An Empirical Analysis of Intra-Firm Product Substitutability in Fashion Retailing

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Abstract

This study offers an empirical investigation of inventory and sales dynamics in a large-scale retail network setting. We infer the impact of product shortages on sales in neighboring outlets using unique data from a large fast fashion retailing chain and an Instrumented Difference-in-Differences (DDIV) methodology. Our analysis reveals that sales for a particular item at a focal store increases when that same item experiences stock-outs in neighboring stores. Our empirical findings suggest that there is substitutability across stores, and that this substitutability is the strongest in the period when the stock-out is observed for the first time, and decreases as time passes following the stock-out. In order to assess the implications of considering the impact of stock-outs on inventory allocation, we develop an optimization model that is calibrated using parameters estimated via our earlier DDIV analysis. The simulation analysis confirms that revenues markedly improve on average by 6% under low demand variance and by 14% under high demand variance when neighboring stock-out information is taken into account for sales forecasting while optimizing initial inventory allocations. Finally, we conduct sensitivity analysis to evaluate how these potential revenue improvements vary with turnover, product price, and inventory.

Keywords: Causal Inference, Customer Search, Fast Fashion Industry, Product Substitutability, Quasi-Experimental Econometrics.
Received: August 2019; accepted: August 2021 by Fred Feinberg after three revisions.

1 Introduction

One of the distinguishing features of retail chains is that they offer very similar (if not identical) products across all of their outlets via standardization, where each outlet itself can be thought of as a channel.[1] An implication of this feature is that customers may be able to find the same item across many stores within their vicinity, thereby propagating customer traffic and thus sales (e.g., Perdikaki et al. (2012)), within the retail network. Consequently, if one store within the retail network runs out of a particular item, the customer then has an option to search for the same item in another (and perhaps slightly more distant) store. Intuitively, the desire to explore other stores to find a specific

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[1]More generally, omnichannel settings may help mitigate lost sales from stock-outs, which relies on the notion that customers switch from one channel to another (e.g., Gallino and Moreno (2014)).

item may be especially pronounced in retail settings for which inventory is rarely replenished (e.g., fast fashion).[2] While such a scenario seems entirely plausible, there exists no concrete empirical evidence that such intra-firm product substitutability has a material impact on sales within a retail network.[3] It is important to study the impact of such substitution patterns as non-negligible portion of lost retail sales is due to items being out-of-stock (e.g., Campo et al. (2000); Corsten and Gruen (2004); Sloot et al. (2005)).[4] Being able to leverage these substitution patterns could ultimately help improve a retail network's productivity, especially if sales and stock-outs in proximal stores can be used in generating demand forecasts for a store in question or distributing the right amount of inventory across all the stores. Leveraging these intra-firm product substitution patterns may even allow retailers to maximize inventory productivity, which in turn has an impact on investor interest towards the retailer (e.g., Alan et al. (2014)). Our research will provide an empirical framework to infer the degree to which these substitution patterns matter, as well as the implication of having access to such information on demand forecasting and inventory allocation.[5]

Our empirical setting is fast fashion, which is often characterized as an industry with quick response and design capabilities (e.g., Cachon and Swinney (2011)). Consequently, this industry sector frequently faces out-of-stock situations, which is further exacerbated by the fact that their products exhibit a high degree of variety and short life-cycles (e.g., Sull and Turconi (2008)) as well as their tendency to not replenish depleted inventories. For example, in our data, only 15% of the sold-out products are replenished. In particular, we analyze a unique data set provided by one of the largest fast fashion retailers in Europe. With this data, we are able to observe weekly sales and inventory levels for all of their products across each of their outlets between 2016 and 2018. Furthermore, as we know exactly where each of their outlets is located, we know the exact distances between each outlet within the network. This spatial dimension ultimately helps us to study empirical patterns of intra-firm product substitutability.

For our analysis, we consider various empirical specifications that test for the impact of stock-outs

[2]For example, "[Zara] will not replenish, but rather replaces sold out styles with new looks. The consumer knows to purchase an item they like as soon as they see it because they will not see it again (*Forbes*, October 23, 2015)." On a similar note, at Zara, "new merchandise displayed in limited quantities and the short window of opportunity for purchasing items motivate people to visit Zara's shops more frequently than they might other stores (*HBS Working Knowledge*, February 21, 2005)."

[3]Our interpretation of "intra-firm" also extends to the notion of branding. That is one may think of "intra-firm" as substitution across products belonging to the same brand.

[4]See, for example, "Retailers are losing $1.75 trillion over this" (*CNBC*, November 30, 2015).

[5]More generally, in operations management, there is disproportionately less empirically-driven research as compared with studies that develop analytical/numerical models (e.g., Musalem et al. (2017)).

in neighboring outlets. The main challenge we face is that the stock-outs (i.e., treatments) for each specific product occur at different times, which makes standard differences-in-differences (DiD) analysis not ideal. Given this challenge, we adopt recent techniques that allow for differences in treatment timing across cross-sections (e.g., Autor (2003)) for our baseline analysis. Furthermore, we leverage pseudo-exogenous variation in the product arrival times across locations for new items to instrument for potential endogeneity in the stock-out treatment via an instrumented differences-in-differences (DDIV) approach (Hudson et al. (2017)). This analysis confirms the presence of positive intra-firm product substitutability effects, in that stock-outs in neighboring outlets lead to higher sales in a focal store. More specifically, we show that in the week of stock-out seen for the first time in the neighboring store, the sales in the focal store increases by 16%, whereas one week after, this effect diminishes to around 3% after two weeks. To explore further a potential explanation behind these effects, we show that the stark increase in sales around the time of a neighboring stock-out is most pronounced for items that are perceived to be limited edition (i.e., products that are never replenished). This difference in the effects suggests that customers are more likely to engage in intra-firm product substitutability when they expect a stocked-out item to not be replenished in the future, thereby creating a transient substitutability effect from neighboring stock-outs, likely due to the heightened sense of urgency for buying an item that is unlikely to be replenished.

Motivated by these empirical patterns, we develop an analytical model to demonstrate how the estimated demand specification that takes into account stock-outs in neighboring stores can be employed to optimize the inventory allocation across the stores. Benchmarking the results from this model on a baseline demand that does not take into account the stock-out information, we perform several counterfactual simulations. Our results show that revenues can be improved markedly by about 6% (under low demand variance) and 14% (under high demand variance) when neighboring stock-out information is taken into account for allocating initial inventory across all the stores; these revenue improvements are especially pronounced for products with high inventory, low turnover and low price. Being able to leverage additional information, such as data coming from the entire network of retail outlets (and not just the focal outlet), seems especially valuable in the fast fashion industry, for which a major challenge in sales forecasting is limited data within a short time duration (e.g., Choi et al. (2014)).

Our research contributes to the stream of literature about efficiency and performance within retail

Table 1: Examples of Research on Product Substitutability and Retail Performance

	Non-product-specific	Product-specific
Inter-firm	Seim (2006); Thomadsen (2005);Cai and Jiang (2020)	Anupindi et al. (1998); Bruno and Vilcassim (2008); Musalem et al. (2010); Vulcano et al. (2012)
Intra-firm	Blevins et al. (2017); Pancras et al. (2012)	**This paper**

networks. Past literature about retail networks has documented two conflicting forces that retailers often have to balance, namely positive forces in the form of economies of density/scale, and negative forces in the form of within-brand business stealing (i.e., cannibalization). For example, studies that have confirmed the presence of such trade-offs include Blevins et al. (2017) and Pancras et al. (2012). Because these studies infer the positive (and negative) effects from proximal outlets within a retail network using store-level sales (or entry) data, such data lacks the granularity to pinpoint precise mechanisms behind these effects beyond the oft-used cost-savings or demand-externalities explanations. In contrast, our study can track exactly sales and inventory over time across all stores within the network, which ultimately allows us to better understand a specific driver (i.e., intra-firm product substitutability) behind potential economies of density. Furthermore, our findings suggest that concerns of cannibalization may be overstated in industries that rely on inventory management, as this store-switching behavior (if used correctly) indeed helps a retail network's capability to better match demand for their products by allowing proximal stores to satisfy excess demand in nearby locations.

This work also builds on past research about customers' responses to product stock-outs.[6] Much of this work focused on quantifying the impact of product unavailability on the demand for substitutable (but different) products (e.g., Xu et al. (2011); Akçay et al. (2020); Lee et al. (2015); Musalem et al. (2010)). Stock-outs will ultimately have implications on assortment planning (e.g., Fisher and Vaidyanathan (2014); Kök and Fisher (2007); Suh and Aydin (2011);Li (2007)). It is worth noting that our analysis investigates stock-outs of a specific product on the sales of that exact same product in proximally near versus far outlets. In particular, we abstract away from consumer demand for differentiated products, which is inherently more challenging to study empirically as the demand analysis for differentiated products is likely to be confounded by product heterogeneity that is exacerbated by

[6]For a comprehensive industry report about the impact of stock-outs on consumers, we refer the reader to Gruen et al. (2002).

cross-product comparisons.

2 Empirical Evidence of Intra-Firm Product Substitutability

In this section, we first describe our empirical setting by providing details about the unique data we use, followed by a discussion about the empirical strategy we employed to identify product substitutability effects.

2.1 Industry and Data Description

Our empirical analysis leverages a unique and extensive panel data-set collected directly from a fast fashion retailer. Compared with traditional fashion retail, fast fashion is often known for having short-cycles, and stock-outs are common occurrences. As a matter of semantics, readers should interpret the term short-cycle as a characteristic of fast fashion retail, while out-of-stock as the event for which an item a customer wanted is no longer available. Note however that out-of-stock events might occur more frequently with short-cycles, as retailers are unlikely to replenish items that have completed their cycle. This interpretation of our terminology applies in both a broad sense (i.e., retail fashion in general), as well as our specific empirical context of fast fashion.

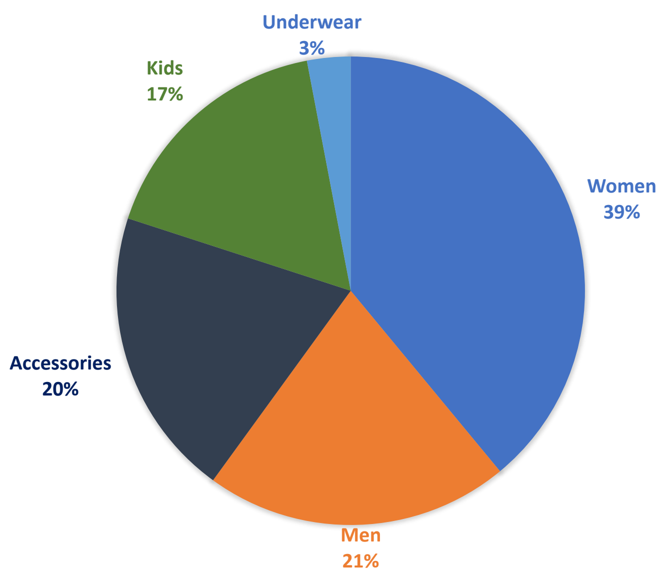
The fast fashion retailer we study was founded in 1988 and is headquartered in Europe. Currently, this company is one of the leading apparel retailers in the region. They carry a wide range of fashion products for women, men, kids, and babies, along with an extensive line of accessories. Typical of other fast fashion brands, they roll-out new products at a high frequency, which makes inventory management especially important to them. Overall, the company has experienced many consecutive years of profit growth as they continue to expand their retail operations. Unlike traditional fashion industry, one nice feature about fast fashion is that the companies typically "work at the item level, rather than using collections", which would allow us to study the sales and inventory dynamics product-by-product, rather than at a more complex collection level that might involve high-dimensional complementarities across products.

The data this retailer has provided us contains detailed information about weekly sales and inventory at SKU level between January 2016 and June 2018 in more than 300 stores.[7] We note that the SKU is the most granular identifier of the products, as the two items of the same style will have a different SKUs

[7] During the time period we study, there was no noticeable fluctuation in the number of outlets (i.e., store openings, store closures).

if their sizes are different. Similar to other fast fashion retailers, the one we study does not normally replenish depleted inventories, as only 15% of the products get replenished.[8] The low replenishment rate suggests that in the event of a stock-out, a customer's best option would be to search for that same item at other nearby stores, as their online channel remains insignificant compared with their network of physical outlets. There are in total more than 50 million records in the main fashion products. The analysis is primarily applied to the sales of women's clothing category in 2016 to work with a less dispersed data-set, though in our robustness checks, we consider other product categories. Another justification for focusing on the women's clothing category is that it generates the largest share of sales for the fast fashion company (see Figure 1).

Figure 1: Share of Sales Across Product Categories



Note: The shares obtained by calculating the total sales that a general product category generates divided by the total sales across all general product categories.

As our main focus is to explore the impact of stock-outs in neighboring outlets, we have chosen a subset of the data where the stores are relatively close to each other in physical distance. A neighboring outlet is defined as one that is closest to the focal outlet. We suspect that in the regions where stores are closer to each other, we would detect more easily the effects in sales due to stock-outs in neighboring stores. Throughout this paper, we use weekly sales of the stores located in one city that contains 72 stores and as such has the highest store density in the region. Even though we are focusing on stores in an urban area, we would like to point out that there is rich variation in how close neighboring outlets

[8]This rate lies somewhere in between H&M (at 23.1%) and Zara (at 2.8%).

are to the focal store. In Table 2, we present the number of stores that have at least 1, 2 and 3 stores in respectively 2, 5 and 10 km radius.[9] According to this table, 36 out of 72 stores have at least one neighboring store within a 2 km vicinity. This means that the customers who visit one of these 36 stores can feasibly find another store only by traveling at most 2 km. Similarly, 60 of the stores are in the 5 km radius of at least one other store. This geographic configuration of outlets enables us to explore whether the stock-outs in neighboring stores indeed have any impact on the sales of focal store.

Finally, we would like to point out that even though our focus is an urban area with high density, the store sizes exhibit a significant amount of variation; that is, our estimation sample includes store sizes that cover the entire spectrum (i.e., smallest size stores to the largest size stores are observed).

Table 2: Number of Neighboring Stores Across Radius Bands

	2 km	**5 km**	**10 km**
At least 1 store	36	60	69
At least 2 stores	16	49	69
At least 3 stores	10	33	64

As we aim to include all the observations throughout the products' life cycles, we eliminate the products that start to sell before January 2016 and continue to sell after June 2018. Among these products, we further reduce the data and focus one season (Autumn/Winter 2016-2017) to minimize the seasonal effects on demand. Table 3 summarizes the descriptions of variables we used for our analysis, while Table 4 displays the means and variances of the variables used in the analysis throughout the

[9] To construct the distances, we make use of a Google Maps API (i.e., distancematrix.api), which calculates the driving distance between stores.

Table 3: Description of Variables

Variable	**Description**
$sales_{ijt}$	Number of products i sold in store j in week t
inv_{ijt}	Inventory level of the product i in store j at the beginning of week t
$price_i$	First selling price of product i
$cost_i$	Cost of product i
$discount_{it}$	The percentage of discount seen on product i during week t
age_{ijt}	Number of weeks passed since product i's arrival to the store j until week t
$stockout_{ijt}$	Dummy set equal to 1 if product i is stocked out in the closest store to store j in week t
$elapsedtimeso_{ijt}$	Elapsed time in weeks since the first stock-out is seen for product i in the closest store to store j in week t
$elapsedtimedisc_{it}$	Elapsed time in weeks since a discount is applied to product i until week t

Table 4: Summary Statistics for Variables ("Women" Category)

Variable	**Mean**	**Std. Dev.**	**Min**	**Max.**
$sales_{ijt}$	0.193	0.537	0	76
inv_{ijt}	1.909	1.998	0	105
$price_{it}$	55.923	32.510	7.99	299.99
$discount_{it}$	0.173	0.224	0	0.95
age_{ijt}	9.323	7.799	0	38
$stockout_{ijt}$	0.002	0.042	0	1
$elapsedtimeso_{ijt}$	0.005	0.183	0	21
$elapsedtimedisc_{ijt}$	6.039	6.041	0	38

selected season. As we expect to see that the boost in the focal store sales due to a stock-out in a neighboring store decreases by time, we use a variable measuring the elapsed time since the first stock-out seen in the neighboring store. Similarly, since the demand increase, that is induced by markdowns, reduces with time (e.g., Pesendorfer (2002), and Ahn et al. (2009)), the variable $elapsedtimedisc_{it}$ is among our control variables. Sales, inventory and first prices are directly observed in the data, while discounts are calculated externally. First, the selling prices of the products obtained by dividing the weekly revenue by the number of products sold on that week. Utilizing selling prices, we observe information about the discounts, which are applied for specific products at a given time at a given store. It should be noted that the pricing decisions are made at the corporate/central level in general. That means, the price for each product is always discounted at the same level for all the stores. Note the reason that weekly sales appear to be low is because we observe a number of observations for which no sales materialize in a given week, which is expected as the retailer carries a very large variety of different clothing items. In addition to price and discount data, our data contains information about each product's unit cost as well.[10] Additionally, the summary statistics stratified across observations based on whether or not a nearby outlet experienced a stock-out can be seen in Table 5. These stratified statistics confirm that the control variables do not vary much across these two possible cases, as the means and standard deviations are very similar.

2.2 Identification Strategy

Our objective is to establish the impact of the stock-outs in the closest neighboring outlet on sales of the focal store which is indexed by j. To identify the impact, we construct various measures that

[10]Due to the highly sensitive nature of this information, the non-disclosure agreement prohibits us from displaying summary statistics for unit costs, as that would reveal the price-cost margins to the company's competitors.

Table 5: Summary Statistics Stratified by Stock-Out Nearby

	Observations	Mean	Std. Dev.	Min	Max
No nearby stock-out					
$sales_{ijt}$	6,500,598	0.193	0.536	0	76
inv_{ijt}	6,500,598	1.909	1.997	0	105
$price_{it}$	6,500,598	55.918	32.506	7.99	299.99
$discount_{it}$	6,500,598	0.173	0.224	0	0.95
age_{ijt}	6,500,598	9.314	7.796	0	38
Nearby stock-out					
$sales_{ijt}$	11,715	0.229	0.728	0	46
inv_{ijt}	11,715	1.548	1.789	0	52
$price_{it}$	11,715	58.406	34.855	7.99	299.99
$discount_{it}$	11,715	0.295	0.232	0	0.95
age_{ijt}	11,715	14.462	8.175	0	38

capture stock-outs in neighboring outlets. In particular, we define $stockout_{ijt}$ to be a dummy variable set to be 1 if SKU i is stocked out in the closest neighboring store of store (j) to focal store j in week t. When the stock-out is observed in the neighboring store for a particular SKU on a particular week, this can be seen as the treatment for the sales of the particular product in the focal store.

Difference-in-differences (DiD) is a commonly used quasi-experimental research design technique to evaluate the effect of any kind of treatment on the relevant outcome variables. It aims to extract the effect of the specific intervention on the outcome by comparing the changes in treatment and control groups over time. In our study, a DiD approach would be helpful to measure the actual effect of a neighboring store's stock-outs. However, due to the nature of our problem, the treatments, in other words, the stock-outs, do not happen in the same week for all of the products and stores. In this case, we need a technique to deal with multiple treatment groups. In similar cases, the general approach is to use a two-way fixed effects regression model with dummy variables for cross-sectional units, time periods, and a treatment dummy. The identifying assumption of utilizing a two-way fixed effect model is "interventions must be as good as random, conditional on time and group fixed effects" (Bertrand et al. (2004)). One way to control for multiple treatment times is to normalize the treatment time as t_0 for all groups and allow for leads and lags of the treatment by adding the interaction of treatment variable and time dummies as suggested by Autor (2003).[11] By including leads, we can analyze the pre-trends to see if the outcome is different for the treated and control groups even before the treatment happens. The lags, on the other hand, show if the treatment effect changes over time. Let t_0 be the

[11] We provide a more detailed discussion about an alternative decomposition approach for staggered DiD settings (Goodman-Bacon (2018)) that corroborates our main findings.

time at which the neighboring store goes to stock-out for store-SKU pair s. Then, we construct the regression model in Equation 2.1 to include the lags and leads as follows.[12]

$$sales_{st} = \alpha_s + \alpha_t + \sum_{\tau=-q}^{p} \delta_\tau DT_{st}(t = t_0 + \tau) + \gamma X_{st} + \varepsilon_{st} \tag{2.1}$$

where s corresponds to store-SKU pairs (i, j), X_{st} represents control variables and DT_{st} where $t = t_0 + \tau$ represents the interactions of time dummies and treatment indicator ($stockout_{ijt}$). Instead of a single treatment effect, the model has "q" leads and "p" lags and δ_τ is the coefficient of τ^{th} lead or lag. The test of difference-in-differences assumption is $\delta_\tau = 0$ when $\tau < 0$. On the other hand, the coefficients for lags shows whether the treatment effect fades or increases over time.

However, there remains an important concern regarding potential endogeneity in nearby store stock-outs. In our context, this bias may arise if unobserved factors have material impact on sales of nearby (or all) outlets alike. For example, stock-outs are generally not thought to be random events and often take place during high demand periods (e.g., Kalyanam et al. (2007)).

To address potential endogeneity concerns, we augment the specification above with instrumental variables estimation by finding instruments for the "q" leads and "p" lags of the stock-out treatment. The literature often refers to this approach as instrumented differences-in-differences (DDIV), as nicely discussed in the econometrics research note by Hudson et al. (2017). There, the authors discuss several well-cited examples of the use of instrumental variables in DID models, where the treatment variable is endogenous (much like the empirical context we face in our paper). Some notable examples include Angrist and Imbens (1995), Bai and Jia (2016), and Duflo (2001), as of these studies make use of 2SLS to estimate the Rubin (1974) causal effects for binary treatments. Here, the instruments should by definition be uncorrelated with unobserved factors that are driving the focal outlet's sales, while at the same time be correlated with the occurrences of neighboring store stock-outs. In the literature about demand estimation, one commonly used identification strategy employs instrumental variables that leverage the panel and spatial structure of data (e.g., Hausman (1996); Nevo (2001)). Inspired by this past literature, we also leverage these spatial structures to form instruments based on the timing of when exactly neighboring stores receive their initial inventory for a new product (i.e., arrival time of a

[12] In theory, a saturated model with more interactions and complex time trends could be considered. However, in practice, the linear specification we use is likely best suited for our empirical setting. First, our specification mimics the ones commonly used in most applications of staggered DiD vis a vis Autor (2003). Second, richer interactions, and especially complex time trends will make the interpretation of our main treatment effect more challenging, as the conceptual interpretation of what we mean by lead/lag periods might be obscured.

product in a given store). In order to construct arrival times, we first establish a baseline time by using the average start dates for each product. Then we take the difference between this baseline time and products' actual arrival dates in each store. We argue that this variable is suitable as an instrument for the following reasons.

First, in light of several extensive interviews with the head of logistics, we have learned that the order in which each store receives a new product is driven by a complex spectrum of factors. Such factors include available capacity in the store's back-room depot, physical shelf-space constraints, logistic considerations for shipping vehicles, and some potentially random factors. Although the new products are distributed to all of the stores in the beginning of the season, the company states that they cannot send as many products as they want to the each store given the capacity constraints. Consequently, at the beginning of the season, some products are not allocated to all of the stores at the same time and instead are shipped only to a subset of stores first. Furthermore, these timing patterns for shipments appear to be random (i.e., there is no specific rule that they follow when choosing which stores get the products first). It is helpful that the arrival times are plausibly random, so we provide further justification below that there is no systematic selection in this instrument. Taken together, these factors would suggest that the timing of initial shipments of new products is not perfectly correlated with unobserved factors that affect current (and future) sales.

Second, we show that there is noticeable variation (across stores and time) in the length of delay to receive the product. As shown in Figure 2a, this variation can typically range from one up to 10 weeks following the official introduction of a new item.[13] Furthermore, the plot also confirms that for a given store, there is a lot of variation in these delays, in that the same store does not persistently receive delayed shipments of new products. To demonstrate that there is no selection with respect to the arrival times across product or store types, we sort the arrival times based on these categories. Similarly, Figures 2b and 2c display the arrival times across stores, that are sorted based on the store's physical size and the store's total sales volume throughout the year, respectively. These plots confirm that certain types of stores (i.e., the stores with large sales volume versus low sales volume, large store size versus small store size) do not systematically receive products faster than the average arrival time. Ultimately, arrival times seem uncorrelated with underlying demand factors. Collectively, these

[13] There are of course outliers, in which stores are delayed shipments of newly released products by up to 40 weeks. There are also 3 stores that do not experience delays of more than 10 weeks ever, but there are no systematic differences in observable characteristics between these 3 stores compared with the general population of stores.

patterns strengthen the power of arrival times as an instrumental variable, provided that it is highly correlated with inventory stock-out incidences, which we confirm to be the case when discussing the main empirical results.

3 Empirical Findings

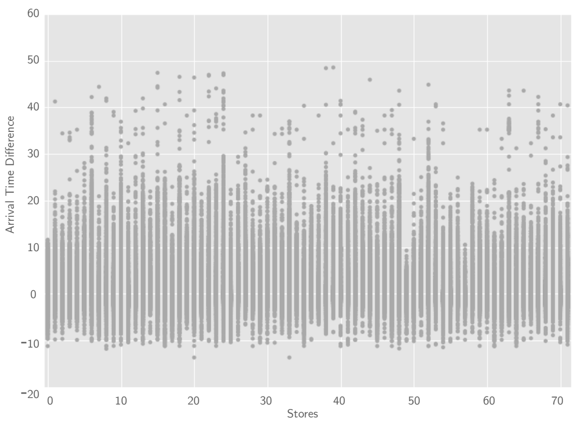
This section provides our main empirical findings about the role of intra-firm product substitution effects in the fast fashion industry. We then provide data-driven mechanism as to a potential explanation behind the patterns we identify. We refer to the the Online Appendix for an alternative staggered DiD approach to demonstrate robustness, alternative linear fixed-effects model specifications, some sub-sample analysis, and specifications with detailed product, store, and/or distance interactions.

3.1 Baseline Estimates from DDIV

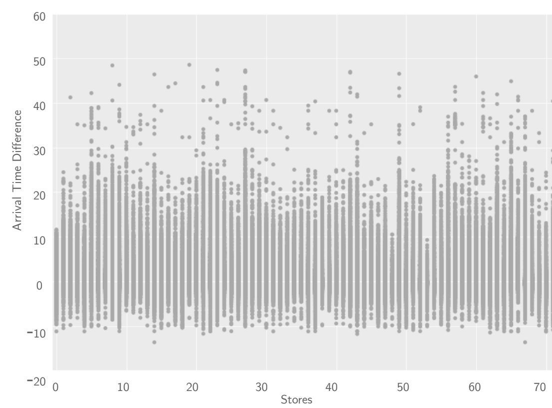
Stock-outs in neighboring outlets may lead to non-trivial dynamic effects on the focal outlet. For example, it is unclear how persistent the nearby stock-out effect will be, and whether or not this neighboring stock-out effect stabilizes after dropping to a point. For this reason, it is important to include lags and leads into our analysis to establish the true direction of the causal effect from nearby stock-outs. To investigate these dynamic effects, we utilize a baseline model described above that includes leads and lags of stock-out events presented in Equation 2.1. Specifically, the indicator values are added for 1,2 and 3 weeks before, 1,2 and 3 weeks after the stock-outs. The lags and leads $t-3$ to $t+3$ are equal to 1 in only one week each per store-SKU pairs with a stock-out in the closest neighboring store. To address the issue of endogeneity, we employ instruments for each of the lead/lag stock-out events constructed using the arrival times. From the first stage of estimation, we find that products arriving to the stores earlier are more likely to experience stock-outs, which is intuitive. For example, a standard deviation increases in the arrival time relative to the reference point (i.e., approximately 20 days) will delay the neighboring stock-out by 0.08% (at a statistically significant level) that constitutes roughly 41% overall likelihood of stock-out. We also note that the F-statistic from the first-stage regression is 88.73, which suggests that weak instruments may not be an issue in our specific empirical context.

The first column of Table 6 presents the base specification with store-SKU and time fixed effects augmented with leads and lags while the second column presents the specification where additional control variables included and SKU fixed effects are excluded. One reason of excluding the SKU

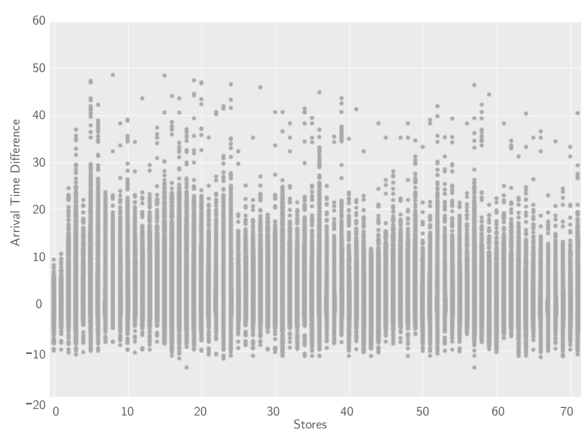
Figure 2: Arrival Times of Products to the Stores



(a) Arrival Times Across the Stores



(b) Arrival Times Across the Stores Sorted in Ascending Order by Store Size



(c) Arrival Times Across the Stores Sorted in Ascending Order by Total Sales Volume Throughout Year

Note: As a reference point, the average start date (across all stores) for each product is used. The difference in weeks between the reference points and products' arrival dates in each store is then calculated.

Table 6: Estimates from Baseline DDIV Specification

	(1)	(2)
$stockout_{t+3}$	0.018	0.026
	(0.031)	(0.031)
$stockout_{t+2}$	0.018	0.035
	(0.021)	(0.009)
$stockout_{t+1}$	0.076^{*}	0.097^{*}
	(0.038)	(0.044)
$stockout_{t0}$	0.138^{***}	0.166^{***}
	(0.025)	(0.029)
$stockout_{t-1}$	0.069^{*}	0.098^{**}
	(0.035)	(0.036)
$stockout_{t-2}$	-0.018	0.011
	(0.030)	(0.029)
$stockout_{t-3}$	0.045	0.070
	(0.06)	(0.066)
Controls	No	Yes
inventory	-	0.029^{***}
	-	(0.001)
price	-	-0.025^{***}
	-	(0.005)
discount	-	-0.059^{**}
	-	(0.013)
elapsedtimedisc	-	-0.003^{***}
	-	(0.001)
Store fixed effects	Yes	Yes
SKU fixed effects	Yes	No
Time fixed effects	Yes	Yes
Observations	6,398,437	6,398,437

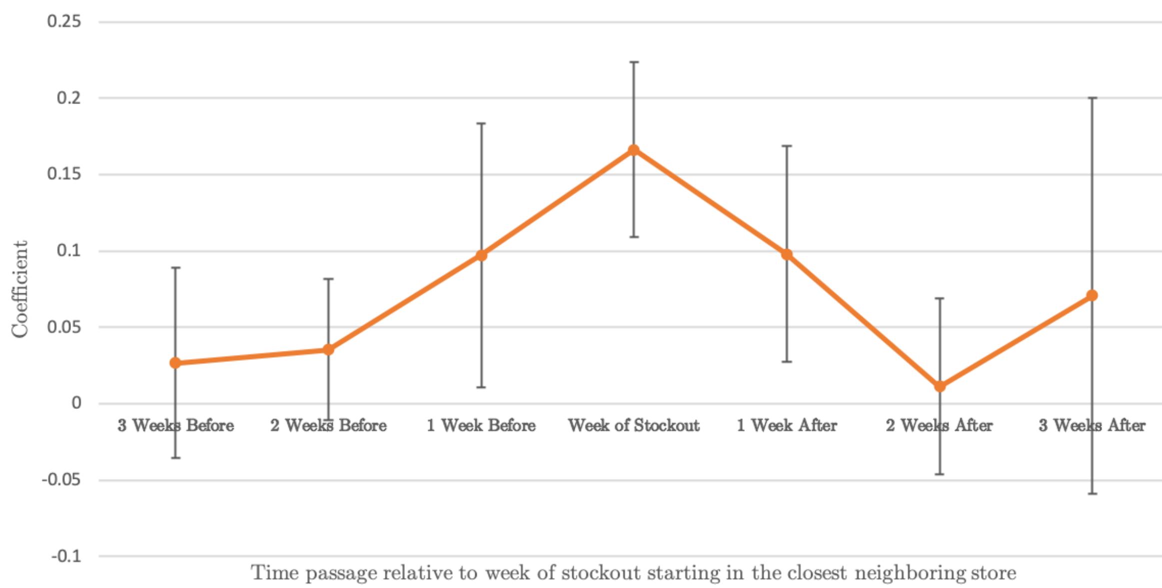
Significance Levels: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.
Numbers in parenthesis indicate standard errors.

fixed effects is some of the control variables already includes SKU specific information. This table confirms that these stock-out effects exist, especially during the week of stock-out. Our results show statistical significance for both the lead and lag effects. A couple remarks are in order. We would like to emphasize that the economic magnitude of the pre-treatment effects is close to 0 as our scale-free effects highlight. Taken together, these patterns remain consistent with the notion of parallel trends, which is an important qualifier for DDIV approaches.

We point out that the pre-treatment effect in the immediate week prior to the stock-out event is statistically significant and of a similar magnitude as the immediate week following the stock-out event. A possible explanation for this pattern is that while a neighboring outlet may still have available inventories for a particular size, the other sizes may start to face stock-outs earlier due to uneven size distribution and/or demand, a phenomenon known as broken assortment effect in the literature (e.g., Smith and Achabal (1998), Fisher and Raman (2010), Caro and Gallien (2012)). Due to this effect, the items that face broken assortment might appear to be stocked-out in the eyes of the consumer, as the customers who prefer trying slightly larger or smaller sizes may not find the appropriate SKUs on the shelf. Therefore, the pre-treatment effect might pick up the spillover demand generated by the customers who face broken assortment in the neighboring store even before the main item gets stocked out.

To get a better sense of percentage change in sales, the outcome is used as log sales. In both of the columns, the coefficients on stock-out leads are smaller than $stockout_{t0}$. This finding can provide some reassurance that there is a little support for reverse causality after controlling for the possible factors affecting the sales. During the week of stock-out seen for the first time in the neighboring store, the sales in the focal store increases by 16% (refer to the coefficient of $stockout_{t0}$ in Table 6). After one week, the effect reduces to 10% and then only 1% after two weeks. This pattern is also depicted in Figure 3. In summary, the causal impact of neighboring stock-out appears to be transient, as its impact is most pronounced within the same week and dampens significantly in subsequent weeks.

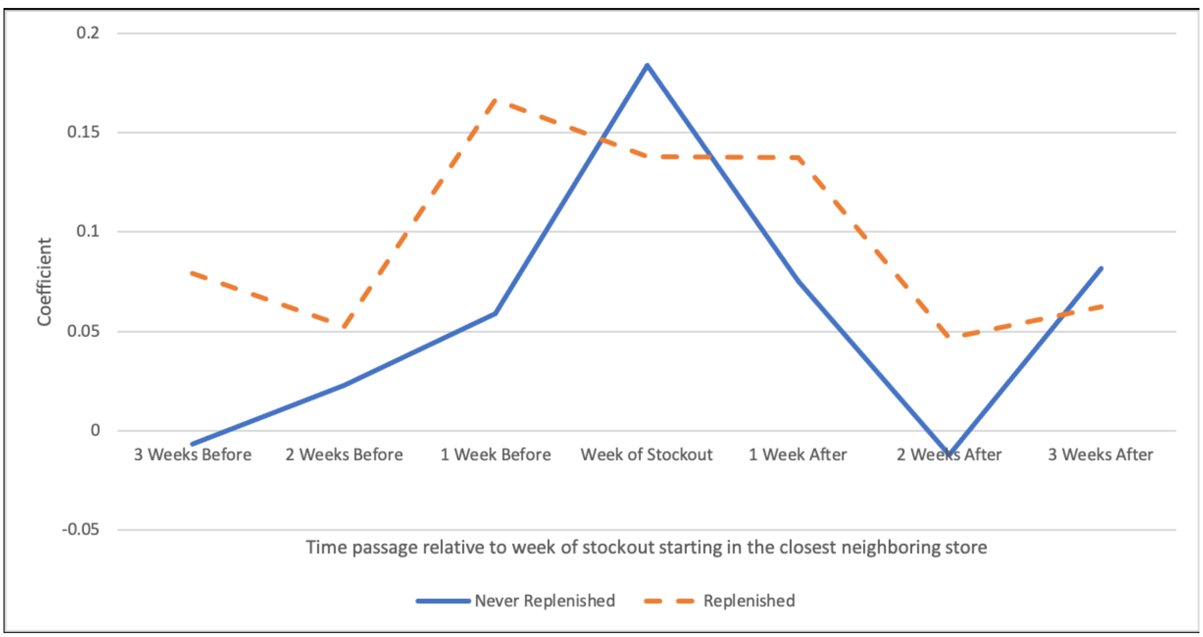
Figure 3: Impact of Stock-outs on log Sales Before, During and After Stock-out Realizations



In light of the patterns exhibited in Figure 3, we now explore potential explanations behind the transient nature of the neighboring stock-out effects on sales. In particular, we consider the possibility that this apparent immediacy of substitutability across stores may be driven by customer expectations about future availability of products experiencing stock-outs. If customers do not expect products to be re-stocked, then this expectation creates a sense of urgency during a stock-out event. The customer's heightened sense of urgency could explain why the impact of neighboring store stock-outs seems concentrated soon after the stock-out event.

We investigate this conjecture by considering a sub-sample analysis of our baseline specification. In particular, we compare the estimated impact of stock-outs for two different sub-samples. The first sub-sample contains products that are replenished (i.e., not limited edition), while the other sub-sample contains products that are never replenished (e.g., limited edition). Figure 4 provides the visualization of the estimated impacts across these two scenarios. The effect is 4.6% larger for items that are never replenished versus those that are replenished during the week of stock-out. These results confirm that the spike in the neighboring stock-out effects (during the week of the stock-out) are more pronounced for products that might be perceived to be limited edition. These findings suggest that the transient nature of the stock-out effects might be driven by customer expectations and their desire to obtain an item before it becomes completely unavailable across all stores.

Figure 4: Impact of Stock-outs on log Sales Before, During and After Stock-out Realizations



One alternative explanation of the transient pattern in our main findings is that consumers may be learning about a stock-out by visiting a store and not finding a product in their own size. Over time, it's less likely that the customer will observe the product in the store in any size, thereby leading to a diminishing effect of stock-outs over time. We leave this potential mechanism as an avenue for future research.

We note that a potential alternative approach to DiD are synthetic controls that leverage the spatial information from neighboring stock-outs. While these methods have strong merits, we believe that this methodology might not be ideal in our specific empirical application. First, many of the well-established techniques do not seem to accommodate for staggered treatments (Cunningham (2020)), which is something our analysis needs to contend with as each product's stock-out in neighboring outlets occurs at different times. The staggered treatment time will have a material impact on the formulation of the synthetic control, as lagged pre-treatment outcomes are typically used to obtain the weights needed for the synthetic control. For cases in which a neighboring stock-out occurs early in a product's life cycle, we might not have reliable synthetic controls. Nevertheless, the spatial patterns from neighboring stock-outs could provide further insights about the spillovers we document. For this reason, we consider a series of empirical specifications that make use of radius-based regression models. In order to identify the impact, various measures that capture stock-outs in neighboring outlets are constructed. In particular, $stockout0\text{-}2_{ijt}$ is defined to be a dummy variable set to be 1 if product i is stocked out in any store which is located within 2 km from store j in week t. Similarly, $stockout2\text{-}5_{ijt}$, $stockout5\text{-}10_{ijt}$, $stockout10\text{-}15_{ijt}$, $stockout15-20_{ijt}$ and $stockout20\text{-}30_{ijt}$ are dummy variables that are equal to 1 if the product$i$ is stocked out in week t in any store located, respectively, within 2 to 5 km range, 5 to 10 km range, 10 to 15 km range, 15 to 20 km and 20 to 30 km range of the focal outlet j. With these measures of neighboring inventory shortages, our radius-based regression model is as follows:

$$
\begin{aligned}
sales_{ijt} = \beta_0 &+ \beta_1 stockout0\text{-}2_{ijt} + \beta_2 stockout2\text{-}5_{ijt} + \beta_3 stockout5\text{-}10_{ijt} + \beta_4 stockout10\text{-}15_{ijt} \\
&+ \beta_5 stockout15\text{-}20_{ijt} + \beta_6 stockout20\text{-}30_{ijt} + \gamma X_{ijt} + \mu_i + \mu_j + \mu_t + \varepsilon_{ijt}, \quad (3.1)
\end{aligned}
$$

where X_{ijt} includes all of the relevant controls about the product (i.e., price, discount, unit costs, inventory, etc.), and μ_i, μ_j, and μ_t are product, store, and time fixed effects, respectively. Note that our radius-based specifications generalize the notion of stock-out, by allowing stock-outs to have different impacts depending on how far away the neighboring outlets are (in physical distance). The results from this analysis can be found in Table 7.

Table 7: Radius Band Results for "Women" Clothing Category

	(1)	(2)
stockout 0-2km	0.054***	0.041***
	(0.002)	(0.002)
stockout 2-5km	0.049***	0.037***
	(0.001)	(0.001)
stockout 5-10km	0.042***	0.030***
	(0.001)	(0.001)
stockout 10-15km	0.043***	0.030***
	(0.001)	(0.001)
stockout 15-20km	0.040***	0.028***
	(0.001)	(0.001)
stockout 20-30km	0.040***	0.027***
	(0.000)	(0.000)
elapsedtimesinceso 0-2km	-0.007***	-0.006***
	(0.000)	(0.000)
elapsedtimesinceso 2-5km	-0.005***	-0.005***
	(0.000)	(0.000)
elapsedtimesinceso 5-10km	-0.005***	-0.004***
	(0.000)	(0.000)
elapsedtimesinceso 10-15km	-0.005***	-0.004***
	(0.000)	(0.000)
elapsedtimesinceso 15-20km	-0.004***	-0.004***
	(0.000)	(0.000)
elapsedtimesinceso 20-30km	-0.004***	-0.004***
	(0.000)	(0.000)
inventory	0.027***	0.026***
	(0.000)	(0.000)
log price	-0.050***	-0.526***
	(0.000)	(0.010)
discount	-0.013***	0.058***
	(0.001)	(0.001)
elapsed time since discount	-0.004***	-0.004***
	(0.000)	(0.000)
product attributes	yes	no
month	yes	no
holiday	yes	no
store fixed effects	yes	yes
product fixed effects	no	yes
time fixed effects	no	yes
Observations	6,465,547	6,360,987
R^2	0.093	0.115

Significance Levels: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

The key results from this analysis are as follows. First, the stock-out effect holds at different radius band indicators, which confirms that our main conclusions are somewhat general, and are not sensitive to how exactly a "neighboring" outlet is flagged (i.e., stock-out in neighboring store that is very close, or a bit further away from focal outlet). Second, the estimated coefficients provide some intuitive insights about how the stock-outs impact the focal store with distance, in that the inferred stock-out effect across different radius-bands dampens with distance. This finding seems intuitive, as the spillovers appear most pronounced if the neighbor is proximally close to the focal store (as opposed to if the neighbor is far away).

Finally, to explore some additional descriptive patterns, we investigate the interaction effects between stock-outs and store/product characteristics. Some of these characteristics include the fashion style (e.g., city fashion, lingerie, party-wear, sportswear), whether the product is discounted, and store features (e.g., mall, size of store). These results are summarized in the Online Appendix. The inferred interaction effects suggest that the stock-out effect might exhibit some heterogeneity across different types of products. More specifically, we show that the effect is especially pronounced for city fashion clothing styles and products sold in mall locations; in contrast, we find dampened effects for products that are discounted as well as those sold in stores that are physically larger.

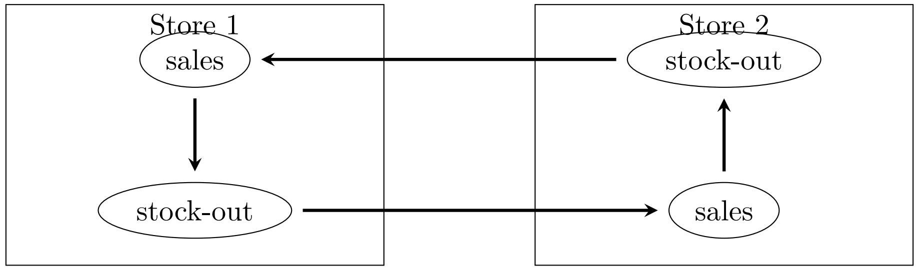
4 Implications: Opportunities to Improve Firm Profitability

An important goal of this research is to analyze the financial impact of considering potentially important information about stock-outs in neighboring outlets. Information about nearby stock-outs is potentially relevant if it can help with sales forecasting, which in turn can lead to better initial inventory allocation decisions and thus, higher revenues. We apply a counterfactual simulation approach to analyze profit implications of neighboring store stock-out information. In some ways, this simulation exercise loosely operationalizes the "demand learning" process (e.g., Sauré and Zeevi (2013)) for the retailer by leveraging information about nearby outlets.

The results obtained from various specifications presented in Section 3 strongly suggest that the stock-outs in neighboring stores create additional demand for the focal store. This information may be valuable for the initial inventory allocation decisions of the firm if we analyze the financial impact of considering potentially important information about stock-outs in neighboring outlets. In this section, we use the demand model constructed in Section 3.1 as it further captures the dynamics between focal store sales and neighboring store stock-outs with respect to time. However, the effect of the neighboring store stock-out on focal store sales becomes obsolete 2 weeks after the first time a stock-out seen in the neighboring store. That is why we do not incorporate the leads and lags after the second week in the simulation model.

Since we work with a dataset at a granular level, we can directly simulate the weekly sales of each SKU according to the stock-out position of the neighboring store as the demand model also utilizes the treatment time (the first time stock-out seen in the closest neighboring store). However, the stock-out

Figure 5: Neighboring Stores' Sales and Stock-out Relationship



period of the neighboring store also depends on the stock-out period of the closest neighboring store of its own as shown in Figure 5. To overcome this issue and obtain joint probabilities, we use Gibbs Sampling algorithm. While working with a pair of nearby stores, we can first simulate the possible demand scheme of Store 1 assuming that we know if and when there is a stock-out in Store 2. Similarly, we can obtain a possible demand pattern of Store 2 assuming that we know the stock-out period of Store 1. Using these two sets of simulations, we calculate the conditional probabilities for stock-out periods for both Store 1 and Store 2 given stock-out periods in their neighboring stores. Finally, given the conditional probabilities, we can re-optimize the inventory position of neighboring stores by keeping their total inventory constant. In order to simulate the stock-out periods for stores 1 and 2, we need to simulate their likelihoods. This is done using a simulation algorithm that incorporates Gibbs sampling, as presented with more detail in the Online Appendix.

To assess the impact of incorporating the neighboring store's stock-out position in the demand model, we change the initial inventory allocations of the two neighboring stores while keeping the total amount of inventory fixed as status quo. Next, using the joint distributions obtained via Gibbs Sampler, we obtain demand realizations throughout the life cycle of each product for any possible stock-out scenario in the neighboring store. We then calculate the expected total revenue for each possible inventory allocation choice. Finally, we choose the optimal inventory allocation yielding the highest revenue. We replicate this simulation exercise and calculate the optimal inventory allocation under a demand model that does not depend on stock-out periods in neighboring stores. Then, we can compare these allocation schemes to assess whether considering stock-out information improves the revenue at a substantial level.

We note that in this simulation analysis, we evaluate the impact of spillover effects on pre-season inventory planning, as such, we estimate demand models (with and without spillover effects) assuming that there is no in-season inventory replenishment activity.

Table 8: Summary Statistics for Randomly Chosen 400 Products in "Women" Category

Variable	**Mean**	**Std. Dev.**
$sales_{ijt}$	0.470	1.192
inv_{ijt}	3.373	4.938
$price_{it}$	57.085	30.133
$discount_{it}$	0.131	0.215
age_{ijt}	9.240	7.505
$stockout_{ijt}$	0.001	0.036
$elapsedtimeso_{ijt}$	0.005	0.188
$elapsedtimedisc_{ijt}$	5.879	6.207

To set up a computationally tractable simulation model, we select pairs of stores as focal and closest neighboring stores. We use a subset of products which are sold at both of these stores. The summary statistics for these 400 products are presented in Table 8. These products are quite representative of the entire set of 6000 products as compared with the summary statistics in Section 2.1 in terms of their price and how long they stay in the stores. The selling speed is higher than the average as we select two stores which are located in a busy area. However, the number of stock-outs seen in these stores is not higher than the average in all stores. We then perturb the initial inventory allocations of the stores by one product while keeping the total amount of inventory fixed as status quo. Next, using Gibbs sampler, we generate stock-out periods in each store and obtain sales forecasts throughout each product's life cycle. Two types of demand models are used to generate demand realizations. Namely, the first one makes use of information about stock-out periods in neighboring store, whereas the second one does not take into account the impact of stock-out periods on the sales. We create two demand variability scenarios by setting the standard deviation of demand in each period at one standard error of residuals, i.e., $\sigma_{low} = 0.5$ and at two standard errors of residuals, i.e., $\sigma_{high} = 1$. For each of these settings, we calculate the expected total revenues that can be generated from each product with different inventory allocations across stores, and choose the best allocation that yields the highest expected revenues among the perturbed set of allocations. As the key point of comparison, we then follow the same steps to evaluate the percentage improvements in expected revenues when additional information about product stock-outs in neighboring stores are included in the sales forecasting.

Our main finding is that the average improvement in revenues for these 400 products varies from 6% (under low demand variance) to 14% (under high demand variance scenario) when inventory allocation decisions are made using information about stock-outs in neighboring stores. The magnitude of this revenue improvement is significant, especially considering that the profit margins for the retail industry are notoriously in the low single digits[14] and that we only focus on the inventory allocation of two stores in a large store network.

We further investigate the relationship between revenue impacts and different product characteristics. We mainly focus on initial price, starting inventory levels and average turnover ratios by creating 3 levels for each of them. We use 33.3% and 66.6% as cut-off points to construct the low, medium and high percentiles.

Table 9 suggests that considering stock-out information in neighboring stores on average yields higher revenue impact under high demand variability scenario. This is intuitive because incorporating the impact of stock-outs in a neighboring store on the sales of a focal store rewards an inventory allocation that captures the extra demand due to stock-outs realized in the neighboring stores. As demand uncertainty increases, such policy becomes more beneficial because if one of the neighboring stores faces a stock-out due to increased demand variability, the other store can still fulfill the demand with its own inventory. It should also be noted that even though the percentage improvement under high demand variance is on average larger than that under low demand variance, the percentage improvement itself becomes more variable as the demand uncertainty increases. We also observe that the products with medium or high inventory levels result in on average higher improvement than those with low inventory levels (especially at low demand variance). One possible reason is that there is not much room for improvement for the products with low inventory because the products with low initial inventory are fully stocked out irrespective of inventory allocation policy. Similarly, the percentage improvement is higher for the products with low turnover than those with medium or high turnover rates. The rationale behind this observation is the same as the previous case because the products with high turnover rates sell very quickly irrespective of inventory allocation policy, therefore, considering the impact of stock-outs at neighboring stores does not make much difference for those products. Finally, when the inventory is low, the improvement is higher for the medium and high priced products for low priced products. From the consumer's perspective, we would expect higher-priced products to be more worthwhile to search in other stores in the presence of stock-outs. Hence, capturing the spill-over demand due to stock-outs yields more value as the product price increases.

Taken together, this simulation analysis generates two new managerial insights. First, there are financial implications of considering information about stock-outs in neighboring outlets, and second,

[14] The exact margins for this company are comparable with other well-known fast fashion brands.

		Low Demand Variability, i.e., σ_{low}			High Demand Variability, i.e., σ_{high}		
Inventory	Price	Low Turnover	Med Turnover	High Turnover	Low Turnover	Med Turnover	High Turnover
Low	Low	0.00%	4.55%	3.39%	17.09%	10.44%	3.24%
	Med	5.49%	5.20%	8.05%	5.07%	7.08%	9.42%
	High	6.88%	7.95%	5.34%	11.88 %	24.18%	6.48 %
Med	Low	9.19%	5.36%	5.82%	7.65%	21.74%	8.30%
	Med	12.87%	5.48%	9.09%	11.47%	14.30%	14.88%
	High	7.14%	3.38%	6.70%	13.08%	9.54 %	7.44%
High	Low	12.61%	6.19%	6.87%	15.55%	41.24%	22.99%
	Med	8.57%	8.28%	3.59%	11.58 %	17.77%	29.63%
	High	3.63%	4.78%	7.01%	7.01%	22.52%	15.64%

Table 9: Percentage Improvements in Profitability for Different Inventory,Price and Turnover Levels at Low and High Level of Uncertainty

the revenue improvements from making use of this information when allocating inventory are likely to be contingent on the product type retailers offer.

5 Conclusion

This paper studies the potential impact of store stock-outs on the sales of neighboring outlets. We conduct empirical analyses using novel big data from a leading European fast fashion retail chain to show that store substitutability patterns exist, as a focal store's sales are impacted by stock-outs in neighboring outlets, especially so during the week of stock-out and dissipate quickly afterward. Next, in order to gauge the managerial implications of stock-outs in neighboring stores, we compare two optimization models. The first optimization model is built on a demand function that takes into account stock-outs in neighboring stores, whereas the second optimization model ignores the effect of stock-outs on demand function. We then optimize the inventory allocations under both models and calculate the percentage improvement in revenue potential due to considering the effects of stock-outs in sales. Our simulation analyses show that making use of nearby stock-out information leads to better sales forecasts and consequently improves inventory positioning among the stores, which in turn generates an average of 6% (under low demand variance) and 14% (under high demand variance) increase in revenues. Our sensitivity analyses with respect to problem parameters show that these revenue improvements are especially pronounced for the products with high demand variability, high inventory, and/or low turnover. Ultimately, this optimization exercise highlights the importance of understanding these spillover effects, as ignoring the impact of neighboring stock-outs will have implications on the retailer's ability to optimize its inventory allocation via improved demand forecasts.

Evidence of positive intra-firm product substitutability effects offers a different perspective about cannibalization in retail, whereby cannibalization is often used as an argument against standardization across locations (e.g., Rigby and Vishwanath (2006)). In past research, literature about cannibalization has focused on its negative effects (e.g., Ngwe (2017)), in the sense that proximal stores take away sales from one another; an implication of these studies is that retail chains need to be cognizant of such negative effects when choosing their outlet locations (e.g., Igami and Yang (2016)). But our findings suggest that this notion of within-firm business stealing may be a moot point if much (or all) of the store-switching behavior is driven by inventory concerns and product availability. In fact, this within-firm business stealing may prevent customers from switching to other firms in light of localized stock-outs as we show that the impact of stock-outs diminishes as the distance between stores decreases, whereby customer switching to other brands is potentially quite costly to the firm (e.g., Matsa (2011)).

Ultimately, our paper offers actionable insights about intra-firm product substitutability.[15] In particular, our findings will likely be most effective if the back-end operations are aligned with the front-end impact, which would be consistent with the idea that both front-end and back-end capabilities contribute towards company value. This alignment can be achieved through optimization of initial inventory allocation, transshipment calibration, and awareness that the profit impact of intra-firm product substitutability is not uniform (i.e., low versus high price items, mall versus street locations, age of product in its life-cycle). We believe these insights need not be restricted to fast fashion alone, as insights from research about this industry sector may be applicable in other retail settings such as consumer electronics.

Acknowledgments

The authors would like to thank the Department Editor, Senior Editor, and anonymous reviewers for their constructive and valuable feedback throughout the review process. This research has improved immensely as a result of their guidance. The work of Mehmet Gümüş was supported in part by research grants from the Natural Sciences and Engineering Research Council of Canada (NSERC RGPIN-2019-06091) and the Institut de Valorisation des Donnèes (PRF-2019-8577073986). The work of Nathan Yang was supported in part by a research/travel grant from Cornell University.

[15] Our research may provide a concrete mechanism behind the oft-vague notion of economies of density/scale that past researchers of retail networks have alluded to (e.g., Holmes (2011)). In our setting, economies of density/scale may materialize from the retail network's ability to satisfy demand by making full use of the network's inventory (and not just the inventory of an individual outlet).

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A Online Appendix

A.1 Motivating the Research Question Using an Analytical Model

Our main research questions are as follows. First, do stock-outs in neighboring outlets have an impact on a focal outlet's sales? Second, do these stock-outs have differential impacts across the radius bands? To further motivate these questions, we develop a stylized model to establish theoretical foundations for the impact of stock-outs in neighboring outlets.

The model consists of two stores denoted by j_1 and j_2. Furthermore, we assume that the stores are located close to each other and the distance between them is denoted by d_{j_1,j_2}. The customers visit their preferred stores to search for a product i_1 whose price is set at p_{i_1}. We assume that customers' valuations v are uniformly distributed between 0 and 1. Without loss of generality, we consider the utility of a customer who first visits store j_1, and finds that the product i_1 is unavailable. In this case, the customer has three choices: (i) opts for another substitutable product i_2 in the same store j_1, (ii) visits the nearby store j_2 to search for product i_1, and (iii) does not buy anything. We can formulate the utility under each choice as follows. Without loss of generality, we normalize the utility under third choice to be 0. Therefore, the customer needs to compare her utilities under the first and second choices to decide whether or not to visit the neighboring store j_2. On one hand, if the customer decides to switch for another product, her utility would be equal to $\alpha v - p_{i_2}$, where α denotes the relative change in her utility due to purchasing product i_2 instead of product i_1. On the other hand, if the customer decides to visit the neighboring store j_2, she receives a net utility equal to $v - p_{i_1} - hd_{j_1,j_2}$, where h denotes the hassle cost incurred while traveling between the focal store j_1 and neighboring store j_2[16]. A simple comparison between utilities under first and second choices would lead to the following threshold on the customer's choice model:

A customer with a hassle cost h will choose one of the following options:

$$\begin{cases} \text{she opts for product } i_2 \text{ in store } j_1, & \text{if} \quad d_{j_1,j_2} \geq \dfrac{v(1-\alpha)+p_{i_2}-p_{i_1}}{h} \\ \text{she visits store } j_2, & \text{otherwise} \end{cases}$$

As expected, this simple choice model implies that customer would incur hassle cost and visit the neighboring store j_2 provided that it is not sufficiently far from the focal store j_1. Therefore, based on

[16] For the sake of simplicity, we assume that the product i_1 is available in the neighboring store j_2. That said, we would like to note that considering the possibility of stock out in the neighboring store j_2 does not change the results.

this characterization, we make the following prediction:

Prediction: *The stock-outs in a store have decreasing impact on the sales of the nearby stores as the distance between neighboring and focal stores increases.*

We next extend the base model presented in Section A.1 by adding a competitor store in the vicinity of neighboring store j_2. More specifically, the extended model consists of focal and neighboring stores, denoted by j_1 and j_2, and a competitor store close to the neighboring store j_2. Let $d_{j_1 j_2}$ denote the distance between stores j_1 and j_2, and h denote the hassle cost for the customer, where h is assumed to be uniformly distributed between 0 and 1. We assume that the distance between store j_2 and the competitor is negligible and there is no hassle cost for a customer to visit the competitor if she already decides to visit store j_2. The customers visit their preferred stores to search for a product i_1 whose price is set at p_{i_1}. We assume that customers' valuations v are uniformly distributed between 0 and 1. Without loss of generality, we consider the utility of a customer who first visits store j_1, and finds that the product is unavailable. Each customer makes her decision in two stages. In the first stage, similar to the base model, the customer has three choices: (i) opts for another substitutable product i_2 in the same store j_1, (ii) visits the neighboring store j_2, and (iii) does not buy. In the second stage, if the customer decides to visit the neighboring store j_2, then she observes the product in the competitor's store and chooses between the following two alternatives: (i) purchases the product i_1 from store j_2, and (ii) purchases the product i_3 from competitor. We can calculate the customer's utility under each choice as follows. Without loss of generality, we again normalize the utility of no-purchase option to be 0. In the first stage, the customer needs to compare her utilities under the first choice with the expected utility under second choice, where the expectation is taken with respect to the valuation of the competitor's product i_3. We define α_{i_3} to denote the relative change in the customer's utility due to purchasing product i_3 instead of product i_1 and assume that α_{i_3} is uniformly distributed between 0 and 1. To summarize, each customer compares the utility of buying a substitutable product i_2 in store j_1 with the expected utility of visiting to store j_2 at the first decision step. After visiting store j_2, the customer also observes the valuation of product i_3 in the competitor's store, and choses to buy either product i_1 from store j_2 or product i_3 from competitor store according to their utilities. The Figure A.1 summarizes the decision tree and corresponding utilities for the customer.

Figure A.2 shows the distribution of the customers when there is no competitor around the neighboring store j_2. The presence of a competitor in the vicinity of store j_2 increases the total number of

Figure A.1: Customer Choices In The Presence of A Competitor

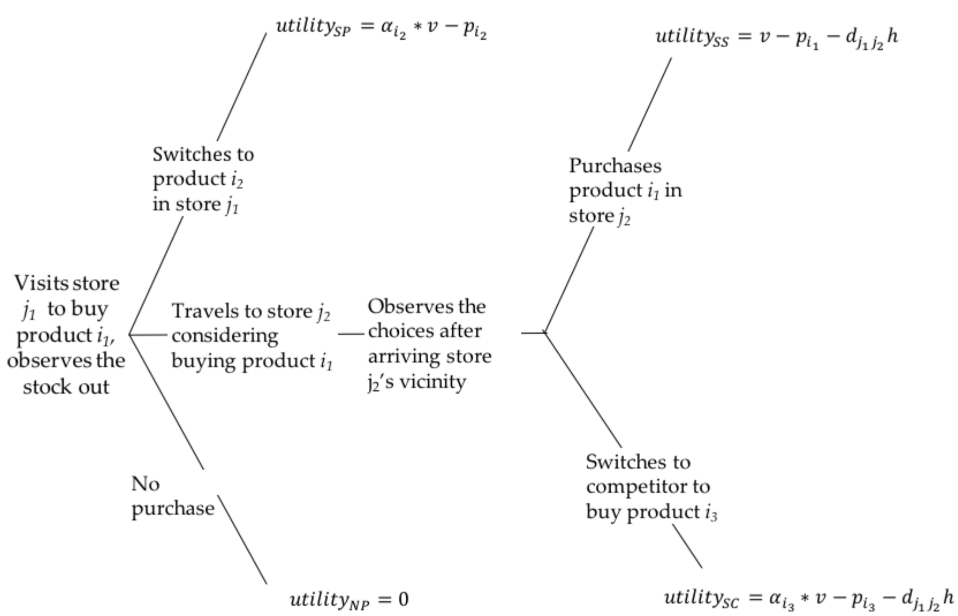


Figure A.2: Distribution of Customers Without a Competitor

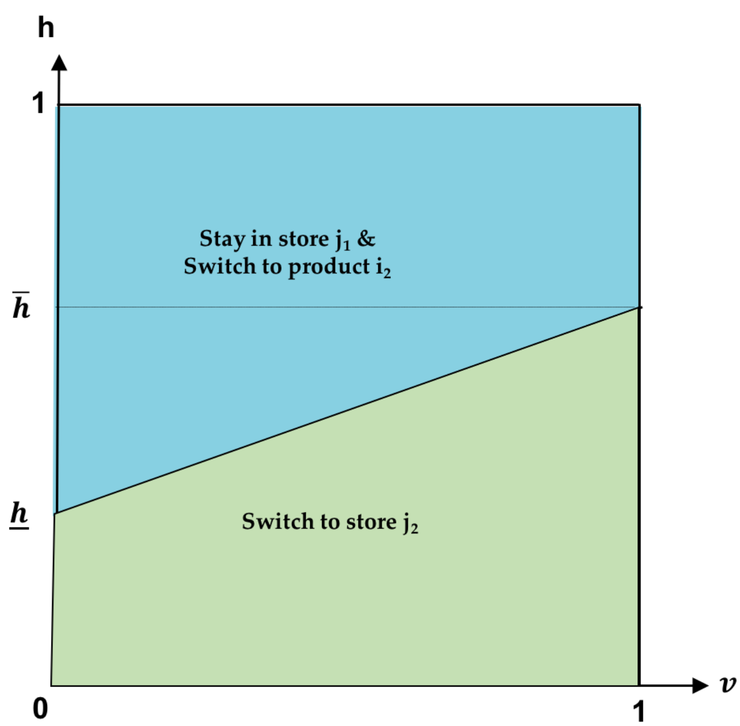
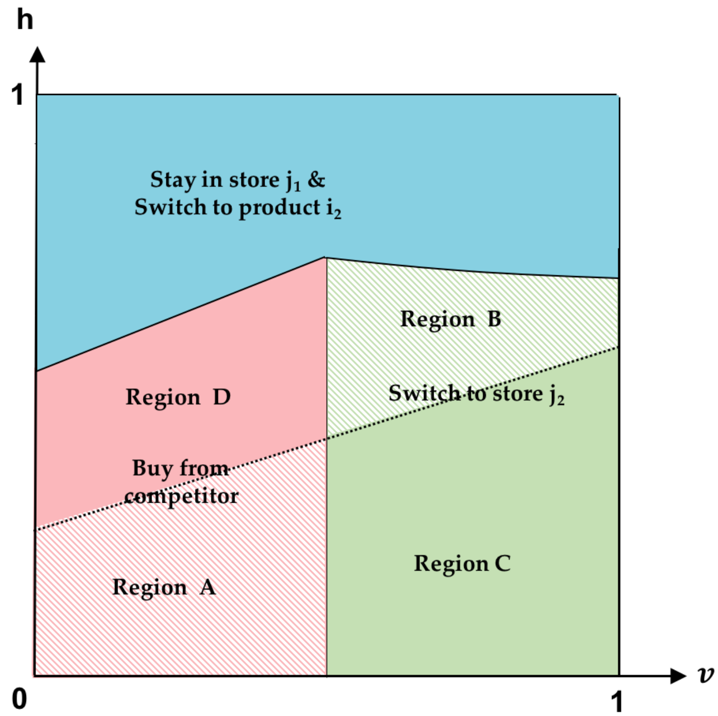


Figure A.3: Distribution of Customers in the Presence of a Competitor Close to Neighbouring Store



customers making the decision of switching to store j_2 according to expected utility as seen in Figure A.3. The total increase is equal to the sum of the regions D and B. However, after arriving store j_2, some of the customers represented by regions A and D in Figure A.3 switches to the competitor's product i_3. Therefore, the net effect of having a competitor in the vicinity of store j_2 can be represented by the difference between the regions A and B. If A is larger than B, then even if we see more customers choosing the second option in Figure A.1, the total demand seen by the store j_2 decreases by A-B. For simplicity, we assume $p_{i_2} > p_{i_1} > p_{i_3}$ and let the valuation of the products change with v, α_{i_3} and α_{i_2}. Additional assumptions made are $0 < \frac{(1-\alpha_{i_2})+(p_{i_2}-p_{i_1})}{d_{j_1 j_2}} < 1$ and $p_{i_1} - p_{i_2} < 1 - \alpha_{i_2}$. These assumptions ensure that there is already a positive number of customers switching to store j_2 even when there is no competitor and that total number of customers choosing to switch to store j_2 increases with the addition of competitor. Using these assumptions, we can calculate the change in store j_2's expected demand when there is a competitor in its vicinity:

$$E\left[\Delta_D\right] = \int_{\frac{p_{i_1}-p_{i_3}}{1-\alpha_{i_3}}}^{1} \frac{(p_{i_3}-p_{i_1})^2 + 2v(v-p_{i_1})}{2vd_{j_1j_2}} + \frac{p_{i_2}-v\alpha_{i_2}}{d_{j_1j_2}} dv$$
$$- [(\frac{\frac{(1-\alpha_{i_2})(p_{i_1}-p_{i_3})}{(1-\alpha_{i_3})d_{j_1j_2}} + 2\frac{p_{i_2}-p_{i_1}}{d_{j_1j_2}} + \frac{1-\alpha_{i_2}}{d_{j_1j_2}}}{2})\frac{(1-\alpha_{i_3})-(p_{i_1}-p_{i_3})}{1-\alpha_{i_3}}] \tag{A.1}$$
$$- [(\frac{\frac{(1-\alpha_{i_2})(p_{i_1}-p_{i_3})}{(1-\alpha_{i_3})d_{j_1j_2}} + 2\frac{p_{i_2}-p_{i_1}}{d_{j_1j_2}}}{2})\frac{p_{i_1}-p_{i_3}}{1-\alpha_{i_3}}],$$

where $E\left[\Delta_D\right]$ denotes the change in store j_2's expected demand. We can numerically calculate the Equation A.1 to investigate the expected demand change in store j_2 due to the stock outs in store j_1 when a competitor closer to store j_2 is added to the system. The values used for the analysis are in the range of $[0, 1]$ with step size 0.1. As shown in Table A.1, the change in the expected demand is negative, which implies that the existence of a competitor decreases the expected demand switching from the focal store to neighboring store. In Table A.1, we present the summary statistics for the change in the store j_2's demand with respect to low and high levels of the parameters. "Low" refers to the values are less than or equal to 0.5 and "High" refers to the values greater than 0.5.

Table A.1: Demand Changes in Store j_2 in the Presence of a Competitor

α_{i_3}	α_{i_2}	$d_{j_1j_2}$	p_{i_3}	p_{i_1}	p_{i_2}	Expected	Std. Dev.	Min	Median	Max
L	L	H	L	L	L	-0.0307	0.0618	-0.444	-0.00946	0.0555
L	L	H	L	L	H	-0.117	0.122	-0.687	-0.0729	0.00365
L	L	H	H	H	H	-0.117	0.146	-0.687	-0.06	0.0555
H	L	H	L	L	L	-0.219	0.183	-0.688	-0.163	0
H	L	H	L	L	H	-0.347	0.208	-0.687	-0.323	0
H	L	H	H	H	H	-0.267	0.186	-0.687	-0.235	-0.00966
L	H	L	L	L	L	-0.00315	0.0788	-0.471	0	0.371
L	H	L	L	L	H	-0.132	0.149	-0.875	-0.0849	0.127
L	H	L	H	H	H	-0.0695	0.175	-0.875	-0.019	0.371
L	H	H	L	L	L	-0.0147	0.0465	-0.333	-0.00247	0.124
L	H	H	L	L	H	-0.12	0.113	-0.833	-0.0873	0.0422
L	H	H	H	H	H	-0.0664	0.118	-0.75	-0.0255	0.124
H	H	L	L	L	L	-0.205	0.202	-0.875	-0.146	0.019
H	H	L	L	L	H	-0.414	0.24	-0.875	-0.375	0
H	H	L	H	H	H	-0.252	0.216	-0.875	-0.199	0.019
H	H	H	L	L	L	-0.154	0.153	-0.833	-0.111	0.00634
H	H	H	L	L	H	-0.351	0.221	-0.938	-0.312	0
H	H	H	H	H	H	-0.183	0.158	-0.75	-0.142	0.00634

A.2 First Stage Regression for Instrumented Difference-in-Difference (DDIV)

Equation A.2 represents the first stage used in the instrumented difference-in-difference (DDIV) estimation:

$$DT_{st} = \rho_s + \tau T_t + \pi Z_{st} * T_t + \eta_{st} \tag{A.2}$$

where DT_{st} represents the endogenous treatment captured by the interaction of time dummies and treatment indicator ($stockout_{ijt}$). Furthermore, Z_{st} represents the instrument (i.e., arrival time of a product in a given store-SKU s), and $Z_{st} * T_t$ is the interaction of time dummies and instrument.

A.3 Decomposed Difference-in-Differences Analysis

This section considers an alternative approach to the main analysis we focus on in the paper. However, as this approach does not easily accommodate for endogeneity concerns, we do not use this specification as our baseline.

In a classical DiD application, the effect of the specific interaction can be represented as $(Y^{POST}_{TREAT} - Y^{PRE}_{TREAT}) - (Y^{POST}_{CONTROL} - Y^{PRE}_{CONTROL})$. It is implemented as an interaction term between time and treatment group dummy variables in a regression model as in Equation A.3 and the estimated coefficient for this interaction term, $\delta^{2\times2}$, gives the DiD estimate.

$$sales_{st} = \delta_0 + \delta_1 TREAT_s + \delta_2 POST_t + \delta^{2\times2} TREAT_s * POST_t + \varepsilon_{st} \tag{A.3}$$

In the classical approach, it is required to divide the data into 2 groups as treatment and control where the treatment time is the same for all the individuals in the treatment group. However, in our setting, the stock-outs are not realized at the same time in all of the stores for all of the products. When there is no one single treatment time, the general approach is to use a two-way fixed effects regression model with dummy variables for cross-sectional units (α_s) and time periods (α_t), and a treatment dummy ($TREAT_{st}$) as in Equation A.4. When there are 2 periods of time and the treatment time is fixed as the 2$^{\text{nd}}$ period, the standard DiD estimator, $\delta^{(2\times2)}$, is numerically equivalent to the linear two-way fixed effects regression estimator, δ_{DD}. Even though the equivalence does not generalize for more than two periods, the researchers still utilize two-way fixed effect estimator by referring to Angrist and Pischke

(2009).

$$sales_{st} = \alpha_s + \alpha_t + \delta_{DD} TREAT_{st} + \varepsilon_{st} \tag{A.4}$$

Goodman-Bacon (2018) confirms that the two-way fixed effects estimator δ_{DD} in Equation A.4 is a weighted average of all possible two-by-two DiD estimators ($\delta^{2\times2}$) that compare timing groups to each other. Goodman-Bacon (2018) also constructs the weights by explaining how they arise from differences in timing and treatment variances. This decomposition approach provides a way of further examining the DiD coefficients. In our study, we use a similar approach and decompose the data into two-by-two groups and obtain every possible two-by-two DiD estimators.

We obtain 3 different types of two-by-two panel data-sets. In the first set, we compare a treated group with the untreated group as in Equation A.5. Group U refers to the collection of store(j)-SKU(i) pairs where we do not see a stock-out in the neighboring store j while group m refers to the cases where there is a stock-out in the neighboring store j. We obtain multiple groups of m as the neighboring store stock-out times are different for different store-SKU pairs.

$$\delta_{mU}^{2\times2} = (Y_m^{POST} - Y_m^{PRE}) - (Y_U^{POST} - Y_U^{PRE}) \tag{A.5}$$

We also construct sets using only the treated units by naming them the earlier (group k) and the later (group l) treated groups. The treatment realizes at time t_l for earlier group and at time t_k for later treated group where $t_l < t_k$. If we create a sub-sample with the observations from these two groups until time t_k, later treated units can act as the control group since the treatment status does not change for later treated groups while the early treated group would be the treatment group.

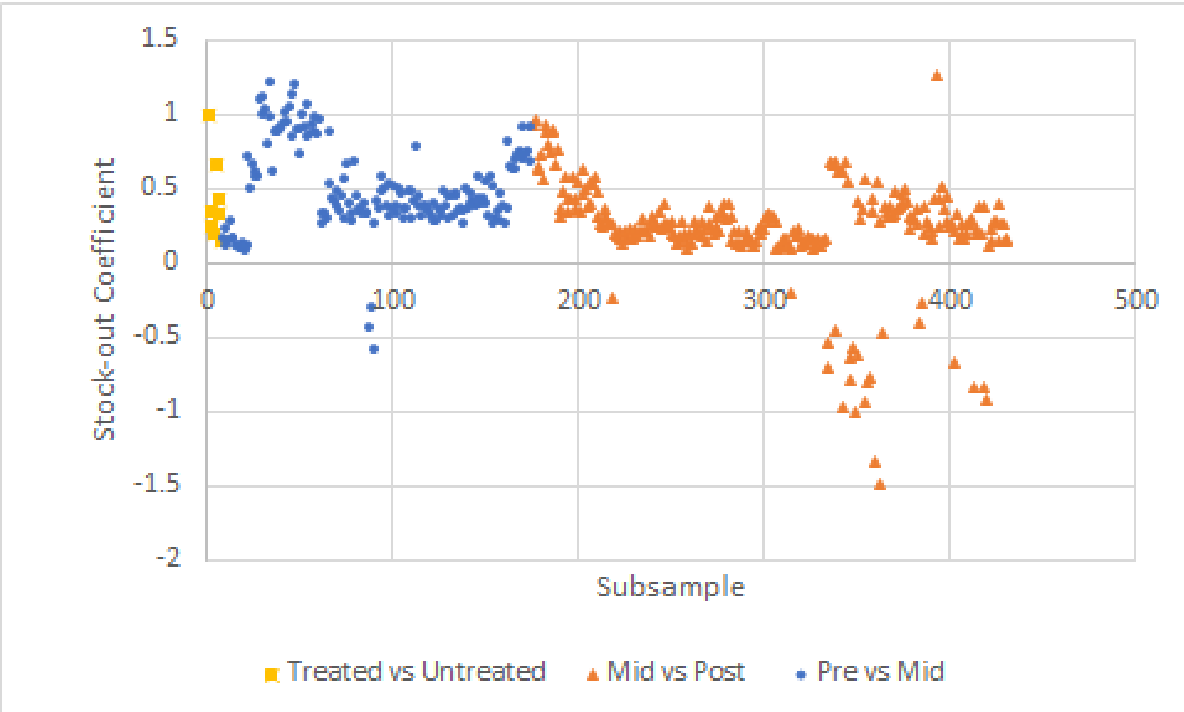
$$\delta_{kl}^{2\times2,k} = (Y_k^{MID(k,l)} - Y_k^{PRE}) - (Y_l^{MID(k,l)} - Y_l^{PRE}) \tag{A.6}$$

If the sub-sample is created starting from time t_l, the later treated group acts as the treatment group while the treatment status of the early treated group does not change throughout this time frame. Even though the early treated group is already treated at time t_l, since its status does not change after this time, it can act as the control group.

$$\delta_{kl}^{2\times2,l} = (Y_l^{POST(l)} - Y_l^{MID(k,l}) - (Y_k^{POST(l)} - Y_k^{MID(k,l)}) \tag{A.7}$$

Figure A.4 depicts all of the significant difference-in-differences coefficients obtained from all possible two-by-two sets in the dataset. The first time the stock-out seen in the closest neighboring store is taken as the treatment time and the untreated, earlier and later treated groups are formed accordingly. Seeing that the replenishments are made rarely, once a store experiences a stock-out, most of the time, its stock-out position does not change afterward, even though the timing of the treatment changes. This enables us to perform the difference-in-differences analysis for the two-by-two groups.

Figure A.4: Difference-in-Differences Decomposition



In Figure A.4, only the significant DiD coefficients are reported. The insignificant coefficients exist because of the lack of observations in some groups. For instance, the number of cases where a stock-out observed in earlier periods is low except for a few exceptional cases. Hence, we focus on the groups with a substantial number of observations. The DiD coefficients obtained by comparing treated and untreated groups are all positive among the significant ones. This result complies with the findings obtained in our main empirical analysis.

The two-by-two sets formed with earlier and later treated groups are analyzed by dividing the time-line into three parts as "Pre", "Mid" and "Post". Figure A.4 presents the DiD coefficients for the comparison of both "Pre" and "Mid" periods and "Mid" and "Post" periods. As explained in Section A.3, the control group used in "Pre" vs. "Mid" comparison is the pre-treatment period of the later treated group. This part can be seen as a better representation of the classical DiD approach in which the control group consists of untreated observations. For the comparisons of "Mid" and "Post" periods, we utilize the post-treatment observations of the earlier treated group as the control group. As long as their treatment position does not change, the treatment effect can be captured using the change in treatment for the later treated group. To establish further confidence in our findings, we emphasize that for both of these two-by-two sets, the DiD coefficients are predominantly positive.

A.4 Descriptive OLS Analysis

This section considers a set of alternative specifications using simpler OLS analysis. By doing so, we can explore some of the more nuanced patterns in the stock-out effects. However, these specifications do not accommodate for the timing considerations and endogeneity concerns, that our baseline analysis can control for.

We also construct various specifications using fixed effects at different levels as the general approach utilized in the literature, as well as utilizing product attributes in a classical OLS specification to further investigate the effect of neighboring store's stock-outs on the sales of focal store while controlling various factors. These specifications are not chosen as our baseline as they do not accommodate for multiple treatment groups as well as endogeneity of the stock-out treatment. Nevertheless, we believe these alternative OLS specifications are worth mentioning as they help confirm that our main findings are qualitatively robust to different econometric assumptions. Below we provide more details about the different specifications we consider.

In Equation A.8, we only include store fixed effects while adding various product attributes, month and holiday to the control variables. Equation A.9 can be seen as a more conservative approach which includes product and time fixed effects instead of product attributes and month and holiday dummies. In these specifications, we also test the impact of the distance between the stores by adding an interaction term of stock-out and distance variables. Referring back to our simple theoretical model presented earlier in the Appendix, its prediction would imply that β_2 is negative, as such a finding would suggest that customers are less willing to choose an alternative store located further away from the focal store they had originally intended to purchase the product from.

$$
\begin{aligned}
sales_{ijt} = \beta_0 + \beta_1 stockout_{i,j,t} + \beta_2 stockout_{i,j,t} * distance_{i,j,t} + \beta_3 stockout_{i,j,t} * elapsedtimeso_{i,j,t} \\
+ \gamma X_{ijt} + \mu_j + \varepsilon_{ijt}, \quad \text{(A.8)}
\end{aligned}
$$

$$sales_{ijt} = \beta_0 + \beta_1 stockout_{i,j,t} + \beta_2 stockout_{i,j,t} * distance_{i,j,t} + \beta_3 stockout_{i,j,t} * elapsedtimeso_{i,j,t} + \gamma X_{ijt} + \mu_m + \mu_j + \mu_t + \varepsilon_{ijt}, \quad \text{(A.9)}$$

where SKUs, options, stores and time (calendar week) are denoted by i, m, j, and t, respectively and j is the closest store to the store j and X_{ijt} includes all of the relevant controls about the product (i.e., price, discount, unit costs, inventory, product attributes, months, holidays etc.), and μ_m, μ_j, and μ_t are product, store, and time fixed effects, respectively.

First column of table A.2 summarizes our baseline regression results obtained from the models presented in Section A.4 for women category products. We have added product attribute dummies, month and holiday indicator and store fixed effects progressively in these models to illustrate the robustness of our results to various dimensions of heterogeneity. The positive effect of stock-outs can be seen immediately starting from the first model. Furthermore, we see that the main findings hold even as we control for various dimensions of heterogeneity. Even at the most conservative level, we see that a stock-out in the closest neighboring outlet can lead to an increase of sales by 0.074; thus, the economic magnitude of this neighboring stock-out effect is non-negligible. To get a better sense of the economic magnitude, we also considered a specification with $log(sales_{ijt})$ as the dependent variable as seen in second column of Table A.2. The results from this alternative specification suggest that stock-out in the closest neighboring outlet can increase sales by about 4%. When customers face a stock-out in focal store, these findings are consistent with the notion that there may be some substitutability across neighboring stores, especially so if the neighboring stores are close in distance. The value of stock-out indicators' estimates decreases as we move further from the focal store as it is indicated by the stock-out and physical distance interaction variable. Note that when product attributes are not controlled for via product fixed effects, the negative coefficient for the negative discount coefficient may reflect the fact that discounts are sometimes applied to products that are not particularly popular (i.e., the clearance rack at clothing stores).

In addition to our main results about the nearby stock-out effects, we also report some other patterns that the empirical analysis picks up. In particular, Table A.2 shows that the level of inventory is positively associated with sales. This pattern is consistent with the previous results in the literature that a larger inventory reflects a higher sales potential. For example, larger inventory may imply more products that are available to be sold (Balakrishnan et al. (2004)). Another empirical finding is that

Table A.2: OLS Results for "Women" Clothing Category

	(1)	(2)	(3)
stockout	0.074***	0.044***	0.041***
stockout*distance	-0.032**	-0.021**	-0.0041***
inventory	0.059***	0.027***	0.058***
price	-0.001***	-0.001***	0.000
discount	-0.003**	-0.012***	0.093***
elapsedtimedisc	-0.006***	-0.003***	-0.007***
elapsedtimestockout*stockout	-0.011***	-0.007***	-0.010***
product attributes	yes	yes	no
month	yes	yes	no
holiday	yes	yes	no
store fixed effects	yes	yes	yes
product fixed effects	no	no	yes
time fixed effects	no	no	yes
age	no	no	yes
Observations	6465547	6413323	6512313
R^2	0.101	0.089	0.125

Significance Levels: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

Note:In Column 1, the sales are used in unit while in Column 2 and 3, log conversion of sales is used. Additionally, for Column 1 and 2 product attributes are included to the model, while for Column 3 product fixed effects are used instead of attributes.

lower prices are associated with higher sales confirming that the additional volume of sales makes up for a potential smaller contribution margin when prices are lower.

We also report another specification where we incorporate product and time fixed effects instead of product attributes, month and holiday indicators. Third column of Table A.2 presents that the positive and significant effect of stock-outs in neighboring outlets on sales of focal outlets remains intact.

In demand estimation models, ignoring the price endogeneity may result in biases and create unsound results. One way to address this issue to use an instrument for the price. Cost of the product is widely used for this purpose Berry et al. (1995). Therefore, in another specification, we use cost as an instrument for the initial price of the product. Table A.3 summarizes the results of this specification both with the model utilizing product attributes, month and holiday indicator, and product and time fixed effects. The results do not differ from our main findings.

In addition to price endogeneity, one may worry about reverse causality between sales and inventory. Such a concern may be relevant if the chosen inventory levels reflect the expected demand for their products. We argue that such confounds are unlikely to play a big role in our specific empirical setting for a few reasons. First, it is unlikely that the firm is able to perfectly predict the inventory levels ex-ante, as these levels (which can take on 6 possible values) are set well in advance by small sets

Table A.3: Use of "Cost" as an Instrument for "Price" in the OLS Specification

	(1)	(2)
stockout	0.074^{**}	0.041^{***}
stockout*distance	-0.032^{**}	-0.040^{**}
controls	yes	yes
product attributes	yes	no
month	yes	no
holiday	yes	no
store fixed effects	yes	yes
product fixed effects	no	yes
time fixed effects	no	yes
age	no	yes
Observations	6465547	6512300
R^2	0.04	0.04

Significance Levels: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

of focus groups (with 16 or fewer participants) and finalized by a team of executive-level employees. This process in setting inventory levels almost surely introduces forecast error via a combination of inadequate focus group foresight and overly crude discretization of the inventories that can be set. Second, it is unlikely that the firm is able to perfectly set the allocation of products across stores. While they have a crude ranking of stores (based on size and their past sales performance), the logistics involved in distributing the products across stores introduce error (i.e., shipping delays because of the truck drivers) into the actual inventories that each store receives when new products are released. Note that both of these explanations give way to exclusion restrictions that can be used to form new instruments for the inventory levels and allocations.[17]

It should also be noted that even though some products are kept in the stores, the data reveals that most of the products sell less over time (see Figure A.5 for the downward trends). On the other hand, since the retailer does not replenish its inventory during the selling season, the possibility of observing stock-outs over time in some of the stores is also increasing due to the decrease in inventories. This feature in the data can create an understatement (i.e., more conservative estimates) of our inferred effects from neighboring stock-outs, since this institutional feature may lead to a negative correlation between stock-outs in neighboring stores and the sales in focal store. To examine if there is such a limiting behavior, we replicate our analysis with the products that are younger than 16 weeks. This value is chosen given that a regular season for a product is approximately 3 to 4 months. As it can be

[17]For brevity, we have left these results out of the paper, although they are available upon request.

seen in the first column of Table A.4, the inferred effects from neighboring stock-outs get stronger when the older products are removed from the analysis since we eliminate the observations that erroneously associate negative effects between nearby stock-outs and focal store sales.

Figure A.5: Change in Average Sales With Respect to Weeks

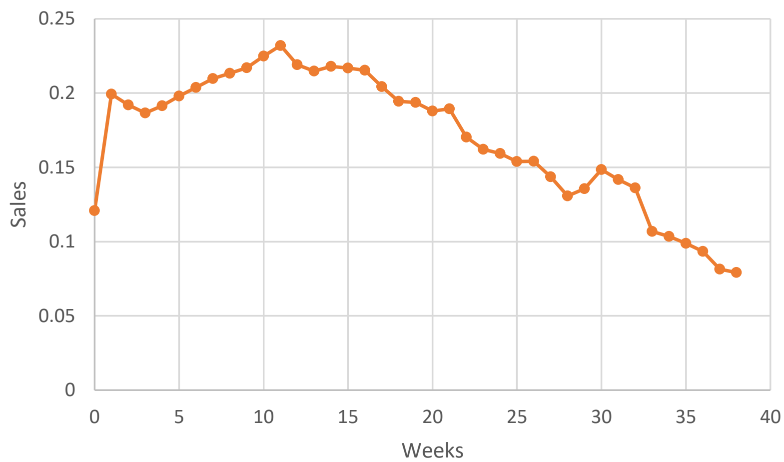


Table A.4: OLS Results for "Women" Clothing Category for the Products Younger than 16 Weeks and with Product-Time Interactions

	(1)	(2)
stockout	0.078***	0.031***
stockout*distance	-0.058**	-0.038***
control	yes	yes
product attributes	yes	no
month	yes	no
holiday	yes	no
store fixed effects	yes	yes
product fixed effects	no	yes
time fixed effects	no	yes
age	no	yes
product dummies x time trend interactions	no	yes
Observations	5301244	6512313
R^2	0.100	0.136

Significance Levels: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

In the final alternative specification, we control for product-time specific heterogeneity as trends may be seen at the product level in fashion. In column 2 of Table A.4, the fixed effect specifications with the addition of interactions between product dummies and time trends are presented. The main finding that the stock-outs in the closest neighboring stores positively affecting the sales in the focal stores is still preserved, even when these product-specific fashion trends are accounted for.

A.5 Robustness Test with Spring/Summer Data

In addition to the OLS analyses applied on Autumn/Winter data, as a robustness test, the baseline specification is applied on a new combined data which consists of original data and Spring/Summer data. With this combined data, we confirm that the main conclusion regarding the stock-out effect is consistent with the baseline results as seen in Table A.5. Note however that in order to include the SKU-level fixed effects with the combined data, we have focused on a sub-category of women's clothing (i.e., fashion sub-category). Without this narrower definition, the number of SKU fixed effects would be prohibitively large, as the number SKUs introduced in spring/summer is far larger than the number of SKUs introduced in fall/winter, despite the fact that a majority of sales occur in the fall/winter sample. For this reason, our main analysis focuses on the fall/winter data, so that the SKU fixed effects can be properly accounted for.

Table A.5: Results for "Women" Clothing Category "Fashion" Subcategory in Autumn/Winter and Spring/Summer Together

	(1)
$stockout_{t+3}$	0.014
	(0.034)
$stockout_{t+2}$	0.013
	(0.030)
$stockout_{t+1}$	0.048
	(0.033)
$stockout_{t0}$	0.161***
	(0.029)
$stockout_{t-1}$	0.071
	(0.043)
$stockout_{t-2}$	0.028
	(0.033)
$stockout_{t-3}$	0.127
	(0.087)
control	yes
seasonal dummy	yes
store fixed effects	yes
SKU fixed effects	yes
time fixed effects	yes
Observations	9393125

Significance Levels: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

A.6 Characteristics of Stocked-out Products

This section summarizes the main findings from our descriptive analysis. Here, we investigate how the stock-out effect differs across different product and store characteristics.

Table A.6: OLS Results for "Women" Clothing Category with Product Characteristics and Stock-out Interactions

	Product Type	Discount
stockout	0.095***	0.123***
	(0.008)	(0.008)
stockout*distance	-0.021*	-0.024**
	(0.008)	(0.008)
stockout*city fashion	0.018*	-
	(0.009)	-
stockout*lingerie	-0.060	-
	(0.031)	-
stockout*partywear	-0.006	-
	(0.035)	-
stockout*sportswear	-0.052	-
	(0.042)	-
city fashion	0.003***	-
	(0.000)	-
lingerie	-0.002	-
	(0.002)	-
partywear	-0.029***	-
	(0.001)	-
sportswear	-0.030***	-
	(0.001)	-
discount	-0.012***	-0.012***
	(0. 001)	(0. 001)
stockout*discount	-	-0.054**
	-	(0.019)
other controls	yes	yes
product attributes	yes	yes
month	yes	yes
holiday	yes	yes
store fixed effects	yes	yes
Observations	6398431	6398431
R^2	0.0884	0.0884

Significance Levels: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

Table A.7: OLS Results for "Women" Clothing Category with Holiday and Stock-out Interactions

	(1)
stockout	0.105***
	(0.006)
stockout*distance	-0.024*
	(0.008)
stockout*holiday	-0.008
	(0.011)
other controls	yes
product attributes	yes
month	yes
holiday	yes
store fixed effects	yes
Observations	6398431
R^2	0.0884

Significance Levels: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

Table A.8: OLS Results for "Women" Clothing Category with Store Characteristics and Stock-out Interactions

	Store Type	Store Size
stockout	0.132^{***}	0.085^{***}
	(0.010)	(0.011)
stockout*distance	-0.022^{*}	-0.028
	(0.008)	(0.015)
stockout*mall	0.018^{*}	-
	(0.009)	-
mall	0.012	-
	(33.310)	-
stockout*storesize	-	-0.017^{*}
	-	(0.06)
storesize	-	19.695^{***}
	-	(0.837)
other controls	yes	yes
product attributes	yes	yes
month	yes	yes
holiday	yes	yes
store fixed effects	yes	yes
Observations	6398431	1926723
R^2	0.0884	0.1099

Significance Levels: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

A.7 Simulation Algorithm with Gibbs Sampling

First, we define the notations for the decision variables. Let I_1 and I_2 be the initial inventory positions for stores 1 and 2. Given the inventory positions of stores, we can calculate the probability of the period in which stores 1 and 2 will face stock-outs. Let S_1 and S_2 denote the stock-out periods for stores 1 and 2, respectively. Based on our estimated stock-out-based demand model, we then define random demands in stores 1 and 2 as a function of the stock-out period in the neighboring store. More specifically, we define $D_t^i(S_j)$ to be the store i's random demand in period t given that neighboring store j is stocked out in period S_j. Finally, since the unit price is uniform across all the stores, we denote it by p. With these notations, we can define the inventory optimization problem under stock-out based demand model as follows:

$$\Pi_{OPT}^{so} = \max_{I_1 \geq 0, I_2 \geq 0} \quad \pi^{so}(I_1, I_2) = \sum_{S_1, S_2} P(S_1, S_2 | I_1, I_2) \left[\sum_{t_1=0}^{S_1} \sum_{t_1=0}^{S_2} E\left[p(D_{t_1}^1(S_2) + D_{t_2}^2(S_1)) \right] \right] \tag{A.10}$$

$$I_1 + I_2 \leq I_{total}$$

where expectation is taken with respect to random demand $D_t^i(S_j)$ realized in store i at period t given store j faces a stock-out in period S_j and I_{total} denotes the total initial inventory to be allocated to

Algorithm 1 Simulation Algorithm with Gibbs Sampling

Initialize I_{total}, T, N_s, and M_{gibbs}, where I_{total} is the total initial inventory to be allocated to the stores, T is the end of planning horizon(i.e., selling season) for the product, N_s is the number of Monte Carlo Simulations, M_{gibbs} is the number of samples drawn for Gibbs Sampling

for $I_1 = 0 : I_{total}$ **do**

  $I_2 = I_{total} - I_1$

  for $S_1 = 1 : T+1$ **do**

    for $j = 1 : N_s$ **do**

      for $t_2 = 1 : T$ **do**

        $D_{t_2} \sim D^2_{t_2}(S_1)$

      $D_{T+1} = \infty$

      $S_2 = min(\tau \geq 0 : \sum_{t_2=0}^{\tau} D(S_{t_2}) \geq I_2)$

    Calculate $P(S_2|S_1)$

  for $S_2 = 1 : T+1$ **do**

    for $j = 1 : N_s$ **do**

      for $t_1 = 1 : T$ **do**

        $D_{t_1} \sim D^1_{t_1}(S_2)$

      $D_{T+1} = \infty$

      $S_1 = min(\tau \geq 0 : \sum_{t_1=0}^{\tau} D(S_{t_1}) \geq I_1)$

    Calculate $P(S_1|S_2)$

  Gibbs Sampling

  Initialize $S_1 = s_1^0$ and $S_2 = s_2^0$

  for $k = 1 : M_{gibbs}$ **do**

    $s_1^k \sim P(S_1 = s_1|S_2 = s_2^{k-1})$

    $s_2^k \sim P(S_2 = s_2|S_1 = s_1^k)$

  Calculate $P(S_1, S_2|I_1, I_2)$

stores 1 and 2. As a benchmark, we also define another optimization problem where we allocate the total initial inventory among the stores assuming that the demand in each store does not depend on the stock-out period of the neighboring store. In this case, we define the random demand realized in each store i at period t independent of the stock-out period of the neighboring store and denote it by D_t^i. Also, the probability of the stock-out period for each store depends only on the inventory level of that store as denoted by $P(S_i|I_i)$. This leads to the following optimization problem:

$$\max_{I_1 \geq 0, I_2 \geq 0} \quad \pi^{noso}(I_1, I_2) = \sum_{i=1}^{2} P(S_i|I_i) \left[\sum_{t_i=0}^{S_i} E\left[pD_{t_i}^i \right] \right] \tag{A.11}$$
$$I_1 + I_2 \leq I_{total}$$

where the expectation is taken with respect to random demand realized in store i at period t. Once we solve the above optimization problem, we then evaluate the total revenue assuming that the actual demand for each store depends on the stock-out periods, i.e., we define

$$\Pi_{OPT}^{noso} = \pi^{so}(I_1^{noso}, I_2^{noso}) \tag{A.12}$$

where I_1^{noso} and I_2^{noso} maximizes $\pi^{noso}(I_1, I_2)$ provided in Problem (A.11). Finally, given Π_{OPT}^{so} and Π_{OPT}^{noso}, we calculate the percentage improvement in revenues by considering the spillover effect between neighboring stores as follows:

$$\Delta_\Pi = \frac{\Pi_{OPT}^{so} - \Pi_{OPT}^{noso}}{\Pi_{OPT}^{noso}} \tag{A.13}$$

To summarize, the optimization problem we study here can be interpreted as a retailer's decision about how to set the initial inventory levels for its two stores, given a total inventory. We note that in this optimization problem, we assume that retailers make decisions based on expected values for demand, as this assumption helps us mimic the fact that they often set initial allocation levels based on forecasts during their pre-season inventory planning.

The sampler generates a sequence of samples $\{(s_1^0, s_2^0), (s_1^1, s_2^1), (s_1^2, s_2^2), ...\}$ from the Markov chain over all possible states. If we draw samples from the Markov chain for sufficiently long periods, i.e., allowing enough time for the chain to reach the stationary distribution, we obtain independent samples from the distribution $f(S_1, S_2)$. This procedure is summarized in Algorithm 1. Once we generate the joint distribution, we can calculate the total expected revenue.