.0016	-0.0004	1

** Correlation is significant at the 0.01 level (2-tailed).

* Correlation is significant at the 0.05 level (2-tailed).

both fixed and random effects. More specifically, we assumed that PROD_COST, PROD_VOL and DEC_EFCT all affect the proportion of ZS; therefore, these regressors' effects are thought to be fixed. However, the regression results might also vary depending on how we sampled the products and time periods and therefore we subsequently assumed the presence of random effects on products and time periods. Our regression model is as follows:

$$ZS\_\mathrm{PROP}_{j,t} = \beta_0 + \beta_1 PROD\_COST_{j,t} + \beta_2 PROD\_VOL_{j,t} + \beta_3 DEC\_EFCT_{j,t} + U_i + V_t + \varepsilon_{j,t} \tag{9}$$

where j and t are product and time indices, respectively, β_0 is the constant, β_k, $k = 1,\ldots,3$ are the fixed effect coefficients, U_i is the product-specific error (unobserved heterogeneity due to products), $V_t \sim N(0, \psi_k^2)$ is the time-specific error component, ψ_k^2 are the variances among the random effects assumed to be constant across times, and finally, $\varepsilon_{j,t} \sim N(0, \sigma^2 \lambda_{j,t})$ affects only the particular observation where $\sigma^2 \lambda_{j,t}$ are the covariances between errors in time t. Note that U_i is unobserved heterogeneity specific to products, while V_t is time specific heterogeneity, which is peculiar to all observations for that time period t. The error term, $\varepsilon_{j,t}$, is uncorrelated with the time specific component and the product specific error. We included these terms to control for possible biases arising from unobserved heterogeneities inherent to sample procedures.

Table 8 presents the results of the linear mixed effect model (9) developed at the market level. The model as a whole has statistically significant predictive capability, as reflected in the strong F-Value (Prob > chi2 = 0.0000). This Wald's chi-squared statistic reveals that all the coefficients in the model are different from zero. The two-tail p-values tests indicate that all of the regressors have a significant influence on the dependent variable, ZS_PROP. Consistent with our analytical results, the proportion of shipping-charge-skeptics (recall that higher PROD_VOL implies lower proportion of skeptics) is negatively related to the proportion of ZS retailers in the market, i.e., β^* decreases in w ($z = -5.31, p < 0.01$). Consequently, Hypothesis 2a was strongly supported. Our data also strongly support Hypothesis 2b, showing that products' total costs are negatively related to the proportion of ZS firms in the market, i.e., β^* decreases in c ($z = -2.48, p < 0.05$). This finding suggests that ceteris paribus, low-cost products are less likely to be subject to S&H charges than their high-cost counterparts.

Table 8 Mixed-effects REML regression (Retailer-Time Level)

Number of obs = 2711, Number of groups = 24, Group variable: product						
Wald chi2(5) = 80.64, Log restricted-likelihood = 3998.254, Prob > chi2 = 0.0000						
ZS_PROP	Coef.	Std. Err.	z	$P > z$	95% Conf. Interval	
PROD_COST	-.0000648	.0000262	-2.48	0.013	-.0001161	-.0000135
PROD_VOL	-.021033	.0039608	-5.31	0.000	-.028796	-.01327
DEC_EFCT	-.0189404	.0034452	-5.50	0.000	-.0256928	-.012188
CONSTANT	.6461126	.0241895	26.71	0.000	.598702	.6935232
Random-effects Parameters			Estimate	Std. Err.	95% Conf. Interval	
product:Independent		sd(time)	.0006454	.0000979	.0004795	.0008687
		sd(_cons)	.0487997	.0080176	.0353646	.0673388
		sd(Residual)	.0531127	.0007282	.0517045	.0545592
LR test vs. linear regression: chi2(2) = 1798.14 , Prob > chi2 = 0.0000						

Dependent variable: Proportion of ZS (ZS_PROP).

Retailer-Product Level Analysis (Hypotheses 3a and 3b)

In Table 9, we present the descriptive statistics and the correlation coefficients of the variables included in this analysis. The mean of FREE_PROPENSITY, which indicates the average propensity that a retailer offers free S&H, was 0.49. The average values of COST_VAR, POPULA and PROD_VOL were 0.035, 8.388 and 5.047, respectively. Based on the bivariate correlation analysis, we found that FREE_PROPENSITY is significantly associated with POPULA, PROD_VOL and CHANNEL at the 95% confidence level. Interestingly, POPULA is positively correlated to COST_VAR. These results suggest that the more popular a retailer is, the higher the degree of price variability within a retailer over time will be. To test Hypotheses 3a and 3b (refer to Table 1), we used a fixed effect model (equation (10)) since the data are collected across retailers and products. Given that time is not of our interest in this test, a mixed effect model with random effects was not necessary. The fixed effect eliminates the heteroskedasticity that is present due to the unobserved retailer characteristics.

$$\begin{aligned} FREE\_\text{PROPENSITY}_{i,j} &= \beta_0 + \beta_1 COST\_VAR_{i,j} + \beta_2 POPULA_i + \beta_3 PROD\_VOL_j \\ &+ \beta_4 CHANNEL_i + \sum \text{Retailer}_\text{i} + \varepsilon_{i,j} \end{aligned} \tag{10}$$

where i and j are retailer and product indices, respectively, β_0 is the constant, β_k, $k = 1, \ldots, 4$ are the fixed effect coefficients, $FREE\_\text{PROPENSITY}_{i,j}$ is retailer i's propensity to offer free shipping

Table 9 Summary statistics and correlation coefficients $N = 978$ (Retailer-Product Level)

	Mean	STDEV	1	2	3	4	5
1. FREE_PROPENSITY	.4887423	.4815146	1				
2. COST_VAR	.0346662	.0882884	-0.0129	1			
3. POPULA	8.388421	3.244646	0.1614**	0.1810**	1		
4. PROD_VOL	1871.508 (5.047)	2734.832 (2.758)	-0.1709**	-0.1911**	-0.034	1	
5. CHANNEL	.3137652	.4642572	0.0959**	0.0185	0.2113**	-0.172**	1

** Correlation is significant at the 0.01 level (2-tailed).

* Correlation is significant at the 0.05 level (2-tailed)

The values in parenthesis indicate log transform of actual values.

for a product j across time, $\sum \text{Retailer}_\text{i}$ is for fixed effects that control for unobserved heterogeneity due to retailer characteristics, and $\varepsilon_{i,j} \sim N(0, \sigma^2)$.

Table 10 Regression Results (Retailer-Product Level)

Retail-Product Level Variables	**Model 1** ($N = 978$)	**Model 2** ($N = 978$)
Cons	.2107363* (.1220657)	.2102114* (.1210264)
COST_VAR	.4530033** (.1772018)	.2998289* (.1797218)
POPULA	.0617392*** (.014693)	.0647958*** (.0145875)
PROD_VOL		-.0000114*** (2.82e-06)
CHANNEL	-.6624503*** (.0878865)	-.6718424*** (.0871691)
F-value	59.67***	60.19***
Adjusted R^2	0.8362	0.8390

Dependent Variable: FREE_PROPENSITY

Numbers in parenthesis indicate standard errors

Significance Levels: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Table 10 shows the regression results at the retailer-product level given by equation (10). We present the results of two models (model 1 and model 2), which differ only in terms of the presence of product volume as a control variable. Hypothesis 3a posits that the degree of uncertainty in costs (COST_VAR) is positively associated with the propensity of the retailer to offer free S&H (Table 1). The results of both models in Table 10 statistically support this hypothesis, albeit at different significance levels. Finally, the results show that the popularity of a retailer (POPULA) is positively associated with its propensity to offer free S&H at the 99 % confidence level ($t = 4.20, p < 0.01$). That is, the more popular a retailer is, the greater its likelihood to offer free S&H for a given product. Regarding the two control variables, in model 2 both PROD_VOL and CHANNEL are negatively associated with the dependent variable.

Before proceeding further, we will summarize our analysis to this point. To this effect, the Table 11 provides a snapshot of our theoretical results and their empirical support.

Table 11 Summary of theoretical and empirical results.

Hypothesis	Unit of Analysis	Theoretical Result	Empirical Support
H1a	Retailer-Product-Time	Corollary 1: $p_{ps}^* < p_{zs}^*$	Supported
H1b	Retailer-Product-Time	Corollary 1: $p_{ps}^* + s_{ps}^* \geq p_{zs}^*$	Supported
H2a	Retailer-Time	Proposition 2: Equilibrium proportion of ZS retailers (i.e., β^*) decreases in w	Supported
H2b	Retailer-Time	Proposition 2: β^* decreases in c	Supported
H3a	Retailer-Product	Proposition 4b: β^* increases in cov_c	Supported
H3b	Retailer-Product	Proposition 5: Equilibrium partitioning strategy (i.e., f_i^*) switches from PS to ZS as γ_i increases	Supported

In addition to the above statistical analysis, our study also offers some economic implications related to retailers' S&H charge strategies (refer to Tables 12 and 13). First, based on over 84,000 samples, we found that the total price of a digital camera, charged by PS firms, is (on average) 3.4% higher than the total price charged by their ZS counterparts; for printers, the total price of PS firms is (on average) 4.5% higher than the one charged by ZS firms. Therefore, ZS retailers with free S&H indeed offer consumers something "free" in the form of lower prices, compared to PS retailers who charge such fees. For example, consumers who buy a \$1,000 printer can save \$45, on average, when they use a ZS retailer. If the empirical regularities found in this study prove to be a general phenomenon across all product categories, PS retailers tend to take advantage of these "opaque" charges to improve their revenues substantially.

To gain additional insights into PS retailers' S&H related tactics, we analyzed the extent to which PS retailers charge S&H for the two product types. Interestingly, the results indicate that PS retailers charge a higher percentage of S&H fees in relation to the total price for large/heavy products, such as printers, than they do for small/light products, such as digital cameras. For example, for a digital camera, S&H fees constitute (on average) 3% of the total price of a given product. However, PS retailers charge almost twice as much (on average, 5.4% of the total price) in S&H fees for printers. Note that, in this comparison, we calculate S&H fees as a percentage of the total price of a given product. Furthermore, we found that, compared to digital cameras, a larger variation exists among PS retailers in S&H charge percentages. This percentage ranges from 1.6% to 4.7% for the twelve digital cameras in our sample. However, the variation increases substantially for printers, ranging from 3.5% to 10.1%. What this might imply is that for larger/heavier products (e.g., printers) depending on where a consumer purchases such products, the S&H fees vary significantly, even within PS retailers. In contrast, a relatively moderate “risk” exists when consumers purchase smaller products (e.g., digital cameras) from PS retailers. All these findings indicate that S&H fees have an important economic implication for consumer welfare and price competition.

Table 12 Product Characteristics (Printer)

Printer	**Volume (cu-in)**	**Shipping Weight (lbs)**	**ZS Total ($)**	**PS Total ($)**	**(PS-ZS)/ZS**	**shipping %**
HP LaserJet 4250N Laser Printer	4590	59	1176.82	1203.98	0.023	0.035
HP LaserJet 4250DTN Laser Printer	7140	84	1714.5	1748.38	0.020	0.037
HP LaserJet 1320 Laser Printer	6101	30	381.49	404.01	0.059	0.057
HP LaserJet 4250TN Laser Printer	6120	78	1430.48	1453.07	0.016	0.036
HP LaserJet 4250 Laser Printer	9903	58	876.96	898.73	0.025	0.044
HP DeskJet 460c Inkjet Printer	274	7	225.6	230.44	0.021	0.053
HP LaserJet 1160 Laser Printer	1918	28	320.5	322.98	0.008	0.077
Epson Stylus Photo R1800 Photo Inkjet	2774	36	536.26	557.8	0.040	0.047
Epson Stylus Photo R2400 Photo Inkjet	2592	37	811.63	853.08	0.051	0.035
Epson Stylus Photo R800 Photo Inkjet	3863	23	381.85	401.79	0.052	0.054
Brother DCP-7020 Laser Printer	3076	30	200.88	225.79	0.124	0.101
Brother MFC-8220 Laser Printer	4284	38	285.68	314.31	0.100	0.073
Median	**4,074**	**37**	**459.055**	**480.905**	**0.033**	**0.050**
Average	**4,386**	**42**	**695.220**	**717.863**	**0.045**	**0.054**

† The volume and weight information for Tables 12 and 13 is collected from Amazon.com.

Price Adjustment Behavior of PS and ZS Retailers

Although studying the price adjustment behaviors of retailers in details is beyond the scope of this study, we provide initial insights into such phenomenon. ZS retailers tend to change prices more frequently than PS retailers. More specifically, for a given product, the retailers in our sample changed the price, on average, 2.2 times in a month, i.e., one price change every 13.6 days. When comparing the two types of retailers in question, ZS retailers changed prices for a particular product, on average, 2.6 times in a month (or every 11.5 days), whereas PS retailers adjusted them, on average, only 1.8 times during the same period (or every 16.7 days). In other words, the prices posted by PS retailers “live” approximately 30% longer than the prices offered by ZS retailers. Interestingly, no significant difference was observed between ZS and PS retailers with respect to the direction (i.e., positive or negative) and magnitude of the price changes. For example, 64% and 62% of price changes by PS and ZS retailers, respectively, were associated with price decreases, while the (absolute) magnitude of price changes was 6.2% for ZS retailers and 7.3% for PS retailers.

Table 13 Product Characteristics (Digital Camera)

Digital Camera	Volume (cu-in)	Shipping Weight (lbs)	ZS Total ($)	PS Total ($)	(PS-ZS)/ZS	shipping %
Canon PowerShot SD600	6.43	2	231.05	240.24	0.033	0.047
Olympus Stylus 720 SW	7.1	2	327.37	341.62	0.053	0.034
Canon PowerShot SD700 IS	7.92	3	331.16	352.23	0.054	0.032
Canon PowerShot u710 IS	15.81	2	321.59	340.40	0.067	0.034
Canon PowerShot SD630	6.34	3	269.33	282.59	0.046	0.041
Canon PowerShot u640	21.24	3	347.73	366.18	0.055	0.033
Canon EOS Digital Rebel XT	46.25	4	617.27	604.33	-0.019	0.023
Canon PowerShot SD900	9.11	2	418.85	422.80	0.017	0.030
Canon Digital Rebel XTi	48.1	4	757.87	783.69	0.028	0.021
Canon PowerShot G7	19.99	3	529.23	552.62	0.045	0.024
Canon EOS 30D	69.43	5	1251.61	1248.08	-0.003	0.016
Olympus EVOLT E-330	46.2	6	839.41	858.99	0.029	0.020
Median	**17.90**	**3.00**	**383.29**	**394.49**	**0.039**	**0.031**
Average	**25.33**	**3.25**	**520.21**	**532.81**	**0.034**	**0.030**

Why do ZS retailers change prices more frequently than PS retailers? We offer two possible explanations for this phenomenon. The first is the negative sentiment of consumers in response to price changes. The kinked demand curve theory (Okun 1981) posits that a decrease in price, unless by a substantial amount (e.g., over 25% of the original price), does not help a firm attract many consumers. In contrast, an increase in price even by small amount is likely to cause it to lose many consumers. We found that a high correlation exists between product prices and S&H fees ($r = 0.85$) for PS retailers, which suggests that for such retailers S&H fees are in proportion to product prices. From a consumer's point of view, price changes occur on both components of the price when dealing with PS retailers. This "double whammy" is likely to have an additional negative impact on consumers' purchasing decisions. Moreover, a small increase in S&H fees might discourage shipping-charge-skeptics from buying the product, but a small decrease in S&H fees is unlikely to have a major impact on their purchase decisions. For these reasons, PS retailers are less active in changing prices relative to ZS retailers. The second explanation is related to the difference in firm size. Based on a survey, Buckle and Carlson (2000) found that large firms are more likely to raise their prices in response to cost increases than small firms, although no significant difference was detected in the case of cost decreases. According to these authors, the shorter price duration in large firms occurs because they, compared to small firms, are quicker to respond to demand and cost changes and gain greater benefits from making price changes. Our data from the popularity analysis show that PS retailers are generally less popular than ZS retailers (implying that PS retailers are also possibly smaller than ZS retailers). This suggests that PS retailers are slower than ZS retailers in responding to market changes and receive relatively little gain from price adjustments. Consequently, PS retailers are less "motivated" than their ZS counterparts in making price adjustments.

5. Managerial Implications

In the last two sections we theoretically and empirically analyzed the price partitioning strategies of online retailers. In this section, we discuss key managerial implications of our results and also show how certain real world phenomena are consistent with our results.

Our analysis suggests that if a retailer wants to offer free shipping, it ought to be aware that such a tactic has a cost associated with it. The retailer in that case is subsidizing, at least partly, the S&H costs, which can erode or even possibly destroy its margins. Moreover, if the retailer adds some of the S&H costs to product price, such an action might adversely affect demand, especially for price-sensitive customers. This is indeed consistent with the opinion of some of the practitioners actually making decisions about whether to offer free shipping or not (Brown 2008; Bolotsky 2008). We suggest that such an offer makes sense if: i) the retailer is operationally efficient so that its S&H costs are low and the "cost of free shipping offers" is not significant; ii) if the retailer is large (i.e., popular) enough so that it can negotiate a low product and/or S&H costs with its suppliers; iii) the demand loss from not providing free shipping is substantial; and iv) there is a high possibility of a risky cost environment in the future.

This means that online retailers need to think about customizing their price partitioning strategies based on the attributes of the products that they are selling as well as retailers' own characteristics. For example, cost(S&H)-to-price ratios for larger/heavier/fragile products are relatively high. For such products, there are less shipping-charge-skeptics since customers will incur high transportation cost if they want to buy the products offline; so, demand loss from charging S&H fees for these products would not be high, and retailers can opt for PS strategy. Free shipping offers perhaps ought to be targeted towards light/small, premium-price products for which customers expect the S&H costs to be relatively low and retailers to bear such costs. At the very least, retailers need to develop an aggregate measure of cost(S&H)-to-price ratio (i.e., if most of the products in their portfolios have high or low cost(S&H)-to-price ratios) in order to decide on the partitioning strategy. Similarly, retailers also need to think about their operational efficiency and/or bargaining power while deciding on shipping policies. When retailers are large or efficient enough such that their S&H costs are low, offering free shipping makes sense. But for a relatively less popular retailer or whose delivery operation is not properly optimized, such offers might be “too costly”. The retailers not only need to worry about average product and/or S&H costs, but also about the potential variability in those costs. For example, if retailers expect that the volatility in fuel costs would make S&H costs charged by carriers like UPS, FedEx and USPS more variable or that the fluctuation in prices of commodity metals would make product costs less stable, our model suggests that we should see more retailers opting for ZS strategy. On the other hand, expectation of a stable cost structure going forward (at least in the short term) would induce most retailers to opt for PS strategy. Indeed, our data suggests that about 40% of the retailers in our sample differentiated their shipping policies across products and across time.

Note that the shipping charge strategy adopted by Amazon.com seems to be consistent with our analysis. Amazon.com now uses a threshold S&H-charge policy; that is, if the total purchase value is above that threshold, then customers get free shipping, otherwise not. However, if we look at the history of Amazon.com, it did not offer any free shipping from its inception in July 1995 until November 1999. Even when it started offering free shipping, the threshold was $100, and only later on (August 2002) Amazon.com reduced the threshold to $25 (Lewis et al. 2006). As indicated earlier, in our setting, a reduction in the threshold value is (approximately) equivalent to changing from the PS to the ZS strategy. So, as our model suggests, Amazon.com indeed started with the PS strategy, but as it established its brand equity and improved its S&H operations, it took advantage of economies of scale/scope and opted for the ZS strategy.[17]

Lastly, recall that, in the last section, we demonstrated that ZS retailers change prices more frequently than PS ones. However, managerial decisions about price adjustments ought to be determined based on the optimal equilibrium between the economic gains from “flexible pricing” and the “consumer costs” (Goodfriend 1997) that arise from the consumer’s negative reaction in response to price changes. ZS retailers should be aware that the benefits accrued from dynamic pricing tactics might be short-lived, as consumers discover the “randomness” in ZS retailers’ prices. One potential *negative* consequence is that this might encourage strategic behavior on the part of consumers. For example, ZS customers might delay their purchases in the anticipation of price cuts. These delayed purchases might eventually result in demand-supply imbalances.

6. Concluding Remarks

Pricing is an important element in any firm’s strategy, whether it operates in online or offline markets. For online firms, inherent S&H costs pose an additional strategic challenge. This study

[17] Refer to Schepp and Schepp (2009) about how Amazon improved the efficiency of its S&H operations over time and Brown (2008) about Amazon’s bargaining power with USPS. Amazon’s strategy contrasts with e-tailers like Webvan who were too aggressive about free shipping offers too early in their lifecycles, and paid the price by actually losing money on most of the items they sold.

sought to provide an analytical and empirical vantage point from which to observe and interpret competitive actions related to S&H charges. More specifically, we developed an oligopoly model of (Nash) price competition among online retailers, which captures the basic trade-off between PS and ZS strategies - the ZS strategy attracts more customers but loses out on S&H charge revenues, while the PS strategy earns more revenue per unit at the cost of lower demand. Analysis of our theoretical model illustrated the effects of both product *and* firm level characteristics on the relative dominance of these two partitioning strategies among the retailers.

Our results first showed that PS retailers should charge *lower* product prices than ZS retailers, but their total prices (including S&H) should be *higher* than ZS ones. As far as the effects of *product* characteristics are concerned, this study showed that the optimal strategy for retailers with a large segment of shipping-charge-skeptics is to offer free shipping to their customers. The same ZS strategy should be ideal for retailers selling low-priced products and products associated with high levels of cost uncertainty. A retailer's popularity and its product portfolio are also important factors in choosing between PS and ZS strategies. When the retailer is relatively less popular or its product portfolio consists of a large number of products with a high cost-to-price ratio, then the optimal strategy should be to add a S&H surcharge. One distinguishing feature of our study is that we validated our theoretical results using an empirical study. Data was collected for four months about actual product and S&H prices charged by a large number of online retailers for two product categories - one of which is relatively small and light but expensive (digital cameras), whereas the other one is relatively larger, heavier and cheaper (printers). All of our theoretical results were supported by the empirical data. Our empirical results also showed that the price adjustment behavior of ZS firms differs from that of PS firms. ZS firms tend to have frequent price changes, while the pricing policy of PS retailers is relatively more stable. But there was no significant difference between PS and ZS firms in terms of magnitude and direction of price changes.

There are three worthwhile avenues for pursuing future extensions of our research. First, we assume the model horizon to be short and so do not include transient behavior of customers and retailers in our framework. A more comprehensive model should capture how a retailer's pricing and partitioning strategy over time affects its demand and brand equity. It would require a dynamic modeling perspective, and in that case the data collection period for the empirical study should also be sufficiently long. Second, it would be useful to empirically study the performance implications of the two shipping modes in terms of financial measure like profitability. Third, a demand model that captures more complex consumer behavior would perhaps be helpful for developing real-life pricing decision support systems, although analyzing it would be a quite challenging endeavor.

Note that one of the assumptions in our theoretical modeling framework is that shipping-charge-skeptics only consider ZS firms while making purchase decisions. An alternative formation might be to assume that all customers consider all firms (i.e., both ZS and PS) while deciding to buy. The only difference between the two customer segments is that skeptics weigh any S&H charge more than non-skeptics (e.g., they weigh this charge by τ_{sc} and τ_{nsc}, respectively, where $\tau_{sc} \geq \tau_{nsc}$). We discuss and analyze such a framework in details in (online) Appendix A.2, which turns out to be quite complex. Some of the results (e.g., parts of Proposition 1 and Corollary 1) can still be analytically established. However, because of the involved expressions for certain equilibrium decisions, analytically proving some other characterizations is quite difficult. Our extensive numerical study clearly demonstrates that almost all the *qualitative theoretical insights of the paper hold true even for that model*, although the equilibrium values are different. As far as our empirical study is concerned, most such studies have several common inherent limitations, such as endogeneity and reverse causality. Regarding endogeneity, our independent variables do not by any means cover all known factors influencing a retailer's free shipping offer. Future research should identify other key factors influencing such a decision. Regarding reverse causality, our empirical models define the causal relationships between the variables in such ways that they provide empirical insights into the findings we obtained from analytical assessments. However, reverse causality could occur. For example, price-partitioning strategies might drive a retailer' popularity rather than the other way around. Although this reverse relationship was not the objective of our study, future research can expand along this line of inquiries.

In closing, we would like to point out that while the reasons for an online retailer's choice between offering free shipping and charging shipping fees can be complex, consumers' attitudes towards shipping fees, product features, and the retailer's market status might be key determinants of a firm's price partitioning strategy and are therefore critical in sustaining a competitive advantage.

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Online Appendix

A.1 Proofs for Propositions and Corollaries

Proof of Proposition 1: We solve the game by backward induction starting from the last stage.

1. Suppose that firm i opts for ZS strategy in the first stage, i.e. $f_i = ZS$. The profit function for firm i then is as follows:

$$\pi^i_{ZS} = (p_i - c)\overbrace{\left(\frac{\alpha(w)}{N_{zs}} e^{-up_i + \sum_{j\in N_{zs}\setminus\{i\}} vp_j + \sum_{j\in N_{ps}} vp_j(1+\tau_{scs})} + \frac{1-\alpha(w)}{N} e^{-up_i + \sum_{j\in N_{zs}\setminus\{i\}} vp_j + \sum_{j\in N_{ps}} vp_j(1+\tau_{nscs})}\right)}^{d^i_{N_{zs},N_{ps}}(p_i,p_{-i}|f_i=\text{ZS})}$$

Note that π^i_{ZS} can be written as multiplication of two terms:

$$\pi^i_{ZS} = (p_i - c)e^{-up_i} \times \left(\frac{\alpha(w)}{N_{zs}} e^{\sum_{j\in N_{zs}\setminus\{i\}} vp_j + \sum_{j\in N_{ps}} vp_j(1+\tau_{scs})} + \frac{1-\alpha(w)}{N} e^{\sum_{j\in N_{zs}\setminus\{i\}} vp_j + \sum_{j\in N_{ps}} vp_j(1+\tau_{nscs})}\right)$$

In order to establish the behavior of π^i_{ZS} in terms of p_i, we focus on the first term, i.e., $(p_i - c)e^{-up_i}$ (since the second term is independent of p_i). Indeed we can show this second term to be quasi-concave in p_i. Specifically, note the following about the first term:

$$\frac{\partial[(p_i - c)e^{-up_i}]}{\partial p_i} = e^{-up_i}(1 + uc - up_i)$$
$$\frac{\partial^2[(p_i - c)e^{-up_i}]}{\partial p_i^2} = -ue^{-up_i}(2 + uc - up_i)$$

Clearly, e^{-up_i} and $(1 + uc - up_i)$ of $\frac{\partial[(p_i-c)e^{-up_i}]}{\partial p_i}$ are decreasing in p_i. However, while e^{-up_i} is always positive, $(1 + uc - up_i)$ is positive for $p_i \le c + \frac{1}{u}$ and then it is negative. This implies that $\frac{\partial[(p_i-c)e^{-up_i}]}{\partial p_i} = 0$ has a unique solution at $p_i = c + \frac{1}{u}$ (it is positive before that and negative after that). In fact, based on the expression for $\frac{\partial^2[(p_i-c)e^{-up_i}]}{\partial p_i^2}$, we can also observe that the first derivative is decreasing for $p_i \le c + \frac{2}{u}$ and then it is increasing ($c + \frac{2}{u} > c + \frac{1}{u}$). Actually the value of the first derivative is $(1 + uc) > 0$ as p_i tends to zero. It decreases in p_i initially, reaching zero at $p_i = c + \frac{1}{u}$. It then becomes negative and continues decreasing until $p_i = c + \frac{2}{u}$. It then starts increasing and approaches zero again as p_i tends towards infinity (however, it never becomes positive). The above discussion implies that $(p_i - c)e^{-up_i}$ is quasi-concave in p_i, and so, given the prices of the other firms, π^i_{ZS} is quasi-concave in p_i. Hence, we can use first order condition to characterize the optimal price for firm i in the second stage, i.e. p^*_i solves the following first order condition:

$$\begin{aligned}\frac{\partial \pi^i_{ZS}}{\partial p_i} &= \frac{\partial}{\partial p_i}(p_i - c) \times d^i_{N_{zs},N_{ps}}(p_i, p_{-i}|f_i = \text{ZS}) + (p_i - c) \times \frac{\partial}{\partial p_i} d^i_{N_{zs},N_{ps}}(p_i, p_{-i}|f_i = \text{ZS}) \\ &= d^i_{N_{zs},N_{ps}}(p_i, p_{-i}|f_i = \text{ZS}) - u(p_i - c)d^i_{N_{zs},N_{ps}}(p_i, p_{-i}|f_i = \text{ZS}) = 0,\end{aligned}$$

where the second equality is due to the exponential demand function. First order condition then implies that $p^*_i = c + \frac{1}{u}$ when $f_i = ZS$.

Now, suppose that $f_i = PS$. The profit function for firm i is then as follows:

$$\pi^i_{PS} = (p_i(1+s)-c)\overbrace{\left(\frac{1-\alpha(w)}{N}e^{-up_i(1+\tau_{nsc})+\sum_{j\in N_{zs}}vp_j+\sum_{j\in N_{ps}\setminus\{i\}}vp_j(1+\tau_{nsc}s)}\right)}^{d^i_{N_{zs},N_{ps}}(p_i,p_{-i}|f_i=\mathrm{PS})}$$

Once again, π^i_{PS} can be written as multiplication of two terms:

$$\pi^i_{PS} = (p_i(1+s)-c)e^{-up_i(1+\tau_{nsc})} \times \left(\frac{1-\alpha(w)}{N}e^{\sum_{j\in N_{zs}}vp_j+\sum_{j\in N_{ps}\setminus\{i\}}vp_j(1+\tau_{nsc}s)}\right)$$

Similar to the above ZS case, π^i_{PS} can be shown to be quasi-concave in p_i by analyzing the first term, i.e., $(p_i(1+s)-c)e^{-up_i(1+\tau_{nsc})}$ (the second term is again independent of p_i). The nature of the analysis remains the same as before except that the first derivative of $(p_i(1+s)-c)e^{-up_i(1+\tau_{nsc})}$ is $(1+s)+uc(1+\tau_{nsc})>0$ as p_i tends to zero. It decreases in p_i initially, reaching zero at $p_i = \frac{c}{1+s}+\frac{1}{u(1+\tau_{nsc})}$. It then becomes negative and continues decreasing until $p_i = \frac{c}{1+s}+\frac{1}{u(1+\tau_{nsc})}$. It then starts increasing and approaches zero again as p_i tends towards infinity (again it never becomes positive). Consequently, $(p_i(1+s)-c)e^{-up_i(1+\tau_{nsc})}$ is quasi-concave in p_i, and so, given the prices of the other firms, π^i_{PS} is quasi-concave in p_i. Then the first order condition would lead to $p^*_i = \frac{c}{1+s}+\frac{1}{u(1+\tau_{nsc}s)}$ when $f_i = PS$.

2. Since β is the proportion of ZS firms in the market, so $\beta = \frac{N_s}{N}$. Let β^* be the equilibrium proportion of ZS firms in the market defined in such a way that none of the firms would profitably change their shipping policies. We can rewrite the first stage profit functions of firm i in terms of β as follows:

$$\begin{aligned}\pi^i_{ZS}(\beta) = \pi^i_{\beta N,(1-\beta)N}(p^*_i,p^*_{-i}|f_i=\mathrm{ZS}) = (p^*_i-c)\Bigg(&\frac{1}{\beta N}\alpha(w)\exp(-up^*_i+\sum_{j\in N_{zs}\setminus\{i\}}vp^*_j+\sum_{j\in N_{ps}}vp^*_j(1+\tau_{sc}s))\\ &+\frac{1}{N}(1-\alpha(w))\exp(-up^*_i+\sum_{j\in N_{zs}\setminus\{i\}}vp^*_j+\sum_{j\in N_{ps}}vp^*_j(1+\tau_{nsc}s))\Bigg)\end{aligned}$$

and

$$\begin{aligned}\pi^i_{PS}(\beta) = \pi^i_{\beta N,(1-\beta)N}(p^*_i,p^*_{-i}|f_i=\mathrm{PS}) = (p^*_i(1+s)-c)\Bigg(&\frac{1}{N}(1-\alpha(w))\\ &\exp(-up^*_i(1+\tau_{nsc}s)+\sum_{j\in N_{ps}\setminus\{i\}}vp^*_j(1+\tau_{nsc}s)+\sum_{j\in N_{zs}}vp^*_j)\Bigg)\end{aligned}$$

Note that $\pi^i_{ZS}(\beta)$ is decreasing in β whereas $\pi^i_{PS}(\beta)$ is constant in β. Therefore, they intersect at unique point given by $\hat{\beta}$, where $\hat{\beta}$ solves

$$\pi^i_{ZS}(\hat{\beta}) = \pi^i_{PS}(\hat{\beta})$$

Clearly, at this intersection point, no firm can increase her profit by changing her shipping policy. After substituting $p_i^* = c + \frac{1}{u}$ and $p_j^* = \frac{c}{1+s} + \frac{1}{u(1+\tau_{nsc}s)}$ into $\pi_{ZS}^i(\hat{\beta})$, we obtain:

$$\pi_{ZS}^i(\hat{\beta}) = \frac{1}{u} e^{-u(c+\frac{1}{u}) + \sum_{j\in N_{zs}\setminus\{i\}} v(c+\frac{1}{u}) + \sum_{j\in N_{ps}} v(\frac{c}{1+s}+\frac{1}{u(1+\tau_{nsc}s)})(1+\tau_{nsc}s)} \left(\frac{1}{\hat{\beta}N}\alpha(w)z + \frac{1}{N}(1-\alpha(w))\right)$$

where $z = e^{(1-\hat{\beta})v\left(\frac{c}{1+s}+\frac{1}{u(1+\tau_{nsc}s)}\right)(\tau_{sc}-\tau_{nsc})s}$. Substituting $p_i^* = \frac{c}{1+s} + \frac{1}{u(1+\tau_{nsc}s)}$ and $p_j^* = c + \frac{1}{u}$ into $\pi_{PS}^i(\hat{\beta})$, we obtain:

$$\pi_{PS}^i(\hat{\beta}) = \frac{(1+s)(1-\alpha(w))}{Nu(1+\tau_{nsc}s)} e^{(-u+\sum_{j\in N_{ps}\setminus\{i\}} v)\left(\frac{c}{1+s}+\frac{1}{u(1+\tau_{nsc}s)}\right)(1+\tau_{nsc}s) + \sum_{j\in N_{zs}} v(c+\frac{1}{u})}$$

Now, equating $\pi_{ZS}^i(\hat{\beta}) = \pi_{PS}^i(\hat{\beta})$, and canceling out the common terms in right and left hand sides, we obtain:

$$\begin{aligned}
\left(\frac{\alpha(w)}{\hat{\beta}}z + (1-\alpha(w))\right)e^{-(u+v)\left[c+\frac{1}{u}\right]} &= \frac{(1+s)(1-\alpha(w))}{1+\tau_{nsc}s} e^{-(u+v)(1+\tau_{nsc}s)\left[\frac{c}{1+s}+\frac{1}{u(1+\tau_{nsc}s)}\right]} \\
\left(\frac{\alpha(w)}{\hat{\beta}}z + (1-\alpha(w))\right)e^{-(u+v)\left[c+\frac{1}{u}\right]} &= \frac{(1+s)(1-\alpha(w))}{1+\tau_{nsc}s} e^{-(u+v)\left[\frac{c(1+\tau_{nsc}s)}{1+s}+\frac{1}{u}\right]} \\
\frac{\alpha(w)}{\hat{\beta}}z + (1-\alpha(w)) &= \frac{(1+s)(1-\alpha(w))}{1+\tau_{nsc}s} e^{(u+v)\left[\frac{cs(1-\tau_{nsc})}{1+s}\right]} \\
\frac{\alpha(w)}{\hat{\beta}}z &= (1-\alpha(w))\left[\frac{1+s}{1+\tau_{nsc}s} e^{(u+v)\left[\frac{cs(1-\tau_{nsc})}{1+s}\right]} - 1\right]
\end{aligned}$$

Substituting $z = \exp((1-\beta)vN(\frac{c}{1+s} + \frac{1}{u(1+\tau_{nsc}s)})(\tau_{sc}-\tau_{nsc})s)$ into the above expression, we obtain

$$\begin{aligned}
\frac{\alpha(w)}{\hat{\beta}} e^{(1-\hat{\beta})vN(\frac{c}{1+s}+\frac{1}{u(1+\tau_{nsc}s)})(\tau_{sc}-\tau_{nsc})s} &= (1-\alpha(w))\left[\frac{1+s}{1+\tau_{nsc}s} e^{(u+v)\left[\frac{cs(1-\tau_{nsc})}{1+s}\right]} - 1\right] \\
\hat{\beta} e^{\hat{\beta}vN(\frac{c}{1+s}+\frac{1}{u(1+\tau_{nsc}s)})(\tau_{sc}-\tau_{nsc})s} &= \frac{\alpha(w)}{1-\alpha(w)} \frac{e^{vN(\frac{c}{1+s}+\frac{1}{u(1+\tau_{nsc}s)})(\tau_{sc}-\tau_{nsc})s}}{\left[\frac{1+s}{1+\tau_{nsc}s} e^{(u+v)\left[\frac{cs(1-\tau_{nsc})}{1+s}\right]} - 1\right]} \\
\hat{\beta} e^{\hat{\beta}vN(\frac{c}{1+s}+\frac{1}{u(1+\tau_{nsc}s)})(\tau_{sc}-\tau_{nsc})s} &= \frac{\alpha(w)}{1-\alpha(w)} \frac{1+\tau_{nsc}s}{1+s} \frac{e^{vN(\frac{c}{1+s}+\frac{1}{u(1+\tau_{nsc}s)})(\tau_{sc}-\tau_{nsc})s}}{e^{(u+v)\left[\frac{cs(1-\tau_{nsc})}{1+s}\right]} - \frac{1+\tau_{nsc}s}{1+s}}
\end{aligned}$$

Since β in our setting is constrained to be between 0 and 1 (while $\hat{\beta}$ can be greater than 1), so the equilibrium proportion of ZS firms in the market β^* would be given by: $\beta^* = \min(\hat{\beta}, 1)$, where $\hat{\beta}$ is as defined above.

Proof of Corollary 1: Recall that equilibrium unit price and S&H charge for PS and ZS retailers are $p_{ps}^* = \frac{c}{1+s} + \frac{1}{u(1+\tau_{nsc}s)}$, $s_{ps}^* = \frac{cs}{1+s} + \frac{s}{u(1+\tau_{nsc}s)}$, $p_{zs}^* = c + \frac{1}{u}$, and $s_{zs}^* = 0$, respectively. Using the price expressions and noting that $s > 0$, it is straightforward to show that $p_{ps}^* < p_{zs}^*$ and $p_{ps}^* + s_{ps}^* \geq p_{zs}^*$.

Proof of Proposition 2: In what follows we perform the sensitivity analysis of $\hat{\beta}$ with respect to w and c. Since, $\beta^* = \min(\hat{\beta}, 1)$, so the same sensitivity analysis holds for β^*. Recall that $\hat{\beta}$ satisfies $g_1 = g_2$, where

$$g_1 = \hat{\beta}\exp(-(1-\hat{\beta})vN(\frac{c}{1+s} + \frac{1}{u(1+\tau_{nsc}s)})(\tau_{sc}-\tau_{nsc})s)$$

and

$$g_2 = \frac{\frac{\alpha(w)}{1-\alpha(w)}\frac{1+\tau_{nsc}s}{1+s}}{\exp\left((u+v)\left[\frac{cs(1-\tau_{nsc})}{1+s}\right]\right) - \frac{1+\tau_{nsc}s}{1+s}}$$

Using g_1 and g_2, we prove the proposition part by part:

u) Note that $\alpha(w)$ is decreasing in w and g_1 is increasing in β. Therefore, it suffices to show that g_2 increases in α. In order to do that, we take the derivative of g_2 with respect to α:

$$\begin{aligned}\frac{\partial g_2}{\partial \alpha} &= \frac{\partial}{\partial \alpha}\left(\frac{\alpha(w)}{1-\alpha(w)}\right)\frac{\frac{1+\tau_{nsc}s}{1+s}}{\exp\left((u+v)\left[\frac{cs(1-\tau_{nsc})}{1+s}\right]\right) - \frac{1+\tau_{nsc}s}{1+s}}\\ &= \left(\frac{1}{1-\alpha(w)}\right)^2\frac{\frac{1+\tau_{nsc}s}{1+s}}{\exp\left((u+v)\left[\frac{cs(1-\tau_{nsc})}{1+s}\right]\right) - \frac{1+\tau_{nsc}s}{1+s}}\end{aligned}$$

which is always positive for all $\alpha(w) \in [0,1]$.

v) Applying implicit function theorem to $g_1 - g_2 = 0$, we obtain the derivative of $\hat{\beta}$ with respect to c as follows:

$$\left[\frac{\partial g_1}{\partial \hat{\beta}} - \frac{\partial g_2}{\partial \hat{\beta}}\right]d\hat{\beta} - \left[\frac{\partial g_2}{\partial c} - \frac{\partial g_1}{\partial c}\right]dc = 0 \rightarrow \frac{d\hat{\beta}}{dc} = \frac{\frac{\partial g_2}{\partial c} - \frac{\partial g_1}{\partial c}}{\frac{\partial g_1}{\partial \hat{\beta}} - \frac{\partial g_2}{\partial \hat{\beta}}}$$

First, we take the derivative of g_2 with respect to c:

$$\begin{aligned}\frac{\partial g_2}{\partial c} &= \frac{\alpha(w)}{1-\alpha(w)}\frac{\partial}{\partial c}\left(\left[\frac{1+s}{1+\tau_{nsc}s}\exp((u+v)\left[\frac{cs(1-\tau_{nsc})}{1+s}\right]) - 1\right]^{-1}\right)\\ &= -(u+v)\frac{\alpha(w)}{1-\alpha(w)}\frac{s(1-\tau_{nsc})}{1+\tau_{nsc}s}e^{(u+v)\left[\frac{cs(1-\tau_{nsc})}{1+s}\right]}\left[\frac{1+s}{1+\tau_{nsc}s}e^{(u+v)\left[\frac{cs(1-\tau_{nsc})}{1+s}\right]} - 1\right]^{-2}\end{aligned}$$

which is always negative for all $c \geq 0$. Next, we take the derivative of g_1 with respect to c:

$$\begin{aligned}\frac{\partial g_1}{\partial c} &= \frac{\partial}{\partial c}\left(\hat{\beta}\exp(-(1-\hat{\beta})vN(\frac{c}{1+s} + \frac{1}{u(1+\tau_{nsc})})(\tau_{sc} - \tau_{nsc})s)\right)\\ &= -\frac{vN(\tau_{sc} - \tau_{nsc})s}{1+s}(1-\hat{\beta})\hat{\beta}e^{-(1-\hat{\beta})vN(\frac{c}{1+s} + \frac{1}{u(1+\tau_{nsc})})(\tau_{sc} - \tau_{nsc})s}\end{aligned}$$

Note that $\frac{\partial g_1}{\partial c} = 0$ for $N = 0$. It then initially decreases and subsequently increases; furthermore, $\frac{\partial g_1}{\partial c} \rightarrow 0$ for large N. On the other hand, $\frac{\partial g_2}{\partial c}$ is independent of N. Moreover, both $\frac{\partial g_1}{\partial c}$ and $\frac{\partial g_2}{\partial c}$ are always negative. Therefore, we can conclude that either $\frac{\partial g_2}{\partial c} - \frac{\partial g_1}{\partial c}$ is always negative or it is negative for large N (actually it will also be negative for very small N). On the other hand, since g_1 is increasing in $\hat{\beta}$ and g_2 is constant in $\hat{\beta}$, we can show that $\frac{\partial g_1}{\partial \hat{\beta}} - \frac{\partial g_2}{\partial \hat{\beta}} \geq 0$. To summarize, we show that for large N, $\left(\frac{\partial g_2}{\partial c} - \frac{\partial g_1}{\partial c}\right) / \left(\frac{\partial g_1}{\partial \beta} - \frac{\partial g_2}{\partial \beta}\right)$ is surely negative (and it can even be negative for small N), i.e., $\hat{\beta}$ is decreasing in c.

Proof of Proposition 3: We solve the game by backward induction starting from the last stage. Since the proof of this proposition is similar to the proof of proposition 1, we will only state the differences.

1. Suppose that firm i's first stage decision is f_i. We assume that firm i observes the cost realization before making pricing decision at each period $t=1,\dots T$. Then the optimal price for firm i in each period in the second stage, i.e. $p_i^*(t)$, solves the first order condition similar to the part 1 of Proposition 1. This implies that $p_i^*(t)=c_t^i+\frac{1}{u}$ when $f_i=ZS$ and $p_i^*(t)=\frac{c_t^i}{1+s}+\frac{1}{u(1+\tau_{nsc}s)}$ when $f_i=PS$.

2. Let β be the proportion of ZS firms in the market. Let β^*_{ran} be equilibrium proportion of ZS firms in the market defined in such u way that none of the firms would profitably change their shipping policies in the expected sense. Let $\bar{\pi}^i_{ZS}$ and $\bar{\pi}^i_{PS}$ be the first stage total expected profit of firm i if firm i opts for ZS and PS strategies, respectively. Then, we can express them as follows:

$$\begin{aligned}\bar{\pi}^i_{ZS}(\beta) &= \sum_{t=1}^{T} E_{c_t^i,c_t^{-i}}\pi^i_{\beta N,(1-\beta)N}(p_i^*(t),p_{-i}^*(t)|f_i=\text{ZS})\\ &=\sum_{t=1}^{T} E_{c_t^i,c_t^{-i}}\Bigg[(p_i^*(t)-c_t^i)\Bigg(\frac{1}{\beta N}\alpha(w)\exp(-up_i^*(t)+\sum_{j\in N_{zs}\setminus\{i\}}vp_j^*(t)+\sum_{j\in N_{ps}}vp_j^*(t)(1+\tau_{sc}s))\\ &+\frac{1}{N}(1-\alpha(w))\exp(-up_i^*(t)+\sum_{j\in N_{zs}\setminus\{i\}}vp_j^*(t)+\sum_{j\in N_{ps}}vp_j^*(t)(1+\tau_{nsc}s))\Bigg)\Bigg]\end{aligned}$$

and

$$\begin{aligned}\bar{\pi}^i_{PS}(\beta) &= \sum_{t=1}^{T} E_{c_t^i,c_t^{-i}}\pi^i_{\beta N,(1-\beta)N}(p_i^*,p_{-i}^*|f_i=\text{PS})\\ &=\sum_{t=1}^{T} E_{c_t^i,c_t^{-i}}\Bigg[(p_i^*(t)(1+s)-c_t^i)\frac{1}{N}(1-\alpha(w))\\ &\Bigg(\exp(-up_i^*(t)(1+\tau_{nsc}s)+\sum_{j\in N_{ps}\setminus\{i\}}vp_j^*(t)(1+\tau_{nsc}s)+\sum_{j\in N_{zs}}vp_j^*(t))\Bigg)\Bigg]\end{aligned}$$

Note that $\bar{\pi}^i_{ZS}(\beta)$ is decreasing in β whereas $\bar{\pi}^i_{PS}(\beta)$ is constant in β. Therefore, they intersect at unique point given by $\hat{\beta}$, where $\hat{\beta}$ solves

$$\bar{\pi}^i_{ZS}(\hat{\beta})=\bar{\pi}^i_{PS}(\hat{\beta})$$

After substituting $p_i^*(t)$ into above equation, using the fact that c_t^i's are i.i.d. random variables, letting $z=E_c\left[\exp((1-\hat{\beta})vN(\frac{c}{1+s}+\frac{1}{u(1+\tau_{nsc}s)})(\tau_{sc}-\tau_{nsc})s)\right]$ and canceling out the common terms in right and left hand sides, we obtain:

$$\begin{aligned}\left(\frac{\alpha(w)}{\hat{\beta}}z+(1-\alpha(w))\right)\frac{E_c\left[\exp(-uc)\right]}{E_c\left[\exp(vc)\right]} &= \frac{(1+s)(1-\alpha(w))}{1+\tau_{nsc}s}\frac{E_c\left[\exp(-\frac{uc(1+\tau_{nsc}s)}{1+s})\right]}{E_c\left[\exp(\frac{vc(1+\tau_{nsc}s)}{1+s})\right]}\\ \frac{\alpha(w)}{\hat{\beta}}z+(1-\alpha(w)) &= \frac{(1+s)(1-\alpha(w))}{1+\tau_{nsc}s}\frac{E_c\left[\exp(-\frac{uc(1+\tau_{nsc}s)}{1+s})\right]}{E_c\left[\exp(-uc)\right]}\frac{E_c\left[\exp(\frac{-vc(1+\tau_{nsc}s)}{1+s})\right]}{E_c\left[\exp(-vc)\right]}\\ \frac{\alpha(w)}{\hat{\beta}}z &= (1-\alpha(w))\left[\frac{1+s}{1+\tau_{nsc}s}\frac{E_c\left[\exp(-\frac{uc(1+\tau_{nsc}s)}{1+s})\right]}{E_c\left[\exp(-uc)\right]}\frac{E_c\left[\exp(\frac{-vc(1+\tau_{nsc}s)}{1+s})\right]}{E_c\left[\exp(-vc)\right]}-1\right]\end{aligned}$$

$$\hat{\beta} = \frac{\alpha(w)}{1-\alpha(w)} \frac{E_c\left[\exp((1-\hat{\beta})vN(\frac{c}{1+s}+\frac{1}{u(1+\tau_{nsc}s)})(\tau_{sc}-\tau_{nsc})s)\right]}{\left[\frac{1+s}{1+\tau_{nsc}s}\frac{E_c\left[\exp(-\frac{uc(1+\tau_{nsc}s)}{1+s})\right]}{E_c[\exp(-uc)]}\frac{E_c\left[\exp(\frac{-vc(1+\tau_{nsc}s)}{1+s})\right]}{E_c[\exp(-vc)]}-1\right]}$$

Since β in our setting is constrained to be between 0 and 1 (while $\hat{\beta}$ can be greater than 1), so the equilibrium proportion of ZS firms in the market β^*_{ran} would be given by: $\beta^*_{ran} = \min(\hat{\beta}, 1)$, where $\hat{\beta}$ is as defined above.

Proof of Proposition 4: Note that we perform the sensitivity analysis of $\hat{\beta}$; since, $\beta^*_{ran} = \min(\hat{\beta}, 1)$, so the same sensitivity analysis holds for β^*_{ran}. We prove the proposition part by part:
a) In order to take the expectation of $\hat{\beta}$ with respect to c, where $c \sim U(\mu_c - \sqrt{3}\sigma_c, \mu_c + \sqrt{3}\sigma_c)$, we need to calculate $E_c[\exp(-uc)]$:

$$E_c[\exp(-uc)] = \frac{1}{2\sqrt{3}\sigma_c}\int_{c=c_1}^{c_2}\exp(-uc)dc = \frac{e^{-u\left(\mu_c-\sqrt{3}\sigma_c\right)} - e^{-u\left(\mu_c+\sqrt{3}\sigma_c\right)}}{2\sqrt{3}\sigma_c u}$$

Recall that $\hat{\beta}$ can be written as follows:

$$\hat{\beta} = \frac{\alpha(w)}{1-\alpha(w)}\exp\left(\frac{(1-\hat{\beta})vN(\tau_{sc}-\tau_{nsc})s}{u(1+\tau_{nsc}s)}\right)\frac{f_1}{\frac{1+s}{1+\tau_{nsc}s}f_2 - 1}$$

where $f_1 = E_c\left[\exp\left(\frac{(1-\hat{\beta})vN(\tau_{sc}-\tau_{nsc})s}{1+s}c\right)\right]$ and $f_2 = \frac{E_c\left[\exp(-\frac{uc(1+\tau_{nsc}s)}{1+s})\right]}{E_c[\exp(-uc)]}\frac{E_c\left[\exp(\frac{-vc(1+\tau_{nsc}s)}{1+s})\right]}{E_c[\exp(-vc)]}$. First, we analyze f_1.

$$\begin{aligned} f_1 &= E_c\left[\exp\left(\frac{(1-\hat{\beta})vN(\tau_{sc}-\tau_{nsc})s}{1+s}c\right)\right] = \frac{e^{x\left(\mu_c+\sqrt{3}\sigma_c\right)} - e^{x\left(\mu_c-\sqrt{3}\sigma_c\right)}}{2\sqrt{3}\sigma_c x} \\ &= \frac{e^{x\mu_c}}{2x}\frac{e^{x\sqrt{3}\sigma_c} - e^{-x\sqrt{3}\sigma_c}}{\sqrt{3}\sigma_c} \end{aligned}$$

where $x = \frac{(1-\hat{\beta})bN(\tau_{sc}-\tau_{nsc})s}{1+s}$. Next, we analyze f_2.

$$\begin{aligned} f_2 &= \frac{E_c\left[\exp(-\frac{uc(1+\tau_{nsc}s)}{1+s})\right]}{E_c[\exp(-uc)]}\frac{E_c\left[\exp(\frac{-vc(1+\tau_{nsc}s)}{1+s})\right]}{E_c[\exp(-vc)]} \\ &= \left(\frac{1+s}{1+\tau_{nsc}s}\right)^2 \frac{e^{-\frac{u(1+\tau_{nsc}s)}{1+s}\left(\mu_c-\sqrt{3}\sigma_c\right)} - e^{-\frac{u(1+\tau_{nsc}s)}{1+s}\left(\mu_c+\sqrt{3}\sigma_c\right)}}{e^{-u\left(\mu_c-\sqrt{3}\sigma_c\right)} - e^{-u\left(\mu_c+\sqrt{3}\sigma_c\right)}} \frac{e^{-\frac{v(1+\tau_{nsc}s)}{1+s}\left(\mu_c-\sqrt{3}\sigma_c\right)} - e^{-\frac{v(1+\tau_{nsc}s)}{1+s}\left(\mu_c+\sqrt{3}\sigma_c\right)}}{e^{-v\left(\mu_c-\sqrt{3}\sigma_c\right)} - e^{-v\left(\mu_c+\sqrt{3}\sigma_c\right)}} \\ &= \left(\frac{1+s}{1+\tau_{nsc}s}\right)^2 e^{\frac{us\mu_c(1-\tau_{nsc})}{1+s}} \frac{e^{\frac{\sqrt{3}u(1+\tau_{nsc}s)}{1+s}\sigma_c} - e^{-\frac{\sqrt{3}u(1+\tau_{nsc}s)}{1+s}\sigma_c}}{e^{\sqrt{3}u\sigma_c} - e^{-\sqrt{3}u\sigma_c}} \frac{e^{\frac{\sqrt{3}v(1+\tau_{nsc}s)}{1+s}\sigma_c} - e^{-\frac{\sqrt{3}v(1+\tau_{nsc}s)}{1+s}\sigma_c}}{e^{\sqrt{3}v\sigma_c} - e^{-\sqrt{3}v\sigma_c}} \end{aligned}$$

b) Recall that $\hat{\beta}$ satisfies the following equation

$$\hat{\beta}\frac{1-\alpha(w)}{\alpha(w)}\exp\left(\frac{-(1-\hat{\beta})vN(\tau_{sc}-\tau_{nsc})s}{u(1+\tau_{nsc})}\right) - \frac{f_1}{\frac{1+s}{1+\tau_{nsc}s}f_2 - 1} = 0$$

Let $g_1 = \hat{\beta}\frac{1-\alpha(w)}{\alpha(w)}\exp\left(\frac{-(1-\hat{\beta})vN(\tau_{sc}-\tau_{nsc})s}{u(1+\tau_{nsc})}\right)$ and $g_2 = \frac{f_1}{\frac{1+s}{1+\tau_{nsc}s}f_2 - 1}$. Then, taking implicit differentiation of the above equation, we obtain

$$\left[\frac{\partial g_1}{\partial \hat{\beta}} - \frac{\partial g_2}{\partial \hat{\beta}}\right]d\hat{\beta} - \left[\frac{\partial g_2}{\partial \sigma_c} - \frac{\partial g_1}{\partial \sigma_c}\right]d\sigma_c = 0 \rightarrow \frac{d\hat{\beta}}{d\sigma_c} = \frac{\frac{\partial g_2}{\partial \sigma_c} - \frac{\partial g_1}{\partial \sigma_c}}{\frac{\partial g_1}{\partial \hat{\beta}} - \frac{\partial g_2}{\partial \hat{\beta}}}$$

First, we show that g_2 is increasing in σ_c. For that, it suffices to show that f_1 is increasing in σ_c and f_2 is decreasing in σ_c. Note that $f_1 = \frac{e^{\frac{x}{\sqrt{3}}\mu_c}}{2x} \frac{e^{x\sigma_c} - e^{-x\sigma_c}}{\sigma_c}$ where $x = \frac{(1-\hat{\beta})vN(\tau_{sc}-\tau_{nsc})s\sqrt{3}}{1+s}$. We take the derivative of $\frac{e^{x\sigma_c}-e^{-x\sigma_c}}{\sigma_c}$ with respect to σ_c, and show that it is always positive.

$$\frac{\partial}{\partial\sigma_c}\left(\frac{e^{x\sigma_c}-e^{-x\sigma_c}}{\sigma_c}\right) = \frac{(xe^{x\sigma_c}+xe^{-x\sigma_c})\sigma_c - (e^{x\sigma_c}-e^{-x\sigma_c})}{\sigma_c^2} \geq 0 \leftrightarrow x\sigma_c \geq \frac{e^{x\sigma_c}-e^{-x\sigma_c}}{e^{x\sigma_c}+e^{-x\sigma_c}}$$

So, it suffices to show that $x\sigma_c \geq \frac{e^{x\sigma_c}-e^{-x\sigma_c}}{e^{x\sigma_c}+e^{-x\sigma_c}} \forall \sigma_c$. First, we will show that $\frac{e^{x\sigma_c}-e^{-x\sigma_c}}{e^{x\sigma_c}+e^{-x\sigma_c}}$ is a concave increasing function of σ_c, and its derivative when $\sigma_c = 0$ is equal to x. So, this implies that $x\sigma_c \geq \frac{e^{x\sigma_c}-e^{-x\sigma_c}}{e^{x\sigma_c}+e^{-x\sigma_c}}$ is always less than $x\sigma_c$ for all σ_c. To show that it is concave increasing, we take its first derivative w.r.t. σ_c:

$$\frac{\partial}{\partial\sigma_c}\left(\frac{e^{x\sigma_c}-e^{-x\sigma_c}}{e^{x\sigma_c}+e^{-x\sigma_c}}\right) = 1 - \left(\frac{e^{x\sigma_c}-e^{-x\sigma_c}}{e^{x\sigma_c}+e^{-x\sigma_c}}\right)^2$$

Note that since $\frac{e^{x\sigma_c}-e^{-x\sigma_c}}{e^{x\sigma_c}+e^{-x\sigma_c}}$ is always less than 1, the first derivative is always positive; hence it is increasing. Also note that since $\frac{e^{x\sigma_c}-e^{-x\sigma_c}}{e^{x\sigma_c}+e^{-x\sigma_c}}$ is increasing, its first derivative is decreasing in σ_c. So, we can establish that $\frac{e^{x\sigma_c}-e^{-x\sigma_c}}{e^{x\sigma_c}+e^{-x\sigma_c}}$ is a concave increasing function of σ_c.

To show that f_2 is decreasing in σ_c, it suffices to show that both

$$\frac{e^{\frac{\sqrt{3}u(1+\tau_{nscs})}{1+s}\sigma_c} - e^{-\frac{\sqrt{3}u(1+\tau_{nscs})}{1+s}\sigma_c}}{e^{\sqrt{3}u\sigma_c} - e^{-\sqrt{3}u\sigma_c}} \text{ and } \frac{e^{\frac{\sqrt{3}v(1+\tau_{nscs})}{1+s}\sigma_c} - e^{-\frac{\sqrt{3}v(1+\tau_{nscs})}{1+s}\sigma_c}}{e^{\sqrt{3}v\sigma_c} - e^{-\sqrt{3}v\sigma_c}}$$

are decreasing in σ_c. Since $s \geq 0$, it implies $\frac{u}{1+s} \leq u$. Therefore, it is sufficient to show that

$$l(\sigma_c) = \frac{e^{y\sigma_c}-e^{-y\sigma_c}}{e^{\sigma_c}-e^{-\sigma_c}}$$

is decreasing in σ_c for all $\sigma_c \geq 0$ and $y \in [0,1]$. First, we calculate the derivative of $l(\sigma_c)$ w.r.t. σ_c:

$$l'(\sigma_c) = \frac{y\left(e^{y\sigma_c}+e^{-y\sigma_c}\right)\left(e^{\sigma_c}-e^{-\sigma_c}\right) - \left(e^{y\sigma_c}-e^{-y\sigma_c}\right)\left(e^{\sigma_c}+e^{-\sigma_c}\right)}{\left(e^{\sigma_c}-e^{-\sigma_c}\right)^2}$$

Now, we will show that $l'(\sigma_c) \leq 0$ for all σ_c. Since denominator of $l'(\sigma_c)$ is always positive for all σ_c, it suffices to show that numerator is always negative, i.e.,

$$y\left(e^{y\sigma_c}+e^{-y\sigma_c}\right)\left(e^{\sigma_c}-e^{-\sigma_c}\right) - \left(e^{y\sigma_c}-e^{-y\sigma_c}\right)\left(e^{\sigma_c}+e^{-\sigma_c}\right) \leq 0$$

Rewriting the expression above, we need to show that for all σ_c:

$$y\frac{e^{\sigma_c}-e^{-\sigma_c}}{e^{\sigma_c}+e^{-\sigma_c}} \leq \frac{e^{y\sigma_c}-e^{-y\sigma_c}}{e^{y\sigma_c}+e^{-y\sigma_c}}$$

Let the left hand side (lhs) of the above expression be denoted by $v(\sigma_c)$. Then, the expression above holds if and only if $yv(\sigma_c) \leq v(y\sigma_c)$ for all $\sigma_c \geq 0$. But $yv(\sigma_c) \leq v(y\sigma_c)$ for all σ_c due to the fact that $v(\sigma_c)$ is concave increasing in σ_c:

$$v'(\sigma_c) = 1 - \left(\frac{e^{\sigma_c}-e^{-\sigma_c}}{e^{\sigma_c}+e^{-\sigma_c}}\right)^2 = 1 - v(\sigma_c)^2 \geq 0$$

since $0 \le v(\sigma_c) \le 1$ for all σ_c. Also, $v''(\sigma_c) = -2v'v \le 0$.

Next, we analyze how g_1 behaves w.r.t. σ_c. Note that g_1 is constant in σ_c; hence $\frac{\partial g_1}{\partial \sigma_c} = 0$. Therefore, $\frac{\partial g_2}{\partial \sigma_c} - \frac{\partial g_1}{\partial \sigma_c} \ge 0$.

Now, we analyze $\frac{\partial g_1}{\partial \hat{\beta}} - \frac{\partial g_2}{\partial \hat{\beta}}$. First, we consider $\frac{\partial g_1}{\partial \hat{\beta}}$. Recall that $g_1 = \hat{\beta}\frac{1-\alpha(w)}{\alpha(w)} \exp\left(\frac{-(1-\hat{\beta})vN(\tau_{sc}-\tau_{nsc})s}{u(1+\tau_{nsc})}\right)$. It is trivial to show that it is increasing in $\hat{\beta}$, i.e., $\frac{\partial g_1}{\partial \hat{\beta}} \ge 0$. Finally, we consider $\frac{\partial g_2}{\partial \hat{\beta}}$. Note that the only term in g_2 that depends on $\hat{\beta}$ is f_1. Recall that $f_1 = \frac{e^{\frac{x}{\sqrt{3}}\mu_c}}{2x} \frac{e^{x\sigma_c} - e^{-x\sigma_c}}{\sigma_c}$ where $x = \frac{(1-\hat{\beta})vN(\tau_{sc}-\tau_{nsc})s\sqrt{3}}{1+s}$, and we already show that it is increasing in σ_c. The same argument can be used to show that it is increasing in x. But since x decreases in $\hat{\beta}$, it implies that f_1 decreases in $\hat{\beta}$, i.e., $\frac{\partial f_1}{\partial \hat{\beta}} \le 0 \rightarrow \frac{\partial g_2}{\partial \hat{\beta}} \le 0$. Therefore, $\frac{\partial g_1}{\partial \hat{\beta}} - \frac{\partial g_2}{\partial \hat{\beta}} \ge 0$.

Proof of Proposition 5:

We focus on finding a pure strategy equilibrium, and solve the game by backward induction starting from the last stage. Since the proof of this proposition is similar to the proof of proposition 1, we will only state the differences.

1. Suppose that firm i's first stage decision is f_i. Then the optimal price for firm i in each period in the second stage, i.e. p_i^*, solves the first order condition similar to the part 1 of Proposition 1. This implies that

$$p_i^* = \begin{cases} c(\gamma) + \frac{1}{u}, & \text{for } f_i = \text{ZS} \\ \frac{c(\gamma)}{1+s} + \frac{1}{u(1+\tau_{nsc}s)}, & \text{for } f_i = \text{PS} \end{cases} \tag{11}$$

2. Now, we can express firm i's profit in the first stage as follows:

$$\pi^i_{f_i,f_j}(p_i^*, p_j^*) = \frac{1}{u}\begin{cases} \alpha(w)\exp(-uc(\gamma) - 1 + \frac{vc(\gamma)(1+\tau_{sc}s)}{1+s} + \frac{v}{u}\frac{(1+\tau_{sc}s)}{(1+\tau_{nsc}s)}) & \\ +\gamma_i(1-\alpha(w))\exp(-uc(\gamma) - 1 + \frac{vc(\gamma)(1+\tau_{nsc}s)}{1+s} + \frac{v}{u}), & \text{for } f_i = \text{ZS}, f_j = \text{PS} \\ \gamma_i \exp(-uc(\gamma) - 1 + vc(\gamma) + \frac{v}{u}), & \text{for } f_i = \text{ZS}, f_j = \text{ZS} \\ \gamma_i(1-\alpha(w))\frac{1+s}{1+\tau_{nsc}s}\exp(-\frac{uc(\gamma)(1+\tau_{nsc}s)}{1+s} - 1 + \frac{vc(\gamma)(1+\tau_{nsc}s)}{1+s} + \frac{v}{u}), & \text{for } f_i = \text{PS}, f_j = \text{PS} \\ \gamma_i(1-\alpha(w))\frac{1+s}{1+\tau_{nsc}s}\exp(-\frac{uc(\gamma)(1+\tau_{nsc}s)}{1+s} - 1 + vc(\gamma) + \frac{v}{u}), & \text{for } f_i = \text{PS}, f_j = \text{ZS} \end{cases} \tag{12}$$

For notational simplicity, let $\pi^i_{f_i,f_j} = \pi^i_{f_i,f_j}(p_i^*, p_j^*)$. Now, we will analyze the first stage game. Let $f_i^*(f_j|\gamma_i)$ be the best response function of firm i given that the firm j chooses to be f_j, i.e.,

$$f_i^*(f_j|\gamma_i) = \arg\max_{f_i \in \{\text{ZS},\text{PS}\}} \pi^i_{f_i,f_j}$$

In this stage, we will consider only pure Nash equilibria, i.e., (f_i^*, f_j^*) pair which solves

$$f_i^* = f_i^*(f_j^*|\gamma_i) \text{ and } f_j^* = f_j^*(f_i^*|\gamma_j).$$

Now, suppose that firm j opts for ZS. Then, the best response of firm i depends on whether the following inequality is satisfied or not:

$$\exp\left(\frac{as(1-\tau_{nsc})}{1+s}c(\gamma_i)\right) \le \frac{1+\tau_{nsc}s}{(1-\alpha(w))(1+s)}$$

Note that we assume that $c(\gamma_i)$ decreases in γ_i. Therefore, the above inequality suggests a threshold for γ_i such that if γ_i is greater than this threshold then inequality holds, otherwise it does not hold. Let $\bar{\bar{\gamma}}$ be the threshold value, i.e.,

$$\exp\left(\frac{as(1-\tau_{nsc})}{1+s}c(\bar{\bar{\gamma}})\right) = \frac{1+\tau_{nsc}s}{(1-\alpha(w))(1+s)}$$

To summarize, if firm i's popularity, i.e., $\gamma_i \geq \bar{\bar{\gamma}}$, then it opts for ZS otherwise PS. Note that left hand side is decreasing in $\bar{\bar{\gamma}}$, and τ_{nsc}, and increasing in s and u, whereas right hand side is decreasing in s and w, and increasing in τ_{nsc}. Therefore, as long as w or u or s is relatively low and/or τ_{nsc} is relatively high, $\bar{\bar{\gamma}}$ becomes small and it becomes quite likely that the condition of Proposition 5 will be satisfied.

Now, suppose that firm j opts for PS. Then, firm i's response depends on whether $\pi^i_{ZS,PS} \geq \pi^i_{PS,PS}$ or not. If $\pi^i_{ZS,PS} \geq \pi^i_{PS,PS}$, then the best response of firm i is to opt for ZS, otherwise PS. Recall that

$$\frac{\pi^i_{PS,PS}}{\pi^i_{ZS,PS}} = \frac{\gamma_i}{\alpha(w)z + (1-\alpha(w))\gamma_i}(1-\alpha(w))\frac{1+s}{1+\tau_{nsc}s}\exp(\frac{as(1-\tau_{nsc})}{1+s}c(\gamma_i))$$

where

$$z = \exp\left(\frac{vc(\gamma)(\tau_{sc}-\tau_{nsc})s}{1+s} + \frac{v}{u}\frac{(\tau_{sc}-\tau_{nsc})s}{(1+\tau_{nsc}s)}\right)$$

Recall that $\gamma_i \geq \bar{\bar{\gamma}}$ implies that

$$(1-\alpha(w))\frac{1+s}{1+\tau_{nsc}s}\exp(\frac{as(1-\tau_{nsc})}{1+s}c(\gamma_i)) \leq 1$$

This in turn implies that

$$\frac{\pi^i_{PS,PS}}{\pi^i_{ZS,PS}} = \frac{\gamma_i}{\alpha(w)z + (1-\alpha(w))\gamma_i}(1-\alpha(w))\frac{1+s}{1+\tau_{nsc}s}\exp(\frac{as(1-\tau_{nsc})}{1+s}c(\gamma_i)) \leq 1$$

for all γ_i. That is, if $\gamma_i \geq \bar{\bar{\gamma}}$, the best response of firm i to firm j's PS strategy is to opt for ZS. So, if $\gamma_i \geq \bar{\bar{\gamma}}$, these results suggest that firm i opts for ZS irrespective of firm j's decision. The same logic also holds for firm j's best response function. Consequently, we have the results of Proposition 5.

Proof of Corollary 2: In this proof, we only consider the case where one of the two firms has a popularity that is less than $\bar{\bar{\gamma}}$. Recall that $\bar{\bar{\gamma}}$ satisfies

$$\exp\left(\frac{as(1-\tau_{nsc})}{1+s}c(\bar{\bar{\gamma}})\right) \leq \frac{1+\tau_{nsc}s}{(1-\alpha(w))(1+s)}$$

Since $c(\gamma)$ is decreasing in γ and $\exp(x)$ is increasing in x, the left hand side of the above equation is decreasing in $\bar{\bar{\gamma}}$, whereas the right hand side is increasing in $\alpha(w)$. This suggests that $\bar{\bar{\gamma}}$ that satisfies the above equation is decreasing in $\alpha(w)$. Recall that from the proof of Proposition 5, the firm whose popularity is less than $\bar{\bar{\gamma}}$ opts for PS strategy. Hence, as $\bar{\bar{\gamma}}$ decreases, the firm with lower popularity switches its strategy from PS to ZS.

A.2 Extension: An alternative formulation of the demand model and its analysis

The model framework in the paper assumes that there are two groups of customers - shipping-charge-skeptics ($\alpha(w)$ proportion) and non-shipping-charge-skeptics ($1-\alpha(w)$ proportion). The weight that the former group puts on S&H charge while making purchase decision is more than the latter. Moreover, while the sizes of the groups depend on the total prices charged by PS and ZS firms, we assume that the shipping-charge-skeptics consider only ZS firms for purchasing, while non-shipping-charge-skeptics consider both ZS and PS firms.

In this extension we consider an alternative model framework. There are still two groups of customers - skeptics ($\alpha(w)$ proportion) and non-skeptics ($1-\alpha(w)$ proportion) and the weight that the former group puts on S&H charge is still more than the latter. However, now both customer groups consider both PS and ZS firms as potential purchase sources. Hence, demand for each ZS and PS firm contains two parts (one for each customer group):

$$\begin{aligned} d^i_{N_{zs},N_{ps}}(p_i,p_{-i}|f_i=\text{ZS}) = & \overbrace{\left(\frac{\gamma_i\alpha(w)}{\sum_{i\in N}\gamma_i}\exp(-up_i+\sum_{j\in N_{zs}\setminus\{i\}}vp_j+\sum_{j\in N_{ps}}vp_j(1+\tau_{sc}s))\right)}^{\text{Demand from shipping-charge skeptics}} \\ & + \overbrace{\left(\frac{\gamma_i(1-\alpha(w))}{\sum_{i\in N}\gamma_i}\exp(-up_i+\sum_{j\in N_{zs}\setminus\{i\}}vp_j+\sum_{j\in N_{ps}}vp_j(1+\tau_{nsc}s))\right)}^{\text{Demand from non-shipping-charge-skeptics}} \end{aligned} \tag{13}$$

$$\begin{aligned} d^i_{N_{zs},N_{ps}}(p_i,p_{-i}|f_i=\text{PS}) = & \overbrace{\left(\frac{\gamma_i\alpha(w)}{\sum_{i\in N}\gamma_i}\exp(-up_i(1+\tau_{sc}s)+\sum_{j\in N_{zs}}vp_j+\sum_{j\in N_{ps}\setminus\{i\}}vp_j(1+\tau_{sc}s))\right)}^{\text{Demand from shipping-charge-skeptics}} \\ & + \overbrace{\left(\frac{\gamma_i(1-\alpha(w))}{\sum_{i\in N}\gamma_i}\exp(-up_i(1+\tau_{nsc}s)+\sum_{j\in N_{ps}\setminus\{i\}}vp_j(1+\tau_{nsc}s)+\sum_{j\in N_{zs}}vp_j)\right)}^{\text{Demand from non-shipping-charge-skeptics}}, \end{aligned} \tag{14}$$

where $0 \le \tau_{nsc} \le 1 \le \tau_{sc}$ represents the relative weight of shipping charge for non-skeptics and skeptics, similar to the original demand model. The main differences between the above demand functions to those in equations (1) and (2) of the paper are two-fold: i) in the above alternate model framework, there is an extra term in the demand function for PS firms representing the demand generated from skeptics (i.e., the first term in equation (14) is not there in equation (2) of the paper); and ii) the demand from shipping-charge-skeptics is now divided among all N firms (in equation (1) of the paper the demand is divided among only ZS firms). Note that the decision variables and the order of the game still remain the same as in the paper. That is, each retailer makes two decisions. In the first stage, each of them simultaneously decides on whether it should adopt the PS strategy or the ZS strategy. Subsequently, in the second stage, each retailer, given its partitioning decision, simultaneously determines the exact values of the product price and S&H fee (if applicable) that consumers need to pay. As our analysis shows below, although the values of the optimal decisions of this alternate model framework might be different from the ones in the paper, the primary analytical qualitative insights of the paper actually do not change.

We analyze the symmetric N-firm version of the two-stage game, by assuming that $\gamma_i = \frac{1}{N}$ for all $i = 1, \ldots, N$. Given that $N_{zs} = \beta N$ and $N_{ps} = (1-\beta)N$ retailers opt for ZS and PS strategy, respectively, in the first stage, equilibrium prices in the second stage can be characterized as follows:

1. **Characterization of unique equilibrium ZS price:** Note that for a ZS firm, the demand function is essentially similar to the original demand function, except the fact that the demand from shipping charge sceptics are now divided among all firms. However, this does not change the equilibrium pricing strategy for a ZS firm, i.e., $p^*_{zs} = c + \frac{1}{u}$ when $f_i = ZS$.

2. **Characterization of unique equilibrium PS price:** The characterization of the equilibrium PS price turns out to be more involved than in the paper. Suppose that $f_i = PS$. Then, the first order condition would lead to the following equation:

$$\begin{aligned}\frac{\partial \pi^i_{N_{zs},N_{ps}}(p_i, p_{-i}|f_i = \text{PS})}{\partial p_i} &= \frac{\partial}{\partial p_i}(p_i(1+s)-c) \times d^i_{N_{zs},N_{ps}}(p_i, p_{-i}|f_i = \text{PS}) \\ &+ (p_i(1+s)-c) \times \frac{\partial}{\partial p_i} d^i_{N_{zs},N_{ps}}(p_i, p_{-i}|f_i = \text{PS})\end{aligned}$$

Let d_{sc} and d_{nsc} denote the demand for a PS firm from the shipping-charge-skeptics and non-shipping-charge-skeptics, respectively. Then, we can rewrite the first order condition as follows:

$$\begin{aligned}&\to (1+s)(d_{sc}+d_{nsc}) = (p_i(1+s)-c)(u(1+\tau_{sc}s)d_{sc} + u(1+\tau_{nsc}s)d_{nsc}) \\ &\to ((p_i(1+s)-c)u(1+\tau_{sc}s) - (1+s))d_{sc} = ((1+s) - (p_i(1+s)-c)u(1+\tau_{nsc}s))d_{nsc} \\ &\to \frac{d_{sc}}{d_{nsc}} = \frac{(1+s) - (p_i(1+s)-c)u(1+\tau_{nsc}s)}{(p_i(1+s)-c)u(1+\tau_{sc}s) - (1+s)} \\ &\to \frac{d_{sc}}{d_{nsc}} = \frac{(1+\tau_{nsc}s)}{(1+\tau_{sc}s)} \frac{\frac{1}{u(1+\tau_{nsc}s)} + \frac{c}{1+s} - p_i}{p_i - \frac{1}{u(1+\tau_{sc}s)} - \frac{c}{1+s}}\end{aligned}$$

Substituting d_{sc} and d_{nsc} into above expression,

$$\begin{aligned}&\to \frac{\alpha(w)}{1-\alpha(w)} \frac{\exp(-up_i(1+\tau_{sc}s) + \sum_{j\in N_{zs}} vp_j + \sum_{j\in N_{ps}\setminus\{i\}} vp_j(1+\tau_{sc}s))}{\exp(-up_i(1+\tau_{nsc}s) + \sum_{j\in N_{ps}\setminus\{i\}} vp_j(1+\tau_{nsc}s) + \sum_{j\in N_{zs}} vp_j)} = \frac{1+\tau_{nsc}s}{1+\tau_{sc}s} \frac{\frac{1}{u(1+\tau_{nsc}s)} + \frac{c}{1+s} - p_i}{p_i - \frac{1}{u(1+\tau_{sc}s)} - \frac{c}{1+s}} \\ &\to \frac{\alpha(w)}{1-\alpha(w)} \exp(-up_i(\tau_{sc}-\tau_{nsc})s + \sum_{j\in N_{ps}\setminus\{i\}} vp_j(\tau_{sc}-\tau_{nsc})s) = \frac{1+\tau_{nsc}s}{1+\tau_{sc}s} \frac{\frac{1}{u(1+\tau_{nsc}s)} + \frac{c}{1+s} - p_i}{p_i - \frac{1}{u(1+\tau_{sc}s)} - \frac{c}{1+s}} \\ &\to \exp((-(u+v) + N(1-\beta)v)(\tau_{sc}-\tau_{nsc})sp_i) = (\frac{1+\tau_{nsc}s}{1+\tau_{sc}s}) \frac{\alpha(w)}{1-\alpha(w)} \frac{\frac{1}{u(1+\tau_{nsc}s)} + \frac{c}{1+s} - p_i}{p_i - \frac{1}{u(1+\tau_{sc}s)} - \frac{c}{1+s}}\end{aligned}$$

Analysis of the above expression shows that the right hand side (rhs) decreases (rapidly) from $+\infty$ to 0 as p increases from $\frac{1}{u(1+\tau_{sc}s)}+\frac{c}{1+s}$ to $\frac{1}{u(1+\tau_{nsc}s)}+\frac{c}{1+s}$. On the other hand, the left hand side (lhs) is equal to 1 as $p_i=0$. If $(u+v)\geq N(1-\beta)v$, then the lhs decreases gradually and approaches 0 as p_i tends towards infinity; if $(u+v)<N(1-\beta)v$, then the lhs increases and approaches ∞ as p_i tends towards infinity (both lhs and rhs are always positive). It is then easy to show that there is a unique equilibrium PS price p^*_{ps} solving the above expression, where $\frac{1}{u(1+\tau_{sc}s)}+\frac{c}{1+s}<p^*_{ps}<\frac{1}{u(1+\tau_{nsc}s)}+\frac{c}{1+s}$. Note that since p^*_{ps} for the demand model of the paper is given by $\frac{1}{u(1+\tau_{nsc}s)}+\frac{c}{1+s}$, so the equilibrium PS price in the alternative demand framework will be lower than that of the model in the paper. Based on the above discussion, we can also show that as $N(1-\beta)$, i.e., the number of PS firms, increases, p^*_{ps} decreases. This makes sense since competition among more PS firms imply that it should result in lower prices for PS firms.

The characterization of unique PS and ZS prices above is *counterpart of Proposition 1* (second and third parts) in the paper. The main difference is that now the complicated form of the first order equation means that there is no closed-form expression for p^*_{ps}.

3. **Comparison of unit and total PS and ZS prices:** Recall that equilibrium unit price and S&H charge for PS and ZS retailers are $\frac{c}{1+s}+\frac{1}{u(1+s)}<p^*_{ps}<\frac{c}{1+s}+\frac{1}{u(1+\tau_{nsc}s)}$, $\frac{cs}{1+s}+\frac{s}{u(1+s)}<s^*_{ps}<\frac{cs}{1+s}+\frac{s}{u(1+\tau_{nsc}s)}$, $p^*_{zs}=c+\frac{1}{u}$, and $s^*_{zs}=0$, respectively. Using the price expressions and noting that $s>0$, we can show that $p^*_{ps}<p^*_{zs}$ and $p^*_{ps}+s^*_{ps}\geq p^*_{zs}$ (we do not provide the details here due to space constraints). This is the *counterpart of Corollary 1* in the paper.

4. **Characterization of unique equilibrium proportion of ZS firms (β^*):** Let β be the proportion of ZS firms in the market. Then $\beta=\frac{N_s}{N}$. Let β^* be the equilibrium proportion of ZS firms in the market defined in such a way that none of the firms would profitably change their shipping policies. We can rewrite the first stage profit functions of firm i in terms of β as follows:

$$\pi^i_{ZS}(\beta)=\frac{1}{N}(p^*_i-c)\Bigg(\alpha(w)\exp(-up^*_i+\sum_{j\in N_{zs}\setminus\{i\}}vp^*_j+\sum_{j\in N_{ps}}vp^*_j(1+\tau_{sc}s))$$
$$+(1-\alpha(w))\exp(-up^*_i+\sum_{j\in N_{zs}\setminus\{i\}}vp^*_j+\sum_{j\in N_{ps}}vp^*_j(1+\tau_{nsc}s))\Bigg)$$

and

$$\pi^i_{PS}(\beta)=(p^*_i(1+s)-c)\frac{1}{N}\Bigg(\alpha(w)\exp(-up^*_i(1+\tau_{sc}s)+\sum_{j\in N_{zs}}vp^*_j+\sum_{j\in N_{ps}\setminus\{i\}}vp^*_j(1+\tau_{sc}s))$$
$$+(1-\alpha(w))\exp(-up^*_i(1+\tau_{nsc}s)+\sum_{j\in N_{ps}\setminus\{i\}}vp^*_j(1+\tau_{nsc}s)+\sum_{j\in N_{zs}}vp^*_j)\Bigg)$$

Recall that the equilibrium proportion is characterized by the following equation

$$\pi^i_{ZS}(\beta^*)=\pi^i_{PS}(\beta^*)$$

At this intersection point, no firm can increase her profit by changing her shipping policy. After substituting p^*_{zs} and p^*_{ps} into above equation, for the left hand side, we obtain:

$$\pi^i_{ZS}(\beta) = \frac{1}{u}\frac{1}{N}\exp(\beta N v p^*_{zs} + (1-\beta)Nvp^*_{ps}(1+\tau_{sc}s)) \\ \big(\frac{\alpha(w)}{1-\alpha(w)} + \exp(-(1-\beta)Nvp^*_{ps}(\tau_{sc}-\tau_{nsc})s)\big)\exp(-(u+v)p^*_{zs})$$

and for the right hand side, we obtain:

$$\pi^i_{PS}(\beta) = (p(1+s)-c)\frac{1}{N}\exp(\beta N v p^*_{zs} + (1-\beta)Nvp^*_{ps}(1+\tau_{sc}s))\exp(-(u+v)(1+\tau_{sc}s)p^*_{ps}) \\ \left(\frac{\alpha(w)}{1-\alpha(w)} + \exp((u+v)(\tau_{sc}-\tau_{nsc})p^*_{ps}s)\exp(-(1-\beta)Nv(\tau_{sc}-\tau_{nsc})p^*_{ps}s)\right)$$

Now, equating $\pi^i_{ZS}(\beta) = \pi^i_{PS}(\beta)$, and canceling out the common terms in right and left hand sides, we obtain the following expression, the solution of which results in β^*:

$$\frac{\frac{\alpha(w)}{1-\alpha(w)} + e^{-(1-\beta)Nvp^*_{ps}(\tau_{sc}-\tau_{nsc})s}}{\frac{\alpha(w)}{1-\alpha(w)} + e^{(u+v)(\tau_{sc}-\tau_{nsc})p^*_{ps}s}e^{-(1-\beta)vN(\tau_{sc}-\tau_{nsc})p^*_{ps}s}} = \frac{(p^*_{ps}(1+s)-c)}{(p^*_{zs}-c)}\frac{e^{-(u+v)(1+\tau_{sc}s)p^*_{ps}}}{e^{-(u+v)p^*_{zs}}} \tag{15}$$

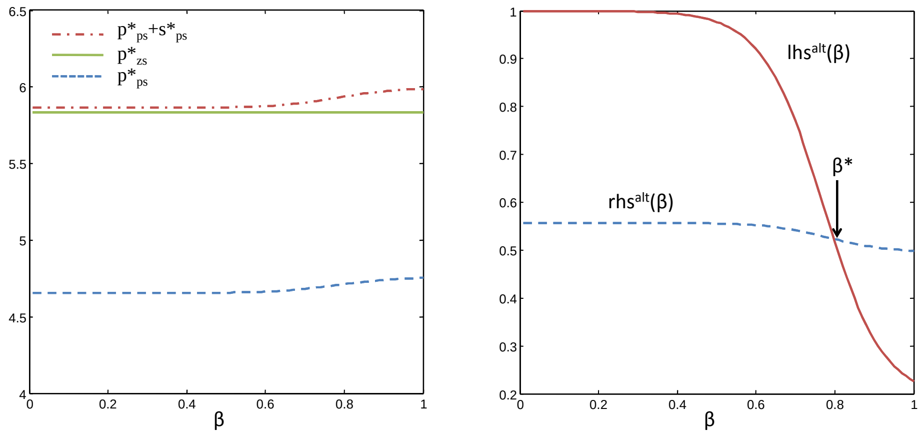
Note that, in the above expression, even though p^*_{zs} does not depend on β, p^*_{ps} is not constant in β anymore (it is so for the model framework in the paper). Hence both left (lhs) and right hand sides (rhs) of the above equation depend on β. For the rhs, it depends both directly through β and indirectly through p^*_{ps}; for the lhs, it depends only indirectly through p^*_{ps}. Moreover, as indicated above, we only have an implicit expression for p^*_{ps}. Consequently, it is very difficult to analytically characterize how the left hand side varies with β (we can analytically argue that the right hand side is decreasing in β). Indeed, our numerical study shows that both left and right hand sides might be decreasing in β. This suggests that a unique characterization of β^* might be intractable from an analytical point of view.

Keeping in mind the above, we resorted to numerical experiments to study the behavior of β^*. The right hand side of (15) is always decreasing in β; the left hand side is also mostly decreasing, although sometimes it might be non-monotone. However, in all our experiments there was a *unique equilibrium proportion of ZS firms (i.e., β^*), which corroborates Proposition 1* (first part) of the paper. An illustrative example of the right and left hand sides of equation (15) as well as β^* is shown below in Figure 1(B) (more details about the numerical study are available from the authors). Note that Figure 1(A) shows how the equilibrium prices for PS and ZS firms behave with respect to β.

5. **Comparison of equilibrium decision variable values with the model framework in the paper:** Since the model framework in this extension is somewhat different from the one in the paper, the values of the equilibrium decision variables (i.e., equilibrium proportion of ZS and PS firms as well as equilibrium ZS and PS prices) in the two are also different. In order to compare

Figure 1 **Equilibrium prices and proportion of ZS retailers for the alternative demand function.**

$(u=1.2; v=0.8; s=0.25; \tau_{sc}=1.1; \tau_{nsc}=0.4; N=20; c=5; \alpha(w)=0.2)$



(A) Equilibrium prices with respect to β (B) Equilibrium Proportion of ZS firms.

Note. lhs^{alt} and rhs^{alt} refer to the left and right hand sides of the equation that defines β^* in alternative demand formulation.

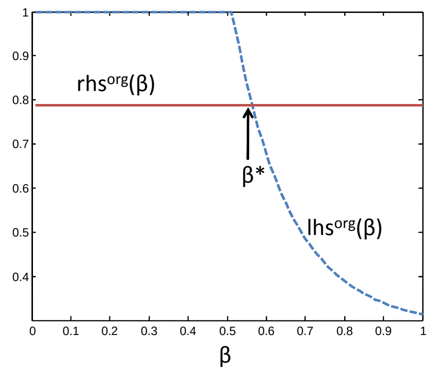
the equilibrium characterizations of two demand models, we first rewrite the equation that defines β^* in the original demand model (i.e., the demand model in the revised version of the paper) as follows:

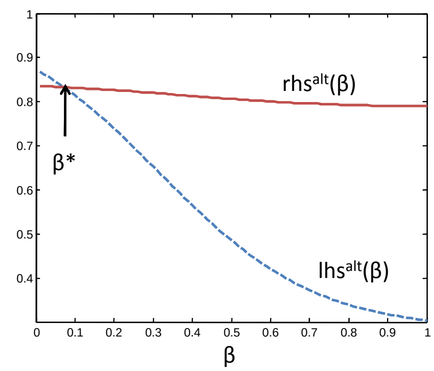
$$\overbrace{\frac{\alpha(w)}{1-\alpha(w)}\frac{e^{(1-\beta)Nvp^*_{ps}(\tau_{sc}-\tau_{nsc})s}}{\beta e^{(u+v)(\tau_{sc}-\tau_{nsc})p^*_{ps}s}}+e^{-(u+v)(\tau_{sc}-\tau_{nsc})p^*_{ps}s}}^{\text{lhs}^{\text{org}}}=\overbrace{\frac{(p^*_{ps}(1+s)-c)}{(p^*_{zs}-c)}\frac{e^{-(u+v)(1+\tau_{sc}s)p^*_{ps}}}{e^{-(u+v)p^*_{zs}}}}^{\text{rhs}^{\text{org}}} \tag{16}$$

Note that the form of right hand side in (16) is similar to the right hand side in (15) except the fact that optimal ZS and PS prices are constant in original demand formulation, whereas they depend on β in alternative demand formulation. Specifically, rhs^{org} is constant in β whereas rhs^{alt} is decreasing in β. Also, left hand sides in both original and alternative demand formulations, i.e., lhs^{org} and lhs^{alt} are decreasing in β. However, note that lhs^{org} is greater than lhs^{alt}, which leads to β^* in original demand formulation being greater than the one in alternative demand formulation. That is, *the equilibrium proportion of ZS firms in the market is higher in the original demand formulation compared to the alternative demand formulation.* This result is driven mainly by the fact that original demand function gives more benefit to a ZS firm than alternative demand formulation does. We present an illustrative example of LHS and RHS for the original and alternative demand formulations with respect to β as well as the values of β^* for the two scenarios in Figure 2. As far as the prices under two formulations are concerned, evidently *ZS product price is the same under both models*. On the other hand, as we have analytically shown above the *PS product price (and hence, S&H charge) is lower for the alternative demand formulation compared to the original one.*

Figure 2 **Comparison of equilibrium proportion of ZS firms (β^*) in original demand formulation to the one in alternative demand formulation.**

$(u = 1.1; v = 0.4; s = 0.25; \tau_{sc} = 1.1; \tau_{nsc} = 0.4; c = 5; N = 30; \alpha(w) = 0.1)$





(A) Equilibrium β^* in original demand formulation (B) Equilibrium β^* in alternative demand formulation

Note. lhs^{org} and rhs^{org} refer to the left and right hand sides of the equation that defines β^* in the original demand formulation.

6. **Behavior of equilibrium proportion β^*:** Although the values of the equilibrium β^* are different in this model compared to the framework in the paper, it turns out that almost all the analytical qualitative insights of the paper still hold true (both for product and retailer levels). Specifically, our numerical experiments show that:[18]

(a) The equilibrium proportion of ZS retailers is lower for bigger and/or heavier products (i.e., β^* decreases in w). (Proposition 2a)

(b) The equilibrium proportion of ZS retailers, i.e., β^*, decreases in the product cost c (it turns out that for high c, this might not be true). (Proposition 2b)

(c) The equilibrium proportion of ZS retailers increases as the coefficient of variation (i.e., $cov_c = \frac{\sigma_c}{\mu_c}$) of the total cost becomes larger. (Proposition 4b)

(d) ZS (PS) becomes a dominant strategy for a firm as its popularity (i.e., γ) increases (decreases). (Proposition 5)

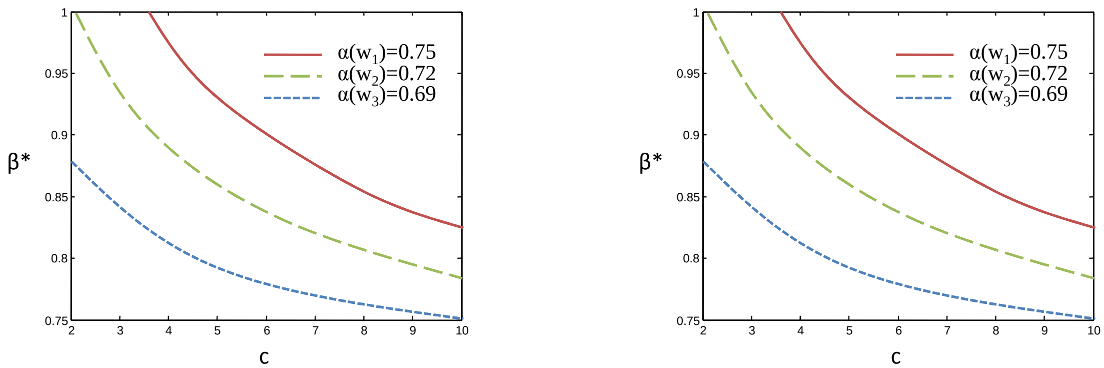
Due to space limitations, we do not present the details of our numerical study here (it is available from the authors). Below we provide some illustrative examples of the above behavior of β^*. Specifically, Figure 3 shows the effects of w, c and σ_c on the equilibrium proportion of ZS firms, i.e., β^*, while Figure 4 establishes that indeed ZS (PS) becomes a dominant strategy as γ increases (decreases).

In summary, our above analysis shows that, although the values of the optimal decisions of this alternate model framework might be different from the original one in the paper, the primary analytical qualitative insights of the paper actually do not change.

[18] Since we cannot analytically establish the uniqueness of β^*, theoretical sensitivity analysis of β^* is also not possible.

Figure 3 **Effects of w, c and σ_c on proportion of ZS firms, i.e., β^*.**

$(u = 1.1; v = 0.05; s = 0.25; \tau_{sc} = 1.4; \tau_{nsc} = 0.25; N = 30)$ $(u = 1.1; v = 0.6; s = 0.2; \tau_{sc} = 1.1; \tau_{nsc} = 0.5; N = 15)$

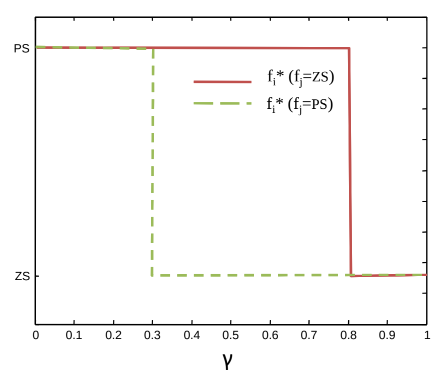


(A) Sensitivity of β^* with respect to c (B) Sensitivity of β^* with respect to σ_c for a given $\mu_c = 2$.

Note. In the figures, $w_1 < w_2 < w_3$.

Figure 4 **Best response functions of firm i as a function of its popularity γ.**

$(u = 1; v = 0.5; s = 0.2; \tau_{sc} = 1.4; \tau_{nsc} = 0.7; c(\gamma) = 5(1 - \gamma); \alpha(w) = 0.45)$



Note. $f_i^*(f_j = ZS)$ and $f_i^*(f_j = PS)$ represent best response functions of a firm i if its opponent (i.e., firm j) adopts ZS and PS strategies, respectively. Note that in this instance, ZS (PS) becomes a dominant strategy for firm i as its popularity (i.e., γ) increases (decreases).