The Impact of Competitive Intelligence Services on Online Marketplaces

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Recent innovations have driven a steady increase in online marketplace transactions. To remain competitive, numerous marketplace platforms and independent data providers offer *Competitive Intelligence Services* (CIS), enabling sellers to explore not only their market potential but also that of their competitors. In this paper, we employ a game-theoretic approach to competitive learning to analyze the impact of CIS on participants with varying market shares in an online marketplace. In the presence of noisy demand signals, the online platform benefits from offering CIS to both sellers. This is because high demand noise makes demand exploration difficult for each seller in the first period. Consequently, price competition under poor knowledge of the price-demand relationship in the second period leads to a lower payoff for each seller as well as the platform. However, as demand uncertainty decreases, the platform prefers inducing CIS exclusively for the seller with the larger market share. This scenario leads to *signal-jamming* behavior between the sellers, which results in a win-win-win situation for both sellers and the platform provider. Finally, we consider various model extensions and discuss the managerial implications for the design and regulation of competitive intelligence services in online marketplaces.

Key words: Online marketplaces; Platform economy; Competitive learning; Mechanism design; Signal-jamming; Price competition

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1. Introduction

Online marketplaces have witnessed significant growth over the years. In 2025, these platforms accounted for more than 23% of global retail sales, and this figure is expected to reach 25% by 2030 (Statista 2025). Online marketplaces differ considerably within the retail sector, depending on whether they facilitate business-to-business (B2B), business-to-consumer (B2C), or peer-to-peer (P2P) transactions (Anastasia 2020); whether they exchange goods, services, or information (Pryhodko 2020); and whether they generate revenue via advertisements, sales commission, or subscription services. Regardless, the value proposition of all online platforms is the same: facilitating transactions between independent supply and demand-side participants (Täuscher and Laudien 2018). As such, their growth will continue in the foreseeable future, and they will continue to attract not only more consumers but also more *sellers*, thereby creating a win-win scenario for all stakeholders.

In order to gain a competitive advantage, many sellers on online marketplaces employ tools that gather, manage, and use information collected not only from their own customers but also from their competitors. In fact, a survey conducted by Forrester Consulting (Yakkundi 2016) among 115 technology, marketing, and business professionals suggests that exploring the customer journey in a competitive customer-facing web environment is the highest strategic priority for sellers. However, not all online sellers have the resources to carry out competitive intelligence activities on their own. According to a 2018 survey of mostly U.S.-based Amazon sellers, 73% of them had only one to five employees (and 93% had fewer than 50 employees)[1]. This is why easy-to-use Competitive Intelligence[2] Services (CIS), powered by artificial intelligence and machine learning, have gained popularity on platforms like Google, Amazon, and others (Mroz 2018). For example, subscribing to Amazon Marketing Web Services (recently revised to Selling Partner API) enables sellers to understand the source of traffic for not only their own products but also for those of their competitors (Malachard 2020). WeChat Marketing Services allow sellers to segment their own customers as well as those of their competitors based on clickstream data (TDA 2019). The need for competitive intelligence services has given rise to a number of third-party data brokers, such as Scrapfly, Competitive Analytics, Liberty Metrics, and Jungle Scout, who provide automated data collection services to sellers in online marketplaces.

However, even though CIS are widely popular, little is known about how these services, provided by the platform to its sellers, affect seller decisions and platform actions over time. On the one hand, consider the decisions taken by the sellers after receiving business intelligence inputs. If, for instance, all the sellers on a platform are subscribed to CIS, then they will all have access to intelligence inputs. Undoubtedly, these inputs will alter their decisions compared to a situation in which only a few sellers are subscribed to CIS. In the latter case, only the sellers receiving CIS inputs will gain a competitive advantage, whereas in the former case, the effect of CIS is likely to be eroded. On the other hand, consider the decisions taken by sellers during the data collection/analysis stage, i.e., before the availability of business intelligence. In that stage, the decisions of all sellers, whether or not they have subscribed to CIS, are likely to directly affect the quality of information gathered from the data collected. Thus, sellers have an opportunity to increase or decrease the quality of intelligence available to the platform. Overall, after considering the ex-ante and ex-post impacts of CIS on all the stakeholders, we arrive at the following research questions:

- How does offering CIS to both sellers affect the payoffs of the platform participants and consumer welfare?

[1] https://www.statista.com/statistics/886904/amazon-seller-business-size-by-employees/

[2] Competitive intelligence means understanding and learning about what is happening outside the firm to enhance its competitiveness. It entails gathering as much information as possible, as quickly as possible, about the external environment, which includes the industry as a whole and relevant competitors.

- What is the impact of offering CIS exclusively to only one seller on the decisions and payoffs of the platform, sellers, and consumers?

To address these research questions, we develop a stylized two-period model that studies two asymmetric sellers (in terms of their market potentials) who sell their products through an online marketplace, which we refer to as the platform[3]. The platform charges a fixed subscription fee for CIS, and the sellers decide whether to subscribe to the CIS or not. The sellers then compete in prices in both periods. In order to capture the value and impact of CIS, we employ a demand-learning framework. Specifically, the market potential (represented by online traffic) of each seller is initially unknown to both sellers and the platform provider. In the first period, each seller sets an exploratory price with the objective of learning the true market potential during the first period. At the end of the first period, each seller observes only her own sales, whereas the platform observes sales of both sellers, and therefore has access to superior information. Based on the observed sales, both the sellers and the platform update their beliefs about the true market potential. If a seller is subscribed to CIS, then, at the end of the first period, that seller receives the updated belief of the platform. Otherwise, she relies only on her own updated belief and decides on the second-period price. Building on the above modeling framework and considering the possibility of each seller's subscription decision for CIS information, we create four scenarios and provide a complete characterization of the equilibrium prices and payoffs in both periods.

To address the first research question, we consider two scenarios: one where both sellers subscribe to the CIS, and the other where neither seller does. Under each scenario, we characterize the subscription fees that are incentive compatible for the sellers. Finally, we compare the equilibrium payoffs from the platform's perspective. Our analysis shows that if the observations from the first period sales are too uncertain for the sellers to accurately identify the true market potential or when the degree of competition between the sellers is low, the platform has an incentive to induce both sellers to subscribe to the CIS. When either of these conditions is not met, the platform actually prefers not to induce any of the sellers to subscribe to the CIS. Consequently, each seller has to find her own balance between exploring the demand parameters and exploiting the available information in a competitive environment. This result is robust to various extensions under which seller asymmetry is observed in marginal cost, the total parameters governing the market potential become state dependent, and the demand noise for each seller is non-uniform and partially correlated.

In order to address the second research question, we consider scenarios in which one seller subscribes to CIS while the other seller does not. Our analysis shows that the platform finds it incentive compatible to induce only the seller with the larger market to subscribe, accomplishing this by setting a subscription fee that makes it profitable for only the larger-market seller to participate. Analyzing the equilibrium, contrary to our intuition, reveals that a seller with a smaller market share may indeed benefit from not subscribing

[3] Although technically and legally the marketplace provider and the platform owner may be different entities we do not make this distinction and assume that platform owner is also the marketplace provider who operates the platform for sellers and buyers.

to CIS. This counterintuitive finding arises due to the nature of first-period price competition between the sellers when only one of them subscribes to CIS. Specifically, the seller who receives updated information about the market potential from the platform has an incentive to charge a price that makes it difficult for the competitor to explore. We call this *signal-jamming* and analyze its impact on equilibrium prices. Interestingly, signal-jamming leads to both sellers charging higher first-period prices, which turns out to be mutually beneficial for all parties involved, resulting in a win-win-win scenario for both sellers and the platform provider. However, from a consumer welfare perspective, this scenario hurts the customers because of the high prices at equilibrium.

Our findings have managerial implications for platforms and regulators. First, our study identifies when and to whom marketplace providers should offer CIS: when the demand information is too uncertain for the sellers, the platform should step in and induce both sellers to subscribe to the CIS by offering this service. Our results show that this would prevent both sellers from charging suboptimal prices and increase profit for the whole platform. We expect that products with short life cycles, such as fashion items or high-tech products, are likely to have more uncertain demand; hence, in such cases, the platform should take the initiative to explore and share the demand *symmetrically* with all marketplace players. However, in the case of products with relatively less uncertain demand, such as commodity items or non-seasonal products, our results show that the platform should either delegate the demand exploration to each seller or, if it must interfere, induce only the seller with the larger market share to subscribe to the CIS.

That being said, from a regulator's perspective, this asymmetric information sharing leads to lower consumer welfare. As such, a marketplace regulator should ensure that platforms do not offer a subscription policy that affects marketplace players asymmetrically. In fact, the recent data-sharing regulations designed by the European Union aim to create a level playing field for all the stakeholders in online marketplaces[4]. Warnings have been issued to ensure proper regulation of platforms and guard against their powerful network effects (Sokol and Van Alstyne 2021).

Along these lines, our findings speak directly to a sharp contrast between how the European Union (EU) and the United States (US) regulate data sharing on digital platforms. The EU adopts an *ex-ante* approach: the Digital Markets Act (DMA), which became applicable in May 2023 with gatekeeper compliance required from March 2024, requires designated platform "gatekeepers" (such as Amazon and Meta) to provide business users with continuous, real-time, non-discriminatory access to the data generated through their own activities on the platform, and prohibits gatekeepers from using non-public business-user data to compete against those same users (Articles 6(2) and 6(10); Commission 2025, Cabral et al. 2021). The US, by contrast, imposes no comparable mandate and instead relies on *ex-post* antitrust enforcement, intervening only after specific conduct is shown to harm competition. These two philosophies embody a fundamental trade-off: the EU's

[4] See, for example, the Digital Markets Act (Commission 2025) of the European Commission dated Dec. 15, 2020.

symmetric-access rule is designed to protect smaller sellers and consumers but constrains the platform's ability to share information selectively, whereas the US's permissive stance preserves the platform's flexibility at the risk of asymmetric outcomes that may disadvantage smaller sellers and consumers. Our model provides an analytical lens for this debate: it identifies the specific market conditions under which the platform's privately optimal information-sharing regime is symmetric (consistent with the DMA) versus asymmetric (permissible only under the US regime), and pinpoints exactly when each regulatory philosophy yields socially desirable outcomes.

The rest of this paper is organized as follows. In § 2, we position our work in the context of other relevant literature. In § 3, we develop our modeling framework. In § 4, we analyze the impact of offering CIS to both sellers. In § 5, we analyze the impact of offering CIS exclusively to only one of the sellers. We extend our results in § 6 and conclude the paper with managerial and theoretical insights in § 7.

2. Related Literature

The design of initiatives for information exchange among members of the distribution channel has always been a crucial topic (Guo and Zhao 2009, Gal-Or et al. 2008, He et al. 2008, Gal-Or et al. 2007, Dukes et al. 2017). More recently, this has become even more critical due to the ability of online platforms to efficiently collect extensive data on consumers' purchasing habits. Consequently, the strategies for deciding to whom and how this information is shared, often hidden from upstream manufacturers and rival sellers, have gained greater significance (Zha et al. 2023, Shi et al. 2023, Long et al. 2022, Caldieraro et al. 2018). Our paper contributes to this research stream by examining how platforms can benefit from competitive learning through the governance of CIS among participants with varying market shares in an online marketplace.

The platform's strategy for sharing sales information is based on how this information influences price competition among retailers and, consequently, the profit of each party, as well as the overall channel profit (Lai et al. 2022). In reality, sellers may not have precise knowledge of the relationship between price and demand. However, they can learn it through price experimentation (exploration) without completely sacrificing their revenue maximization goal (exploitation). This exploration-exploitation trade-off has been examined in prior research (Chen and Chen 2015). With this in mind, our primary goal is to provide context for our work and highlight its value and relevance. Accordingly, we categorize existing research into three groups based on the market structures they analyze: (i) a monopolistic seller, (ii) competing sellers, and (iii) a platform and competing sellers.

Early work on the exploration-exploitation trade-off analyzed the conditions under which a monopolistic seller would eventually obtain complete information about the underlying demand environment, while it focused only on exploitation (Rothschild 1974, McLennan 1984, Easley and Kiefer 1988, Aghion et al. 1993, Harrington 1995). Later, the focus shifted to developing algorithms with provable worst-case bounds based on minimizing regret. One approach to categorizing this stream of literature is by the mechanism used to estimate the demand parameters: Bayesian updating (Narayan et al. 2011, Lin 2006, Araman and Caldentey 2009, Gallego and Talebian 2012, Ching et al. 2013), realization probability (Cao and Zhang 2021), and linear least squares (Misra et al. 2019, Bertsimas and Perakis 2006). More recent work considers a variety of revenue management issues, such as a limited range of prices available to the seller (Perakis and Singhvi 2019), customized pricing decisions (Chen and Iyer 2002, Liu and Zhang 2006, Chen and Gallego 2018), and the impact of markdown policies for demand learning with forward-looking customers (Birge et al. 2019). Our work differs from the monopoly stream in that we study seller competition. We identify the subgame-perfect equilibrium prices, which take into account the optimum exploration-exploitation trade-off given the competitive dynamics between sellers and the platform.

Accounting for competition in a demand-learning framework not only requires a rich underlying demand model and a convoluted game-theoretic analysis, but also complicates the exploration-exploitation trade-off faced by the sellers. Each seller needs to decide on the level of exploration while being aware that their experimentation can benefit rival sellers by increasing their profits or by providing them with free demand information (Bolton and Harris 1999). The majority of the works in this stream deploy a two-period model with two sellers. Sellers have imperfect information about the demand model when making their first-period decisions. In the second period, sellers make use of Bayesian updating based on the observed decisions and outcomes. This research focuses on identifying how the subgame-perfect equilibrium decisions compare with the myopic case (no exploration), the monopoly case, or the case of perfect demand information. Thus, the most relevant categorization of the existing work from our perspective is based on the distribution of information among the stakeholders. Specifically, it is either assumed that all sellers observe all the sales and decisions (Belleflamme and Bloch 2001, Keller and Rady 2003, Tsunoda and Zennyo 2021) or that the first-period decisions (output in Cournot models and price in Bertrand models) are private information (Bernhardt and Taub 2015, Bonatti et al. 2017, Stern and Birge 2020). When first-period decisions are common knowledge, sellers tend to adjust their decisions to acquire more information at the end of the first period. However, when first-period decisions are private information, an opportunity arises to manipulate the information available to opponents, i.e., signal-jamming. The novelty of this work lies in its use of a platform (i.e., an online marketplace) through which all the sellers compete. As an entity that has access to the sales individually observed by each seller, the platform can also explore the true demand by itself and share this information with all or none of the sellers or even exclusively with one seller, which is a unique and novel scenario. By explicitly considering platform-mediated interactions, we explore the interaction between the platform's exploration and sellers' pricing decisions.

Our model is also related to Operations Management literature that explores the impact of sharing demand information in supply chains (Chen 2003, Wang et al. 2022, Guo et al. 2014, Jain 2022). Previous studies often examine bilevel supply chain structures and involve vertical (Cachon and Lariviere 2001) or horizontal competition (Ha and Tong 2008), as well as information-sharing games among competing supply chains (Ha et al. 2011, Shamir and Shin 2016). The credibility of shared demand information is a central concern in this literature, addressed through screening (Ha and Tong 2008) or signaling mechanisms (Gümüş 2014). Our work differs in two main ways. First, while many studies assume one supply chain partner possesses *ex-ante* private information about demand, in our model, demand information is *ex-ante* unknown to all parties. Second, prior research often assumes that private information remains unaffected by other supply chain firms' decisions. In contrast, our model considers how each party's (sellers and platform) knowledge of true demand information is influenced by competing firms' decisions (specifically pricing). These unique features allow us to analyze the exploration-exploitation trade-off in the context of platform-mediated competition.

Recent works point to a growing interest in modeling the exploration-exploitation trade-off over a duration longer than two periods (Perakis and Sood 2006, Stern and Birge 2020). Perakis and Sood (2006) propose a robust optimization approach that maximizes the revenue for each seller under the most adverse instances of unknown parameters of the demand function. They prove the existence of equilibrium policies and develop an iterative learning algorithm for finding them. In the work by Stern and Birge (2020), each seller estimates the underlying demand curve using least squares estimation. Interestingly, the results suggest that sellers may prefer to stay ignorant, as being informed may reduce prices (ignorance may lead to collusion). The analysis of a strategic game shows the existence of an equilibrium in which firms actively avoid learning (by avoiding experimentation) the true value of demand and attain a collusive outcome even in finite-time horizons.

Our work differs from the literature on information sharing in retail platforms. First, unlike much of the literature (e.g., Bimpikis et al. (2019); Liu et al. (2021); Wang et al. (2022); Zha et al. (2023)), we do not assume that the platform or sellers initially hold private demand information; instead, we introduce a novel self-learning channel whereby sellers can infer demand from their own sales. Second, while most existing studies adopt a single-period or single-decision setting in which uninformed parties remain passive, we allow sellers to make two sequential pricing decisions, which creates a new intertemporal trade-off between exploration and exploitation. Finally, relative to empirical work such as Huang et al. (2022), which studies adaptive learning in real markets, our contribution lies in a game-theoretic analysis of how subscription-based and self-learning channels jointly shape equilibrium pricing, information acquisition, and welfare outcomes.

Last but not least, our model is also related to the literature on Bayesian persuasion and strategic information design (Kamenica and Gentzkow (2011)). In our model, the platform effectively commits to an information disclosure policy—implemented through its CIS subscription design—that shapes sellers' posterior beliefs and pricing behavior. Unlike classical persuasion models, however, information in our setting is endogenously generated through sellers' pricing decisions and sales realizations, and competing sellers can strategically manipulate the informativeness of this data. This interaction gives rise to signal-jamming incentives that are unique to platform-mediated competition and highlight new limits and possibilities of information design in competitive marketplaces.

3. Model Framework

Consider two sellers, A and B, who are selling their products through an online platform, O, over two periods. Both sellers are uncertain about their market sizes. To capture this uncertainty, we assume that the market size of each seller depends on the demand state $\boldsymbol{\theta}$, where $\boldsymbol{\theta}$ can be high (i.e., $\boldsymbol{\theta} = H$) or low (i.e., $\boldsymbol{\theta} = L$) with equal probability. The true value of the demand state is set by nature and is a priori unknown to both sellers and the platform. We assume that sellers A and B vary in terms of their potential market sizes. Let $\alpha_i(\boldsymbol{\theta})$ denote the market size of seller i under demand state $\boldsymbol{\theta}$, where $i \in \{A, B\}$. Without loss of generality, we assume that seller A's market size is stochastically larger than that of seller B, i.e., $\alpha_A(\boldsymbol{\theta}) \geq \alpha_B(\boldsymbol{\theta})$ for any realization of the demand state $\boldsymbol{\theta} \in \{H, L\}$.

Note that, at the end of the first period, each seller observes her own sales, while the platform observes the sales of both sellers. This highlights the significance of CIS benefits; the platform has a comprehensive view of total sales, resulting in a more accurate updated belief about the demand state. Consequently, the platform can share its updated belief with the seller(s) who have subscribed to the CIS. To model this, we define ϕ to represent the subscription fee, a lump-sum amount required to be paid by sellers if they desire access to CIS benefits. Let $y_i \in \{0, 1\}$ represent seller-i's decision to subscribe to CIS (i.e., $y_i = 1$) or not (i.e., $y_i = 0$). Consequently, four subscription regimes emerge: (i) regime $J = \emptyset$ under which neither of the sellers subscribe (i.e., $y_A = y_B = 0$), (ii) regime $J = AB$ under which both sellers subscribe (i.e., $y_A = y_B = 1$), (iii) regime $J = A$ under which only the larger seller subscribes (i.e., $y_A = 1; y_B = 0$), and (iv) regime $J = B$ under which only the small seller subscribes (i.e., $y_A = 0; y_B = 1$). In order to avoid trivial cases, in which a seller can always learn the true demand state, we introduce a random shock in the demand function in a multiplicative form[5] via a random variable ϵ_t for $t \in \{1, 2\}$, where ϵ_t is independent and identically distributed uniformly $\epsilon_t \sim U[0, 1]$. The sales observed by each seller in period t under regime J have the following linear functional form[6]:

$$d_{i,t}\left(p^J_{i,t}, p^J_{-i,t}\right) = \alpha_i(\boldsymbol{\theta}) - p^J_{i,t} + 2\gamma\epsilon_t(p^J_{-i,t} - p^J_{i,t})$$

where $p^J_{i,t}$ and $p^J_{-i,t}$ denote the prices set by seller i and the competitor $-i$, respectively, in period t, given the subscription regime $J \in \{AB, A, B, \emptyset\}$. Note that parameter γ controls the magnitude of both own- and cross-price elasticities, and as such, influences the degree of price competition between the sellers. When γ is set to zero, the cross-elasticity term vanishes, and therefore, the price set by the competitor firm has no effect on the demand for the firm's product. However, as γ increases, a change in the competitor's price has a progressively greater impact on the firm's demand curve. Note also that the absolute value of the own-price elasticity of the demand function is greater than that of the cross-price elasticity in expectation, which is consistent with the findings in economics literature (e.g. Vives 1999).

[5] Both multiplicative and additive cases are extensively considered in the literature. We refer to Petruzzi and Dada (1999) for a detailed discussion of this issue.

[6] Consistent with game theory terminology, we denote the competing seller by subscript $-i$ throughout the paper.

We assume that the marginal cost of the product is normalized to zero and consider the non-zero marginal cost in §6.1. Given the subscription decisions $y_i, y_{-i} \in \{0,1\}$, which result in regime J, the expected channel profit generated by seller i can be expressed as follows:

$$\pi^J_{C|i}\left(P^J_i, P^J_{-i}\right) = E_{\theta,\epsilon_t}\left[\sum_{t=1}^{2} d^J_{i,t}\left(p^J_{i,t}, p^J_{-i,t}\right) p^J_{i,t}\right]$$

where P^J_i represents the price trajectory of seller i over two periods; i.e., $P^J_i \equiv (p^J_{i,1}, p^J_{i,2})$, the expectation is taken with respect to the a priori distribution on the demand state $\boldsymbol{\theta}$ and the multiplicative noise $\boldsymbol{\epsilon}_t$ for $t \in \{1,2\}$. Consequently, the payoff of seller i can be written as follows:

$$\pi^J_i\left(P^J_i, P^J_{-i}\right) = (1-\beta)\pi^J_{C|i}\left(P^J_i, P^J_{-i}\right) - y_i\phi \tag{1}$$

where β denotes the commission rate charged by the platform[7] for each unit sold. To induce the information regime $J \in \{AB, A, B, \emptyset\}$, the platform needs to solve the following optimization problem:

$$\max_{\phi}\ \pi^J_O = \beta \sum_{i\in\{A,B\}} \pi^J_{C|i}\left(P^J_i, P^J_{-i}\right) + \sum_{i\in\{A,B\}} y_i\phi \tag{2}$$

$$\text{Subject to}\quad \pi^J_i\left(P^J_i, P^J_{-i}\right) \geq \pi^{\check{J}}_i\left(P^{\check{J}}_i, P^{\check{J}}_{-i}\right),\ \check{J} \neq J,\ i \in \{A,B\} \tag{3}$$

where constraint (3) represents the incentive compatibility constraint, ensuring that seller i is better off by choosing a subscription decision aligned with regime J. The detailed account of events is depicted in Figure 1. At the outset, nature determines the true demand state $\boldsymbol{\theta} \in \{H, L\}$. The game is then played in the following stages:

– *Subscription stage*: Without knowledge of the value of $\boldsymbol{\theta}$, the platform sets the CIS subscription fee ϕ. Sellers then rely on their prior beliefs to decide whether to subscribe to CIS (i.e., $y_i = 1$) or not (i.e., $y_i = 0$). Consequently, the subscription regime $J \in \{AB, A, B, \emptyset\}$ is established and remains valid for two periods. A key practical consideration is whether, in the presence of CIS services, a seller consents to the platform sharing her sales information with the other seller[8]. In §4, we characterize the equilibrium under subscription regimes AB and $\emptyset$. Under regime AB, the platform offers the CIS subscription to both sellers, and it is implemented only if both sellers provide their consent. In §5, we extend the analysis to subscription regimes A and B, where the CIS subscription is offered unilaterally to a single seller, and thus, consent from the other seller is required.

– *Price competition in period 1*: Sellers A and B use their a priori beliefs about the true demand state and simultaneously set their prices $p^J_{A,1}$ and $p^J_{B,1}$, respectively. At the end of period 1, each seller observes her own sales, $d_{i,1}, i \in \{A,B\}$, while the platform observes each seller's sales; $d^J_{A,1}$ and $d^J_{B,1}$.

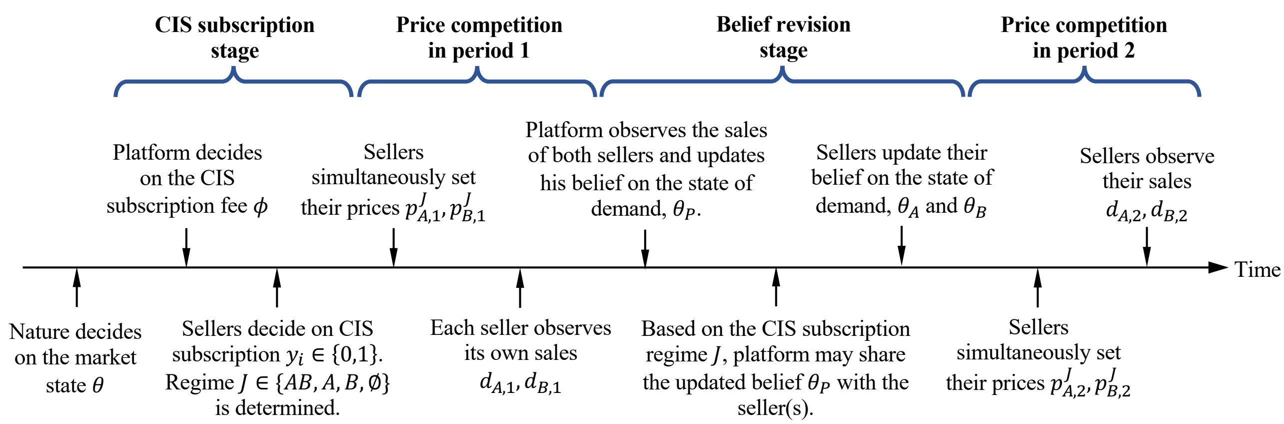
[7] For instance, according to Amazon pricing policy (`https://sell.amazon.com/pricing`), Amazon charges a fixed commission fee of 15% for categories like backpacks, handbags and luggage, as well as toys and games, 17% for clothing and accessories, and 8% for consumer electronics such as cell phone devices and computers.

[8] We assume that if a seller subscribes, the seller will provide consent. However, if a seller chooses not to subscribe to the CIS, she may either provide consent or withhold it from the platform.

– *Belief revision stage*: The platform updates its belief $\boldsymbol{\theta}_O$ and learns the true demand state; i.e., $Pr(\boldsymbol{\theta}_O = \boldsymbol{\theta}) = 1$. It shares these updated beliefs $\boldsymbol{\theta}_O$ with seller(s) if they opted for subscription to the CIS. Specifically, let $\boldsymbol{\theta}_i$ denote the posterior belief of seller $i \in \{A, B\}$ regarding demand at the end of the first period. Therefore, $\boldsymbol{\theta}_i = \boldsymbol{\theta}_O$ if $y_i = 1$.

– *Price competition in period 2*: Sellers A and B use their posterior beliefs $\boldsymbol{\theta}_i$ regarding the true demand state and simultaneously set their prices $p^J_{A,2}$ and $p^J_{B,2}$, respectively. The sales $d^J_{A,2}$ and $d^J_{B,2}$ would be observed at the end of the second period.

Figure 1 **Sequence of events**



To simplify the notation, in our baseline analysis, we assume that the total market size is normalized and equal to 1 under both demand states[9]. Accordingly, we use $\alpha_A(\boldsymbol{\theta}) = \alpha_{\boldsymbol{\theta}}$ and $\alpha_B(\boldsymbol{\theta}) = 1 - \alpha_{\boldsymbol{\theta}}$ for $\boldsymbol{\theta} \in \{H, L\}$.

3.1. First-Best Solution

In this section, we analyze the *first-best solution* assuming that the platform controls the pricing decision of both products[10]. Let $p^{\text{fb}}_{i,t}$ denote the first-best prices charged by the platform for product of seller $i \in \{A, B\}$ and in period $t \in \{1, 2\}$. Then, the platform solves the following optimization problem:

$$\max_{p^{fb}_{A,t}, p^{fb}_{B,t}} \; \pi^{fb}_O = \sum_{t\in\{1,2\}} \pi^{fb}_{O,t}(p^{fb}_{A,t}, p^{fb}_{B,t}) \tag{4}$$

The optimal solution is fully characterized in the following proposition. Note that all proofs are provided in the e-Companion under Section EC.2. For ease of reference, each proposition has a dedicated subsection, i.e., the proof of Proposition 1 appears in EC.2.1, the proof of Proposition 2 in EC.2.2, etc.

PROPOSITION 1. *Under the first-best scenario:*

– *In the first period, the platform sets* $p^{fb}_{A,1} = \overline{p}^{fb} + z^{fb}_p \Delta$ *and* $p^{fb}_{B,1} = \overline{p}^{fb} - z^{fb}_p \Delta$ *where* $\Delta \equiv \frac{\alpha_H - \alpha_L}{4\gamma}$, $\overline{p}^{fb} = \frac{1}{4}$, *and* $z^{fb}_p = \frac{\gamma}{1+2\gamma} \frac{\alpha_H + \alpha_L - 1}{\alpha_H - \alpha_L}$,

[9] We relax this assumption in §6.

[10] In the first best solution, sellers A and B sell differentiated products and we refer to them as product A and B.

– *The learning is perfect; the platform learns the true demand state, i.e.,* $\boldsymbol{\theta}_O = \boldsymbol{\theta}$*,*

– *In the second period, the platform charges the state-dependent prices, i.e.,*

$$p^{fb}_{A,2} = \frac{1}{4} + \frac{2\alpha_{\boldsymbol{\theta}} - 1}{4(1+2\gamma)}, \quad \textit{and} \quad p^{fb}_{B,2} = \frac{1}{4} - \frac{2\alpha_{\boldsymbol{\theta}} - 1}{4(1+2\gamma)}.$$

Recall that, by assumption, seller A's product is expected to achieve a higher market share than seller B's product. Therefore, both first- and second-period prices for product A are set higher than those for product B. This price distinction is evident in the optimal price structure described in Proposition 1, where the price for product A surpasses that of product B. Furthermore, the price difference between the products diminishes as the degree of substitution (measured by γ) increases. This is intuitive because the platform prefers to narrow the price gap to prevent demand cannibalization between the products.

3.2. Preliminaries

Before characterizing equilibrium under different CIS regimes J, we first formulate the problem and define the solution concept for the equilibrium. Under a specific CIS regime, each seller sets her price in each period by solving the following price optimization problem:

$$\max_{p_{i,t}} \pi^J_i = \pi^J_{i,1}\left(p_{i,1}, p_{-i,1}\right) + \pi^J_{i,2}\left(p_{i,2}, p_{-i,2}\right) \tag{5}$$

given that $p_{-i,t}$ solves the competing seller's optimization problem:

$$\max_{p_{-i,t}} \pi^J_{-i} = \pi^J_{-i,1}\left(p_{-i,1}, p_{i,1}\right) + \pi^J_{-i,2}\left(p_{-i,2}, p_{i,2}\right) \tag{6}$$

We utilize the concept of Perfect Bayesian Equilibrium (PBE) (Fudenberg and Tirole 1991) to solve the game described above. Given the subscription regime J, PBE requires the satisfaction of the following two conditions:

- Subgame perfection condition: Given the updated (i.e., a posteriori) beliefs $\boldsymbol{\theta}_A$ and $\boldsymbol{\theta}_B$, there is no profitable deviation for any seller from the equilibrium prices sequentially in the sense that:

— In period $t = 2$, there is no profitable deviation for a seller given the equilibrium first-period prices and the competitor's second-period price.

— In period $t = 1$, there is no profitable deviation for a seller given the equilibrium subscription decisions, and that sellers follow their best responses in period $t = 2$.

- Bayesian updating condition: Given the first-period prices, sellers update their beliefs about the true demand state in line with the Bayesian updating rule.

4. Model Analysis

In this section, our focus is on the competitive setting, and we analyze the impact of inducing both sellers to subscribe or not to subscribe to CIS on their equilibrium decisions and payoffs. To accomplish this, we first examine the equilibrium in the case of $J = AB$ in §4.1, where both sellers choose to subscribe to CIS. Subsequently, in §4.2, we characterize the equilibrium prices in the case of $J = \emptyset$, where neither of the sellers opts to subscribe to CIS. Next, in §4.3, we characterize the full equilibrium and evaluate the impact of the equilibrium on payoffs.

4.1. Regime $J = AB$: Inducing Both Sellers to Subscribe to CIS

In this case, the platform can induce both sellers to subscribe to CIS by finding a subscription fee ϕ in Eq. (2) that satisfies the following two incentive compatibility constraints:

$$\pi_B^{AB}\left(P_B^{AB}, P_A^{AB}\right) \geq \pi_B^{A}\left(P_B^{A}, P_A^{A}\right) \tag{7}$$

$$\pi_A^{AB}\left(P_A^{AB}, P_B^{AB}\right) \geq \pi_A^{B}\left(P_A^{B}, P_B^{B}\right) \tag{8}$$

The above constraints collectively ensure that both sellers are better off by subscribing to CIS, thus forming the information regime $J = AB$. Moreover, since the platform has access to both sellers' sales data and can always identify the true demand state at the end of the first period, both sellers are guaranteed to learn the true state of demand in the first period. This means that, unlike the no-CIS regime analyzed later in §4.2 (Proposition 4), where the first-period prices are shaped by an exploration-exploitation trade-off, in the presence of CIS, the sellers focus purely on *exploitation*, i.e., maximizing their first-period profit through price competition.

PROPOSITION 2. *The platform can induce subscription regime $J = AB$ only if $\alpha_H \in (\alpha_L, \alpha_H^{AB}]$, where α_H^{AB} is the solution of $\pi_{C|B}^{AB}\left(P_B^{AB}, P_A^{AB}\right) - \pi_{C|B}^{A}\left(P_B^{A}, P_A^{A}\right) = 0$. The CIS subscription is offered at a cost $\phi^{AB} = (1-\beta)\left[\pi_{C|B}^{AB}\left(P_B^{AB}, P_A^{AB}\right) - \pi_{C|B}^{A}\left(P_B^{A}, P_A^{A}\right)\right]$; both sellers subscribe and set the following equilibrium prices for the first period:*

$$p_{A,1}^{AB} = \overline{p}^{AB} + z_p^{AB}\Delta, \quad \text{and} \quad p_{B,1}^{AB} = \overline{p}^{AB} - z_p^{AB}\Delta,$$

where $\overline{p}^{AB} = \frac{1}{2(2+\gamma)}$, $z_p^{AB} = \frac{2\gamma(\alpha_H+\alpha_L-1)}{(2+3\gamma)(\alpha_H-\alpha_L)}$, and $\Delta = \frac{\alpha_H-\alpha_L}{4\gamma}$. After the first period, the platform learns the true demand state, i.e., $\boldsymbol{\theta}_O = \boldsymbol{\theta}$, and shares the information with both sellers, i.e., $\boldsymbol{\theta}_A = \boldsymbol{\theta}_B = \boldsymbol{\theta}$. In the second period, sellers set the following equilibrium prices:

$$p_{A,2}^{AB} = \frac{\gamma + (2+\gamma)\alpha_A(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)}, \quad \text{and} \quad p_{B,2}^{AB} = \frac{\gamma + (2+\gamma)\alpha_B(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)}.$$

The main takeaways from Proposition 2 are as follows. First, to induce both sellers to subscribe, the platform sets the subscription fee high enough to capture the entire surplus that the smaller seller would gain from subscribing to CIS. However, the larger seller, i.e., seller A, can still retain a portion of the surplus after accounting for the subscription fee. Second, inducing both sellers to subscribe to CIS is only feasible when the difference between the demand states under the a priori distribution is sufficiently low (i.e., $\alpha_L < \alpha_H \leq \alpha_H^{AB}$). Note that the first-period price of seller A (and seller B) increases (decreases) with α_H. When this difference becomes significant, i.e., $\alpha_L < \alpha_H^{AB} < \alpha_H$, it becomes infeasible for the platform to satisfy the incentive compatibility constraint (7). This is because the price competition intensifies in the first period; as α_H increases, seller A becomes more optimistic about a favorable market state, leading her to raise her price to take advantage of the opportunity. To mitigate the price competition, seller B prefers not to subscribe, opting to wait, observes her own sales, and makes decisions based on her posterior beliefs in the second period. Lastly, under regime $J = AB$, the average equilibrium price in the first period is lower than the first-best average price in Proposition 1 (i.e., $\overline{p}^{AB} < \overline{p}^{fb}$). However, the difference between equilibrium prices is higher than that of the first-best prices (i.e., $z_p^{AB} > z_p^{fb}$). These results are due to the price competition, leading to lower and more differentiated price levels compared to the first-best scenario, where the platform controls sellers' prices.

4.2. Regime $J = \emptyset$: Inducing No subscription to Both Sellers

In this section, we analyze the scenario under which the platform induces both sellers not to subscribe to the CIS. To achieve this, the platform needs to satisfy the following incentive compatibility constraints:

$$\pi_B^{\emptyset}\left(P_B^{\emptyset}, P_A^{\emptyset}\right) \geq \pi_B^B\left(P_B^B, P_A^B\right) \tag{9}$$

$$\pi_A^{\emptyset}\left(P_A^{\emptyset}, P_B^{\emptyset}\right) \geq \pi_A^A\left(P_A^A, P_B^A\right) \tag{10}$$

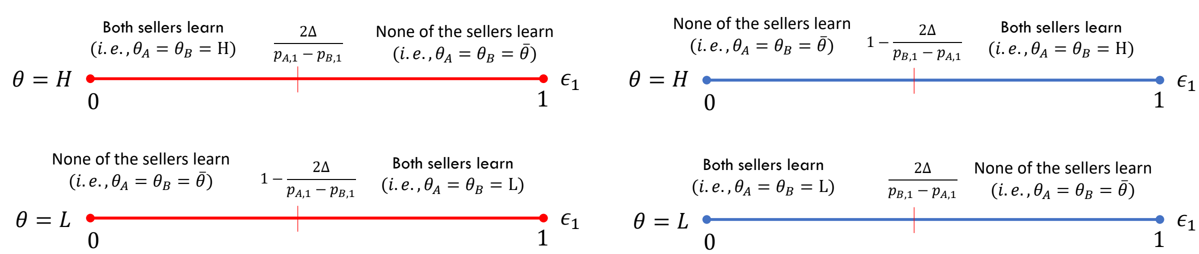
The above constraints collectively ensure that both sellers are better off by not subscribing to CIS, thus forming the information regime $J = \emptyset$. Note that, without CIS, sellers only observe their own sales at the end of the first period. Using backward induction, we first establish the beliefs regarding the true demand state for each seller based on their first-period prices. Depending on the ordering of first-period prices between sellers A and B, two cases arise: (i) $p_{A,1} \geq p_{B,1}$ and (ii) $p_{A,1} \leq p_{B,1}$. Throughout the paper, we focus on the first case, because, as shown in the e-Companion, it is the only scenario that can be sustained on the equilibrium path. Given the price difference in the first period and realized first-period demand, one can characterize the belief revision of each seller.

PROPOSITION 3. *Without subscribing to CIS, seller i updates her beliefs about the true demand state $\boldsymbol{\theta}$ according to Figure 2, which is illustrated as follows:*

- *If the difference between the first-period prices is less than 2Δ, where $\Delta = \frac{\alpha_H - \alpha_L}{4\gamma}$, i.e., $z_p^{\emptyset} = \frac{|p_{A,1}^{\emptyset} - p_{B,1}^{\emptyset}|}{2\Delta} \leq 1$, then, irrespective of the demand shock, both sellers A and seller B always learn the true demand state, i.e., $\boldsymbol{\theta}_i = \boldsymbol{\theta}$ for both $i \in \{A, B\}$.*
- *If the price difference is larger than 2Δ, i.e., $z_p^{\emptyset} > 1$, then, seller i's learning depends on the true demand state (i.e., whether $\boldsymbol{\theta} = H$ or $\boldsymbol{\theta} = L$) and the magnitude of the first-period demand shock, i.e.,*

$$\boldsymbol{\theta}_i = \begin{cases} H & \text{if } \epsilon_1 \leq \frac{2\Delta}{p_{A,1}^{\emptyset} - p_{B,1}^{\emptyset}} \text{ and } \boldsymbol{\theta} = H \\ L & \text{if } \epsilon_1 \geq 1 - \frac{2\Delta}{p_{A,1}^{\emptyset} - p_{B,1}^{\emptyset}} \text{ and } \boldsymbol{\theta} = L \\ \bar{\boldsymbol{\theta}} & \text{otherwise.} \end{cases}$$

Figure 2 **Sellers' belief updating based on first-period prices, true demand state θ and demand shock ϵ_1**



(a) $p_{A,1} \geq p_{B,1}$ (b) $p_{A,1} \leq p_{B,1}$

Proposition 3 clarifies that a seller's ability to learn the true demand state depends on the magnitude of the first-period price difference and demand shock. As shown in Figure 2, if the first-period prices are sufficiently close to each other (i.e., $z_p^{\emptyset} \leq 1$), both sellers are *guaranteed* to learn the true demand state through individual self-exploration; i.e., they can do so on their own. The rationale behind this is that the random noise gets amplified in the price difference. Therefore, if the price difference exceeds a certain threshold, it becomes difficult (with a smaller probability) to determine whether the observed demand is driven by the true demand state or random demand noise. If the magnitude of the first-period price difference is sufficiently large, then sellers can infer the true demand state depending on the magnitude of demand shock and true demand state. Namely, sellers can infer the true demand state only when their individual demand realization is at an extreme: either the demand shock is sufficiently low (i.e., $\boldsymbol{\epsilon}_1 \leq \frac{2\Delta}{p_{A,1}-p_{B,1}}$) or the demand shock is sufficiently high (i.e., $\boldsymbol{\epsilon}_1 \geq 1 - \frac{2\Delta}{p_{A,1}-p_{B,1}}$). This makes sense because when the price difference is large, the demand noise also becomes significant. Therefore, each seller needs to observe a signal that is sufficiently strong to differentiate between the high demand state and the low demand state.

The above observation has an important implication for the pricing policy of the sellers during the first period. On the one hand, if sellers A and B want to learn (i.e., *explore*) the true demand state by observing only their own sales, they have to converge their prices closer to each other, resulting in a smaller $z_p^{\emptyset}$. On the other hand, the desire to *exploit* their first-period profit triggers price competition, compelling sellers to differentiate their products by widening the price gap. This creates conflicting incentives for the first-period prices and introduces an exploration and exploitation trade-off for the sellers. The following proposition fully characterizes the equilibrium first-period prices under this exploration-exploitation trade-off:

PROPOSITION 4. *The platform can induce both sellers not to subscribe by setting subscription fee* $\phi^{\emptyset} \in [\max\{S_B^{\emptyset}, S_A^{\emptyset}\}, +\infty)$, *where* $S_B^{\emptyset} = (1-\beta)[\pi_{C|B}^{B} - \pi_{C|B}^{\emptyset}]$ *and* $S_A^{\emptyset} = (1-\beta)[\pi_{C|A}^{A} - \pi_{C|A}^{\emptyset}]$.

$$p_{A,1}^{\emptyset} = \overline{p}^{\emptyset} + z_p^{\emptyset}\Delta, \quad \textit{and} \quad p_{B,1}^{\emptyset} = \overline{p}^{\emptyset} - z_p^{\emptyset}\Delta$$

where $\Delta = \frac{\alpha_H - \alpha_L}{4\gamma}$, *and* $(\overline{p}^{\emptyset}, z_p^{\emptyset})$ *is characterized in the EC.2.4. At the end of the first period, the sellers observe the sales and update their own beliefs according to Proposition 3. Two scenarios may occur:*

(i) If the sellers learn the demand, i.e., $\boldsymbol{\theta} = H$ *or* $\boldsymbol{\theta} = L$, *then they set the following equilibrium second-period prices:*

$$p_{A,2}^{\emptyset} = \frac{\gamma + (2+\gamma)\alpha_A(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)}, \quad \textit{and} \quad p_{B,2}^{\emptyset} = \frac{\gamma + (2+\gamma)\alpha_B(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)}.$$

(ii) If the sellers do not learn the demand, i.e., $\boldsymbol{\theta} = \bar{\boldsymbol{\theta}}$, *they set the following equilibrium second-period prices:*

$$p_{A,2}^{\emptyset} = \frac{1}{2(2+\gamma)} + \frac{\alpha_H + \alpha_L - 1}{2(2+3\gamma)}, \quad \textit{and} \quad p_{B,2}^{\emptyset} = \frac{1}{2(2+\gamma)} - \frac{\alpha_H + \alpha_L - 1}{2(2+3\gamma)}.$$

4.3. Characterization of Equilibrium

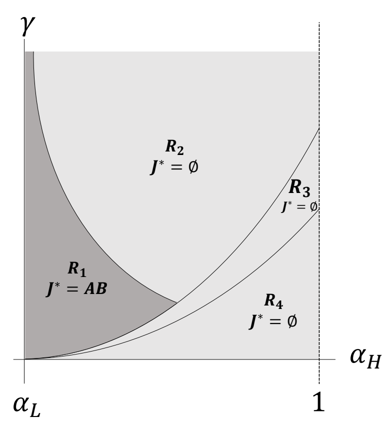
Now, we are ready to analyze the subscription decision of the platform. By comparing Propositions 2 and 4, we can characterize the full equilibrium. Leaving the details to the e-Companion, we characterize the equilibrium subscription decision of the platform and equilibrium prices and belief-updating of the sellers in the following Proposition 5:

PROPOSITION 5. *The equilibrium is divided into four regions, in each of which the platform's equilibrium regime J^* is characterized as follows:*

- *Region R_1 ($J^* = AB$): Sellers are induced to subscribe to the information-sharing service and set their first-period prices accordingly. The platform observes sales, learns the true demand state, and shares this information with the sellers.*
- *Region R_2 ($J^* = \emptyset$): The platform sets the subscription fee high enough to discourage sellers from subscribing. Sellers set their first-period prices and update their beliefs based solely on their own demand observations.*
- *Regions R_3 and R_4 ($J^* = \emptyset$): Sellers do not require a subscription to learn the true demand state. They set their first-period prices and always infer the true demand by observing their own sales.*

The corresponding equilibrium subscription fees, prices, and belief-updating rules are characterized in EC.2.5 and depicted in Figure 3.

Figure 3 **Platform's equilibrium subscription regime between $J = AB$ and $J = \emptyset$ as a function of the demand spread α_H (horizontal axis) and the substitution intensity γ (vertical axis)**



As characterized in Proposition 5, the full equilibrium is categorized into four regions depending on the value of system parameters. To gain a deeper insight into the equilibrium analysis, we can evaluate each region by defining a ratio denoted by $n = \frac{\alpha_H - \alpha_L}{\alpha_H + \alpha_L - 1}$. Delegating the details to the e-Companion, this ratio can be shown to equal the spread between the demand states under the a priori distribution divided by the difference between the sellers' average demands. Hence, it can be referred to as the *signal-to-noise ratio* (SNR)[11]. Since

[11] A similar term is employed by Hansen et al. (2021) to demonstrate how long-run prices depend on informational value.

the SNR (n) increases in α_H, we can rank the regions by their SNR values: region R_1 corresponds to the lowest SNR, region R_2 to a medium SNR, and regions R_3 and R_4 to the highest SNR. Using this measure, we can assess each region as follows:

- Region R_1: The SNR is extremely small, indicating only a narrow gap between the two demand states. In this setting, sellers find it difficult to identify the true demand state on their own. It is therefore up to the platform to determine whether it is beneficial to induce them to subscribe to the information-sharing service (CIS), a decision that involves a trade-off between first- and second-period outcomes. Inducing both sellers to subscribe to CIS benefits the platform in the second period: by sharing information, the platform enables sellers to fully learn the demand state at the end of the first period, allowing them to set more efficient prices and ultimately increase their second-period profits. The impact on first-period profits, however, is less straightforward. If both sellers know they will receive updated beliefs before the second-period price competition, they have no incentive to bring their prices closer together in the first period. This reduces price competition and lowers the platform's first-period payoff. Nevertheless, the gain in the second-period payoff from more efficient pricing outweighs this loss. Accordingly, in region R_1, the platform has an incentive to set the subscription fee low enough to encourage both sellers to subscribe to the information-sharing service.
- Regions R_2, R_3, and R_4: The SNR is sufficiently large for sellers to potentially learn the true demand state without subscribing. They can do so via one of two strategies: (i) bring their prices closer together (i.e., set $z_p \leq 1$), which -per Proposition 3- allows both sellers to always learn the true demand state, or (ii) maintain distinct prices in the first period (i.e., set $z_p > 1$) and infer the true demand state only probabilistically. The first strategy intensifies competition, which benefits the platform during the first period, while the second may result in suboptimal second-period prices due to the possibility that the demand state remains unidentified, which hurts the platform during the second period. Nevertheless, we can show that regardless of which strategy sellers choose, it is always optimal for the platform not to share demand information. Hence, in Regions R_2, R_3, and R_4, the platform has no incentive to induce sellers to subscribe to CIS. From the sellers' perspective, however, there is a trade-off between these two strategies, which depends on the level of γ, as explained below:

—Region R_2: In this region, γ is relatively high, which increases the degree of competition between sellers. Therefore, the sellers prefer to keep their first-period prices different ($z_p > 1$). Consequently, in some cases, the sellers cannot determine the true demand state by the end of the first period and must rely on their a priori beliefs when setting second-period prices.

—Region R_3: With sufficiently low γ, the cost of bringing prices closer is smaller due to weaker competitive pressure. Sellers, therefore, set their prices close enough ($z_p = 1$) to fully eliminate noise and always learn the true demand state.

—Region R_4: The SNR is already very high, so sellers set $z_p < 1$, which enables them to identify the true demand state from their own demand observations without having to adjust their first-period prices.

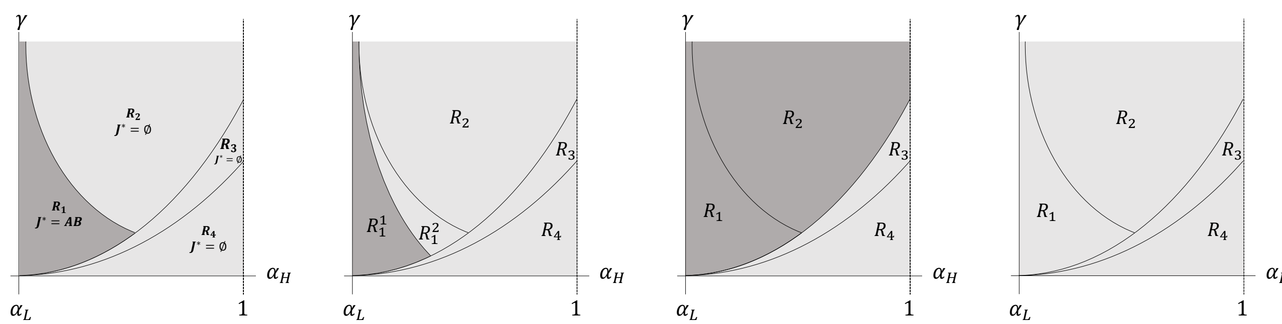
4.4. The Impact of Equilibrium on Sellers' Payoffs and Consumer Surplus

Up to this point, our analysis has focused on the platform's perspective. Since the platform's payoff depends on the combined profits of both sellers, it can tolerate a reduction in one seller's profit as long as the overall change in total profit is positive. We now extend the analysis to examine how the equilibrium decisions affect the individual payoffs of sellers A and seller B, as formalized in the following proposition.

PROPOSITION 6. *As shown in Figure 4, the comparison between each seller's payoff under $J = AB$ and $J = \emptyset$ scenarios results in the following characterizations:*

1. *Seller A's preference is similar to that of the platform, albeit with slightly different regions, i.e., seller A prefers $J = AB$ over $J = \emptyset$ in region R_1^1, otherwise seller A prefers $J = \emptyset$ over $J = AB$.*
2. *Seller B prefers $J = AB$ over $J = \emptyset$ in regions R_1 and R_2, otherwise seller B prefers $J = \emptyset$ over $J = AB$.*
3. *Consumer surplus (CS) is maximized by regime $J = \emptyset$ in all regions.*

Figure 4 **Payoff comparison between regime $J = AB$ (both sellers subscribe) and regime $J = \emptyset$ (no seller subscribes), shown for the platform, each seller, and consumer surplus, as a function of the demand spread α_H (horizontal axis) and the substitution intensity γ (vertical axis) (In each panel, the dark-shaded area indicates where the corresponding party is better off under $J = AB$, and the light-shaded area where it is better off under $J = \emptyset$).**



(a) Platform (b) Seller A (c) Seller B (d) Consumer Surplus

Proposition 6 characterizes the preferences of sellers and consumer surplus between regime AB and regime $\emptyset$. Do they prefer exploration mediated by the platform (i.e., regime AB) or independent self-exploration (i.e., regime $\emptyset$)? Firstly, as shown in Figure 4, seller B prefers CIS regime in both regions R_1 and R_2. But seller A prefers CIS regime only in a subregion inside R_1, where the SNR is very low. Note that the smaller the SNR, the more difficult it becomes for seller A to independently explore the demand state, therefore, Proposition 6 can be interpreted as follows: If seller A has to significantly reduce the price in order to self-learn, seller A can benefit from CIS, while seller B consistently benefits from CIS in both regions R_1 and R_2. The rationale behind this interesting result is, again, tied to the impact of the method of demand exploration on competitive prices. When sellers need to explore demand themselves, seller A ends up decreasing the price, while seller B increases it. This leads to different consequences for sellers' first- and second-period profits. For both seller

A and seller B, self-exploration during the first period might not always reveal the true demand state in the second period. This is because they rely solely on their own observations and may not determine the true demand state if the actual sales differ only slightly from their expectations (for detailed information, refer to Proposition 4). However, if the sellers subscribe to CIS, they receive the true demand state from the platform, regardless of the sales realization. Consequently, both sellers are worse off with self-exploration from the perspective of second-period profits.

However, self-exploration, while it may harm both sellers in the second period, can actually benefit seller A in the first period. To understand this difference, let us first consider the CIS scenario. When the platform provides information about the true demand state, sellers have complete knowledge in the second period, leading to intense competition between seller A and seller B in the first period. Seller B tends to undercut seller A. This benefits seller B, as seller B always benefits from receiving CIS, both in the short term (i.e., first period) and in the long term (i.e., second period). However, seller A prefers the no-CIS scenario over CIS in the first period, primarily because it leads to reduced price competition in the form of less aggressive undercutting by seller B. There is, however, a caveat. Under the no-CIS regime, seller A needs to lower her price to engage in self-exploration, which results in a reduced profit margin in the first period. If the SNR is very low (i.e., subregion R_1^1 in R_1), seller A needs to significantly decrease the profit margin; thus, overall, the seller benefits from CIS subscription in the first period. In summary, in subregion R_1^1, seller A's short-term loss from no-CIS is compensated by her long-term gain from receiving CIS.

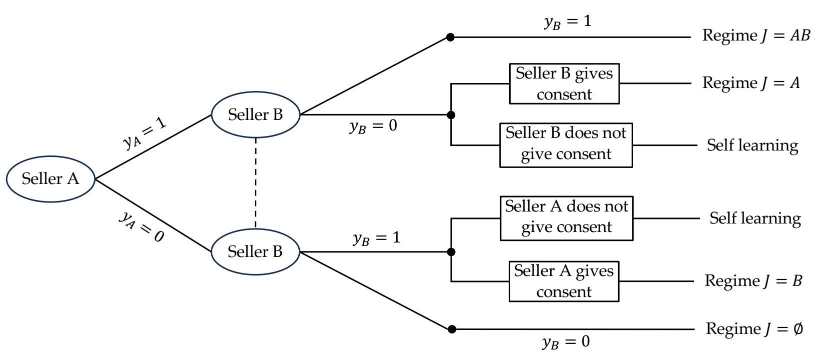
The above analysis also has direct implications for the ongoing regulatory debate between the European Union (EU) and the United States on data access in digital platforms. Recent regulations in the EU increasingly emphasize improving sellers' access to data generated through their own platform activities. Notably, the EU's Digital Markets Act (DMA) (Commission 2025) adopts a non-discriminatory perspective by requiring designated platform "gatekeepers" to provide business users with continuous, high-quality access to data related to their sales and performance, without discriminatory treatment across sellers of different sizes (Cabral et al. 2021). Although our model treats the information-sharing regime as a choice variable of the platform provider, the fact that smaller sellers benefit disproportionately from information sharing relative to larger sellers yields insights that are consistent with this regulatory approach. Specifically, region R_1 (where $J^* = AB$) corresponds precisely to the symmetric, non-discriminatory regime that the DMA mandates: when the SNR is low, the platform's privately optimal choice happens to coincide with the symmetric-access rule, so the EU mandate is not binding. As we discuss in detail at the end of §5, the tension between the two regulatory philosophies arises only once the asymmetric regime $J = A$ becomes available, which is precisely the case the US framework permits but the DMA forecloses.

Figure 4(d) also shows that consumer surplus is maximized under regime $\emptyset$. This outcome stems from how information-sharing regimes affect equilibrium prices, as described in Proposition 5. Between regimes AB and $\emptyset$, the latter yields equilibrium prices that are more favorable for consumers. Intuitively, when both sellers must rely solely on their own sales to update beliefs, they bring their prices closer together to improve demand learning. This leads seller B to increase her price while seller A lowers hers. Although the increase in seller B's price reduces consumer surplus, the decrease in seller A's price has a larger positive effect, since seller A serves a larger share of the market. The net result is an overall improvement in consumer welfare under regime $\emptyset$.

5. The Impact of Inducing Only One Seller to Subscribe to CIS

In §4, we analyzed the scenario in which the platform induces both sellers to subscribe to CIS. In this section, we extend the analysis to cases where the platform may offer the CIS subscription unilaterally to a single seller - either seller A or seller B - provided that the chosen seller agrees to pay the subscription fee. Since the platform derives demand information from the realized sales of both sellers, sharing this information with only one seller requires obtaining consent from the other. Consequently, as shown in Figure 5, unilateral information sharing is implemented only if the non-subscribing seller grants consent.

Figure 5 Sellers' subscription and consent decisions



Now, we analyze the equilibrium under unilateral subscription regimes. We have two cases: Regime $J = A$ and Regime $J = B$. If the platform wants to induce CIS subscription for seller A only, i.e., forming the subscription regime $J = A$, then the subscription fee offered by the platform should satisfy the following incentive compatibility constraints of the sellers:

$$\pi_B^A\left(P_B^A, P_A^A\right) \equiv (1-\beta)\pi_{C|B}^A\left(P_B^A, P_A^A\right) \geq (1-\beta)\pi_{C|B}^{AB}\left(P_B^{AB}, P_A^{AB}\right) - \phi^A \equiv \pi_B^{AB}\left(P_B^{AB}, P_A^{AB}\right) \tag{11}$$

$$\pi_A^A\left(P_A^A, P_B^A\right) \equiv (1-\beta)\pi_{C|A}^A\left(P_A^A, P_B^A\right) - \phi^A \geq (1-\beta)\pi_{C|A}^{\emptyset}(P_A^{\emptyset}, P_B^{\emptyset}) \equiv \pi_A^{\emptyset}\left(P_A^{\emptyset}, P_B^{\emptyset}\right) \tag{12}$$

where constraint (11) ensures that when seller A subscribes to CIS, seller B has no incentive to subscribe. Similarly, constraint (12) ensures that if seller B decides not to subscribe, seller A has no incentive to deviate and not subscribe. These two constraints together define subscription regime $J = A$ where $y_A = 1$ and $y_B = 0$. Similarly, the following constraints should be met if the platform intends to induce only seller B to subscribe to CIS, hence forming regime $J = B$:

$$\pi_B^B\left(P_B^B, P_A^B\right) \equiv (1-\beta)\pi_{C|B}^B\left(P_B^B, P_A^B\right) - \phi^B \geq (1-\beta)\pi_{C|B}^{\emptyset}\left(P_B^{\emptyset}, P_A^{\emptyset}\right) \equiv \pi_B^{\emptyset}\left(P_B^{\emptyset}, P_A^{\emptyset}\right) \tag{13}$$

$$\pi_A^B\left(P_A^B, P_B^B\right) \equiv (1-\beta)\pi_{C|A}^B\left(P_A^B, P_B^B\right) \geq (1-\beta)\pi_{C|A}^{AB}(P_A^{AB}, P_B^{AB}) - \phi^B \equiv \pi_A^{AB}\left(P_A^{AB}, P_B^{AB}\right) \tag{14}$$

Before characterizing the equilibrium, we first show that inducing subscription only to seller B (i.e., the seller with the smaller market size) is not feasible. This issue stems from conflicts between the subscription incentives. On one hand, constraint (13) represents the maximum allowable value of the subscription fee to ensure that seller B retains incentives to subscribe. On the other hand, constraint (14) signifies the minimum value of a subscription fee needed to prevent seller A from subscribing. These two conditions together establish bounds on the subscription fee when inducing $J = B$, namely: $\pi^{AB}_{C|A} - \pi^{B}_{C|A} < \phi^B < \pi^{B}_{C|B} - \pi^{\emptyset}_{C|B}$. However, the lower bound is greater than the upper bound. Therefore, as characterized below, implementing the subscription regime $J = B$ is not possible.

PROPOSITION 7. *Regime $J = B$ is not implementable, i.e., the platform cannot induce only seller B to subscribe.*

Next, we consider the case where the platform aims to induce only seller A to subscribe to CIS. At the end of the first period, seller A receives an updated belief from the platform, while seller B relies solely on her own sales. Our analysis reveals that seller B's belief updating follows the same process as the one characterized under the no-CIS scenario (regime $J = \emptyset$). In other words, as outlined in Proposition 4, seller B updates her beliefs as follows:

$$\theta_B = \begin{cases} H & \text{if } \epsilon_1 \leq \frac{2\Delta}{p_{A,1} - p_{B,1}} \text{ and } \theta = H \\ L & \text{if } \epsilon_1 \geq 1 - \frac{2\Delta}{p_{A,1} - p_{B,1}} \text{ and } \theta = L \\ \bar{\theta} & \text{otherwise,} \end{cases} \tag{15}$$

where $\Delta = \frac{\alpha_H - \alpha_L}{4\gamma}$. Since ϵ_1 is bounded between 0 and 1, the updating rule implies that seller B can learn the true demand state when the first-period price difference between sellers A and B is sufficiently small. In §4.2, we showed that both sellers have an incentive to bring their prices closer together to reduce demand noise and facilitate demand learning. However, under regime $J = A$, seller A is induced to subscribe to CIS and thus receives the true demand state directly from the platform, while seller B must still rely on her first-period sales to update her beliefs on the true demand state. In this case, seller B continues to have an incentive to align her price with seller A's, but seller A no longer benefits from doing so. In fact, as we characterize below, seller A now has an incentive to increase the first-period price difference, which in turn makes demand learning more difficult for seller B. We refer to this as signal-jamming and analyze its impact on equilibrium prices.

PROPOSITION 8. *The equilibrium under regime A is characterized as follows:*

– *The platform can induce only seller A to subscribe by offering* $\phi^A = (1-\beta)\left[\pi^{A}_{C|A}\left(P^A_A, P^A_B\right) - \pi^{\emptyset}_{C|A}\left(P^{\emptyset}_A, P^{\emptyset}_B\right)\right]$.

– *Sellers A and B set the following prices in the first period:*

$$p^A_{A,1} = \overline{p}^A_1 + z^A_p \Delta, \quad \textit{and} \quad p^A_{B,1} = \overline{p}^A_1 - z^A_p \Delta$$

where $(\overline{p}^A_1, z^A_p)$ *is described in EC.2.8 and* $\Delta = \frac{\alpha_H - \alpha_L}{4\gamma}$.

– *After the first period sales, seller A receives the true demand state from the platform and seller B updates her own beliefs according to Eq.(15).*

– *In the second period, the following equilibrium prices are set if seller B learns the true demand state:*

$$p^A_{A,2} = \frac{\gamma + (2+\gamma)\alpha_A(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)}, \quad \textit{and} \quad p^A_{B,2} = \frac{\gamma + (2+\gamma)\alpha_B(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)}$$

and the following equilibrium prices are set if seller B does not learn:

$$p^A_{A,2} = \frac{\gamma + (2+\gamma)\alpha_A(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)} + \frac{\gamma[\alpha_H + \alpha_L - 2\alpha_A(\boldsymbol{\theta})]}{4(1+\gamma)(2+3\gamma)}, \quad \textit{and} \quad p^A_{B,2} = \frac{4(1+\gamma) - (2+\gamma)(\alpha_H + \alpha_L)}{2(2+\gamma)(2+3\gamma)}.$$

The first part of Proposition 8 establishes that inducing subscription from seller A, i.e., the seller with the larger market size, is feasible provided the subscription fee ϕ^A is chosen appropriately. Specifically, the fee must be high enough to deter seller B from subscribing to CIS, yet low enough to ensure that seller A finds subscription profitable. By setting ϕ^A accordingly, the platform can satisfy the incentive compatibility constraints of sellers A and B. The second part of Proposition 8 characterizes the first-period equilibrium prices under regime $J = A$. Since seller A receives updated demand beliefs from the platform, she has no incentive to engage in exploration and instead focuses solely on exploitation in the first period. This behavior has two key implications. First, it increases the price gap between sellers A and B. As characterized in the below Proposition 9, we compare the first-period equilibrium prices across all three regimes and demonstrate that the price difference between A and B is larger under regime A than under regime $\emptyset$. This wider gap between the first-period prices generates a signal-jamming effect: the greater the initial price difference, the less likely is seller B to update her beliefs about the true demand state. Second, as seller A raises her first-period price to hinder seller B's learning, seller B responds by increasing her own price. Consequently, the average first-period price level under regime A exceeds the price levels characterized under both the full-information regime ($J = AB$) and the no-information regime ($J = \emptyset$).

PROPOSITION 9. *The following statements hold for first-period equilibrium prices under the three regimes:*

– *The ranking of first-period price differences between the sellers is* $z_p^{AB} > z_p^A > z_p^{\emptyset}$.

– *Let* $\overline{p}^J = \frac{p^J_{A,1} + p^J_{B,1}}{2}$ *denote the average first-period price in regime J. Then,* $\overline{p}^A > \overline{p}^{AB} = \overline{p}^{\emptyset}$.

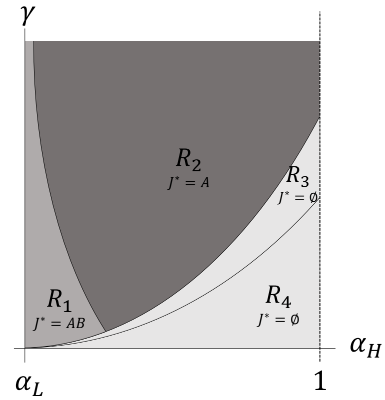
Note that the above proposition not only reveals the signal-jamming behavior but also has implications for the platform's preference in implementing regime A, regime AB or regime $\emptyset$. By comparing the platform's payoffs under these three regimes, we can now characterize the equilibrium as follows:

PROPOSITION 10. *The equilibrium is divided into four regions, as summarized below:*

- *Region* R_1*: Both seller A and seller B are induced to subscribe to the information-sharing service.*
- *Region* R_2*: Only seller A is induced to subscribe to the information-sharing service.*
- *Regions* R_3 *and* R_4*: Neither seller A nor seller B is induced to subscribe to the information-sharing service.*

The corresponding equilibrium subscription fees, first- and second-period prices, and belief-updating rules are characterized in EC.2.10 for each region and depicted in Figure 6.

Figure 6 **Platform's equilibrium subscription regime (J^*) in the full model as a function of the demand spread α_H (horizontal axis) and the substitution intensity γ (vertical axis)**



The comparison between the equilibria characterized in Proposition 10 and Proposition 5 reveals both similarities and differences. First, similar to Proposition 5, the equilibria in Proposition 10 also consist of four distinct regions, although regions R_1 and R_2 differ slightly from those in Proposition 5. Second, in all regions except R_2, the same regime is implemented in both propositions. The key distinction arises in region R_2, where regime $\emptyset$ in Proposition 5 is replaced by regime A in Proposition 10.

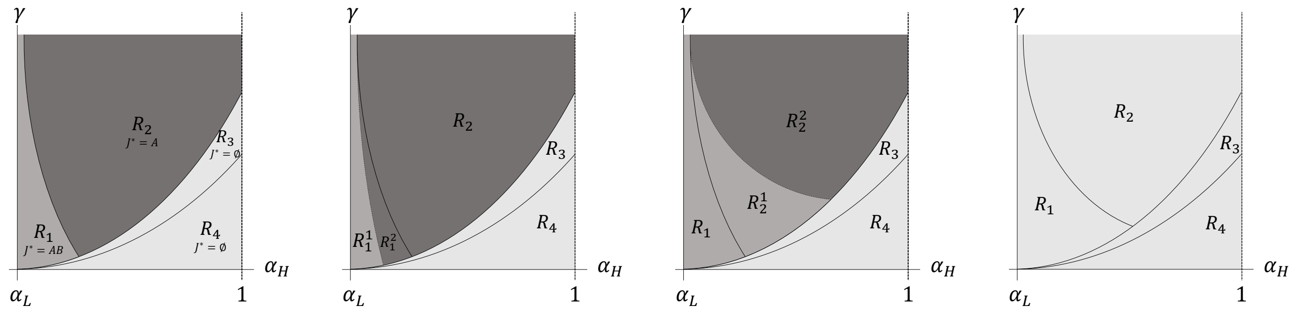
This difference stems from the signal-jamming behavior of seller A toward seller B. In region R_2 under Proposition 5, the platform benefits from not sharing demand information, as the sellers are incentivized to align their prices more closely, thereby intensifying first-period price competition. In contrast, under regime A, the platform can selectively set the subscription fee to incentivize only seller A to subscribe to the information-sharing service. With access to information, seller A increases her price to jam seller B's demand learning, prompting seller B to increase her own price in response. As a result, both sellers charge higher prices. Even though this results in lower demand for both sellers, the increase in equilibrium prices ultimately increases the platform's total revenue.

Although closed-form expressions for equilibrium prices in both periods can be derived, the complicated nature of these closed-form expressions makes direct comparisons of sellers' profits and consumer surplus across regions intractable. To address this, we provide a graphical comparison of profits and consumer surplus in Figure 7 (for $\alpha_L = 0.65$).

Since the platform's preferences have already been discussed, we now restrict attention to the effects of different regions on the profits of sellers A and B, as well as consumer surplus:

- **Seller A's preference:** As shown in Figure 7(b), seller A's preferences are generally aligned with those of the platform. This is intuitive: the platform shares demand information exclusively with seller A, which benefits her in two ways. During the first period, seller A can sustain higher prices, and at the start of the second period, she always has access to the true demand state, whereas seller B may not. This ensures that seller A maintains a competitive advantage over seller B in the second period. Indeed, our numerical analysis

Figure 7 **Payoff comparison in the full equilibrium for the platform, each seller, and consumer surplus, as a function of the demand spread α_H (horizontal axis) and the substitution intensity γ (vertical axis) (In each panel, dark shading marks where the corresponding party is best off under $J = A$, medium shading under $J = AB$, and light shading under $J = \emptyset$).**



(a) Platform (b) Seller A (c) Seller B (d) Consumer Surplus

confirms that seller A prefers regime A over regime AB in region R_1, since under regime AB the true demand information is shared with both sellers, while under regime A it is available only to seller A. Thus, seller A preserves a competitive edge in regime A, making it more favorable than regime AB in region R_1.

- **Seller B's preference:** Figure 7(c) reveals a more surprising result: within a subregion of R_2, seller B also prefers regime A over regime $\emptyset$. Although regime A puts seller B at a competitive disadvantage in the second period - since seller A holds exclusive access to the demand state - seller B nonetheless gains in the first period. The reason is that seller A's signal-jamming behavior increases not only her own equilibrium price but also that of seller B. The resulting increase in seller B's first-period revenue can outweigh the loss from reduced learning in the second period. In summary, seller B prefers regime AB over regime A only when both the signal-to-noise ratio (SNR) and the degree of competition are very low - corresponding to region R_1 and the left portion of region R_2 (i.e., lower α_H and lower γ). In contrast, she prefers regime A when competition is more intense and the SNR is relatively low - corresponding to the right portion of region R_2. This right-hand side of region R_2 is precisely where the signal-jamming effect is strongest, leaving seller B worse off in relative terms in the second period, yet still benefiting her due to higher equilibrium prices in the first period.

- **Consumer surplus:** Figure 7(d) shows that the consumer surplus is maximized under regime $\emptyset$ even after we add regime A into the consideration set. The rationale behind this is due to the effect of information-sharing regimes on equilibrium prices, as characterized in Proposition 9. Among all regimes, regime $\emptyset$ yields the most favorable prices from the consumers' perspective. Intuitively, when both sellers must rely solely on their own sales to update beliefs, they bring their prices closer together to improve demand learning. This basically implies that seller B increases her price, while seller A decreases her price. Although the increase in seller B's price reduces consumer surplus, the decrease in seller A's price generates a larger positive effect, since seller A serves a larger market. As a result, the net impact is an overall improvement in consumer surplus under regime $\emptyset$.

We conclude this section by noting that the information-sharing regimes characterized here are also consistent with practices observed in digital markets. To illustrate how such regimes may appear in practice, consider a platform (such as Amazon.com or Google.com) that simultaneously operates as a gatekeeper and competes as a seller on its own platform. While the EU's Digital Markets Act (DMA) (Commission 2025, Cabral et al. 2021) takes a strict stance toward such dual-role platforms, these arrangements can readily arise under more permissive regulatory frameworks, such as those adopted in the United States. Moreover, in the United States, platforms commonly allow sellers to opt into analytics services that process seller-level data and generate insights that may be informative to competing sellers. For example, Amazon offers opt-in analytics tools (e.g., Brand Analytics (Amazon 2025)) that use seller-level sales data to produce pricing, demand, or inventory recommendations, and platforms may also provide insights derived from aggregated activity across many sellers, such as market-basket analyses identifying products that are frequently purchased together.

The contrast between the US and EU regulatory approaches maps directly onto our equilibrium analysis, and our model helps clarify precisely what is at stake in choosing between them. The two jurisdictions embody fundamentally different philosophies. The EU relies on an *ex-ante* rule: under the DMA, designated gatekeepers must grant business users continuous, real-time, and non-discriminatory access to the data they generate, and are prohibited from using non-public business-user data to compete against those users (Commission 2025, Cabral et al. 2021). In the context of our model, this rule effectively forces a symmetric information regime—either $J = AB$ or $J = \emptyset$—and forecloses the asymmetric regime $J = A$, in which the platform shares demand information with the larger seller only. The US, by contrast, relies on *ex-post* antitrust enforcement: asymmetric data practices are permitted to emerge and are challenged only after specific conduct is shown to harm competition.

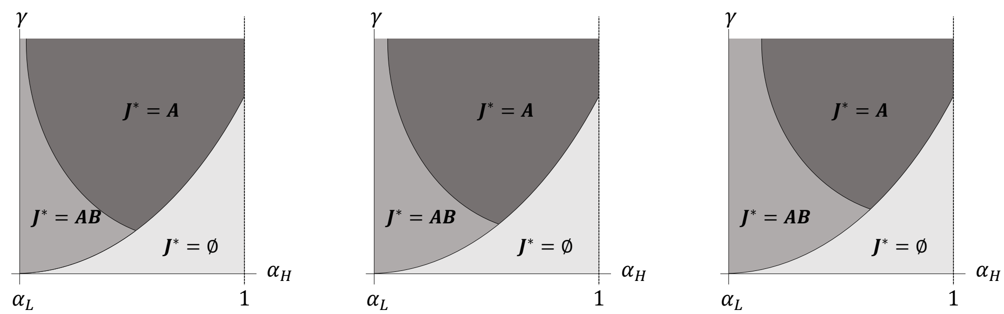
6. Robustness Check of Analytical Results

While §5 analytically established the impact of competitive demand exploration-exploitation on pricing strategies and payoffs, the results were obtained under the following assumptions required for tractability: (*i*) $c_A = c_B = 0$, i.e., the marginal cost is homogeneous and zero for both sellers, (*ii*) $\alpha_A(\boldsymbol{\theta}) + \alpha_B(\boldsymbol{\theta}) = 1$ for $\boldsymbol{\theta} \in \{L, H\}$, i.e., the total market size under both demand states is normalized to be equal to 1, (*iii*) the noise in demand for both periods is distributed uniformly between 0 and 1, i.e., $\boldsymbol{\epsilon}_t \sim U[0,1]$, and (*iv*) $\boldsymbol{\epsilon}_1$ and $\boldsymbol{\epsilon}_2$ are independent. It turns out that relaxing any of these assumptions makes it challenging to characterize equilibrium decisions and, more importantly, to analytically establish the effects of different CIS arrangements. Delegating the details in EC.3, in this section, we numerically investigate whether or not the managerial insights presented in §4 and §5 remain valid if we relax the above assumptions. While our numerical study can validate the robustness of all our results, below we focus on the main insights about how heterogeneous marginal costs and state-dependent total demand affect the platform's incentive to provide CIS.

6.1. Heterogeneous Marginal Costs

In this section, we use the same model framework as in §3 and consider the case where the marginal costs for sellers are heterogeneous. It is reasonable to assume that $c_A > c_B$, since seller A's market share is larger than seller B's market share for both demand states. For expositional brevity, we set $c_A = c > 0$ and $c_B = 0$. By varying the value of c, we focus on the impact of c on the equilibrium regions. Specifically, we consider three cases: the baseline model ($c = 0$), a medium cost ($c = 0.05$), and a high cost ($c = 0.1$). We pick these values because when c exceeds 0.1, seller A finds it unprofitable to remain in the market, which leads to a monopolistic outcome for seller B and thus trivial results for our model. The results are presented in Figure 8:

Figure 8 **Comparison of the equilibrium information regimes with various seller A's marginal cost c (seller B's cost is normalized to 0). As c increases, the $J^* = AB$ region expands.**



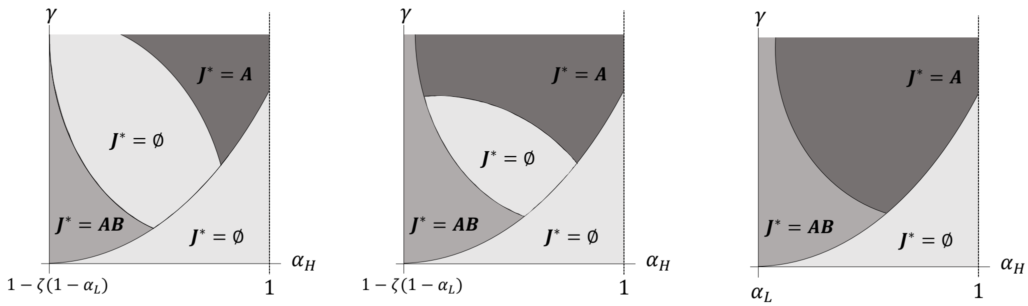
(a) Baseline model ($c = 0$) (b) Medium cost ($c = 0.05$) (c) High cost ($c = 0.1$)

Recall that in §4 and §5 we show that under both regime $J = \emptyset$ and regime $J = A$, sellers narrow the first-period price gap in order to increase the probability of learning. This implies that seller A must decrease her first-period price, while seller B must increase her price. However, when seller A faces a positive marginal cost, her ability to reduce the price in pursuit of learning is limited. Consequently, both regimes $J = \emptyset$ and $J = A$ become more costly for the platform. As illustrated in Figure 8, our numerical results demonstrate that as marginal cost c for seller A increases, regime AB becomes relatively more advantageous for the platform compared to both regime $J = \emptyset$ and regime $J = A$.

6.2. State-dependent Total Demand

In the baseline model, we normalized the total demand under both demand states to be equal to 1. In this section, we relax this assumption. That is, for the high-demand state, we still normalize the total market size to 1, but for low demand state, we normalize it to be equal to ζ, where $\zeta \in (0, 1]$. Similar to §6.1, we only focus on the impact of ζ on the comparison between equilibrium information-sharing regimes. We consider three values for ζ: low ζ (i.e., $\zeta = 0.6$), medium ζ (i.e., $\zeta = 0.8$) and high ζ (i.e., $\zeta = 1$). The results are presented in Figure 9.

Figure 9 **Equilibrium information regimes when low-state market size is scaled by $\zeta \in (0,1]$. As ζ decreases, demand variation across states demand grows, and the $J^* = \emptyset$ region emerges due to price collusion enabling self-exploration.**



(a) Low ζ ($\zeta = 0.6$) (b) Medium ζ ($\zeta = 0.8$) (c) High ζ ($\zeta = 1$)

Unlike in §6.1, the state-dependent aggregate demand extension changes the possible learning cases. When each seller self-explores the demand state by observing only their own sales, it is possible that seller A learns the true demand state while seller B does not. Moreover, by setting a price sufficiently far from the competitor's, seller A can ensure that she always learns the true demand state, regardless of demand realization, whereas seller B cannot.

This new possibility has two implications. First, unlike in §4.2, seller A no longer needs the platform's demand information to engage in signal-jamming against seller B in the first period. Second, this dynamic leads to price collusion, which raises the average equilibrium price in the first period. Consequently, regime $J = \emptyset$ emerges as the equilibrium in Figure 9, since it is preferred from the platform's perspective. However, as α_H increases, sustaining signal-jamming under regime $J = \emptyset$ becomes harder: the growing price gap between sellers A and B makes the signal-jamming region increasingly small. Hence, for higher values of α_H, the platform prefers to intervene and implement regime $J = A$.

6.3. Non-uniform and correlated noise distributions

In the baseline model, we assumed that the demand shocks ϵ_t follow independent uniform distributions on $[0,1]$. To assess the robustness of our findings, we relax this assumption along two dimensions and summarize our key findings below.

Non-uniform noise. We first allow $\epsilon_t \sim \text{Beta}(\alpha, \beta)$, which nests the uniform case ($\alpha = \beta = 1$) and accommodates right-skewed, left-skewed, and more concentrated symmetric shapes. The equilibrium regions remain qualitatively consistent with the baseline. When the distribution becomes more concentrated around its mean (e.g., $\alpha = \beta = 5$), demand uncertainty diminishes and the region in which both sellers share information (regime AB) shrinks. Skewness shifts the mean of ϵ_t: a right-skewed distribution reduces effective demand noise and further contracts regime AB, whereas a left-skewed distribution amplifies it and expands regime AB. A new feature emerges in the left-skewed case: because the distribution is also more concentrated than the uniform, sellers can sometimes infer the demand state from their realized sales even absent information sharing, making regime $\emptyset$ an equilibrium when α_H is moderate and γ is relatively low.

Correlated noise. We next allow ϵ_A and ϵ_B to be correlated via a truncated bivariate normal distribution with correlation $\rho \in [-1, 1]$. The baseline (uniform, independent) case corresponds to a benchmark of strong positive correlation ($\rho = 0.9$); we additionally examine $\rho \in \{-0.5, 0, 0.5\}$. As correlation weakens, the signals observed by the two sellers diverge, producing asymmetric learning outcomes in which only one seller learns the true demand state. The resulting signal-jamming behavior cuts in opposite directions: seller A's signal-jamming raises prices and benefits the platform, while seller B's signal-jamming depresses prices and harms it. In the upper-right portion of the parameter space (high α_H and γ), the platform benefits on average from seller A's signal-jamming, so regime $\emptyset$ emerges as the equilibrium. As α_H decreases, seller B's signal-jamming becomes too costly and the platform intervenes by sharing information with seller A.

To summarize, these extensions confirm that the main qualitative insights of our baseline analysis are robust to richer distributional assumptions. We refer the reader to EC.4 for the detailed analysis and discussion.

7. Conclusion

Inarguably, the exponential growth in online marketplaces will attract not only more consumers but also more *sellers*, creating a win-win scenario for all stakeholders. In this paper, we focus on the Competitive Intelligence Services (CIS) that a platform can offer to its sellers. These services leverage the platform's ability to learn about the unknown price-demand relationship faster than the sellers themselves. We examine the demand exploration-exploitation trade-off faced by two sellers, competing over two periods, with varying market shares on an online platform. Each seller updates their estimate for the price-demand relationship based on individual sales observations. However, by observing the sales of both sellers, the platform excels at exploring demand, turning this demand information into valuable competitive business intelligence. As a result, the platform can mediate the demand exploration of the sellers in the first period by influencing their decision to subscribe or not to subscribe to CIS. We analytically establish the specific conditions that determine the optimal approach to demand exploration and the optimal CIS arrangement in online marketplaces. First, we characterize whether the platform should induce sellers to subscribe to CIS or not. Second, if the platform undertakes demand exploration, should it induce both sellers to subscribe to CIS, or just one of them? Finally, if the platform induces only one of the sellers to subscribe, which one should it be?

To address these research questions, we characterize closed-form expressions for the associated equilibrium prices and payoffs for the platform and the two sellers under all demand-exploration scenarios. From the platform's perspective, our analysis suggests that by offering CIS to one or both sellers, the platform alters the nature of pricing decisions in the second period. More importantly, it also changes the nature of self-exploration and, consequently, the equilibrium pricing decisions in the first period. This transformation may have either a beneficial or a detrimental impact on the profitability of both the sellers and the platform.

We also establish that the signal-to-noise ratio (SNR) of the random demand faced by the sellers is a key factor in determining the optimal CIS subscription strategy in a competitive environment. Specifically, it is optimal for the platform to offer CIS to both sellers when the SNR is low. This is because a low SNR makes self-exploration difficult for each seller in the first period. Consequently, the poor knowledge of the price-demand relationship in the second period leads to lower payoffs for both the sellers and the platform. This result appears to be rather robust and remains valid under general assumptions about the degree of demand substitution between sellers and heterogeneity in their marginal costs. In fact, as the degree of demand substitution and heterogeneity in their marginal costs increase, the provision of CIS by the platform becomes even more critical. Finally, when the SNR is high, the identity of the seller who subscribes to the CIS determines the level of price competition during the first period. Our results indicate that inducing CIS subscription solely for the smaller seller is not incentive-compatible. However, the larger seller benefits when she receives the demand information. Surprisingly, our analysis shows that the smaller seller, as well as the platform, also gain an advantage when the demand information is shared with the opponent. This is due to price collusion triggered by the signal-jamming behavior of the larger seller. Our robustness check confirms that the larger seller engages in signal-jamming even if demand exploration is pursued independently by each seller.

Last but not least, our results highlight the inherent trade-offs faced by policymakers when regulating data sharing in digital markets. Highly restrictive regimes may limit platforms' ability to leverage data-driven efficiencies, potentially slowing market expansion, whereas overly permissive approaches risk facilitating anti-competitive behavior and disadvantaging smaller sellers and consumers. This trade-off is precisely the one that distinguishes the EU's ex-ante DMA mandate from the US's ex-post antitrust approach. Our analysis shows that the two philosophies coincide when the signal-to-noise ratio of demand is either low or high, and diverge only in the intermediate range, where the asymmetric regime $J = A$ raises producer surplus at the expense of consumers. Our analysis suggests that carefully designed information-sharing rules are essential for the healthy development of digital markets. Along these lines, several important design issues remain open for future research. First, while many regulatory frameworks allow information to be shared in aggregated form, an open question concerns how such aggregation should be designed. Our model focuses on market size as the relevant information dimension, but in practice platforms may share more granular information, such as clickstream data capturing product views, add-to-cart actions, and purchases. Second, another important question is whether information-sharing regimes should be implemented on an opt-in or opt-out basis. Studying these design choices would help inform emerging regulatory frameworks that aim to account for the perspectives of multiple stakeholders. We leave these issues for future research.

References

Aghion, Philippe, Maria Paz Espinosa, Bruno Jullien. 1993. Dynamic duopoly with learning through market experimentation. *Economic Theory* **3**(3) 517–539.

Amazon. 2025. Amazon brand analytics: Get data to grow your business Accessed: 2025-01-03.

Anastasia, D. 2020. Types of online marketplaces: Complete categorization with examples URL `https://web.archive.org/web/20250420104202/https://rubygarage.org/blog/types-of-online-marketplaces`.

Araman, Victor F, René Caldentey. 2009. Dynamic pricing for nonperishable products with demand learning. *Operations research* **57**(5) 1169–1188.

Belleflamme, Paul, Francis Bloch. 2001. Price and quantity experimentation: a synthesis. *international journal of industrial organization* **19**(10) 1563–1582.

Bernhardt, Dan, Bart Taub. 2015. Learning about common and private values in oligopoly. *The RAND Journal of Economics* **46**(1) 66–85.

Bertsimas, Dimitris, Georgia Perakis. 2006. Dynamic pricing: A learning approach. *Mathematical and computational models for congestion charging*. Springer, 45–79.

Bimpikis, Kostas, Davide Crapis, Alireza Tahbaz-Salehi. 2019. Information sale and competition. *Management Science* **65**(6) 2646–2664.

Birge, John R, Hongfan Chen, N Bora Keskin. 2019. Markdown policies for demand learning with forward-looking customers. *Available at SSRN 3299819* .

Bolton, Patrick, Christopher Harris. 1999. Strategic experimentation. *Econometrica* **67**(2) 349–374.

Bonatti, Alessandro, Gonzalo Cisternas, Juuso Toikka. 2017. Dynamic oligopoly with incomplete information. *The Review of Economic Studies* **84**(2) 503–546.

Cabral, Luis, Justus Haucap, Geoffrey Parker, Georgios Petropoulos, Tommaso M Valletti, Marshall W Van Alstyne. 2021. The eu digital markets act: a report from a panel of economic experts. *Cabral, L., Haucap, J., Parker, G., Petropoulos, G., Valletti, T., and Van Alstyne, M., The EU Digital Markets Act, Publications Office of the European Union, Luxembourg* .

Cachon, Gérard P, Martin A Lariviere. 2001. Contracting to assure supply: How to share demand forecasts in a supply chain. *Management science* **47**(5) 629–646.

Caldieraro, Fabio, Jonathan Z Zhang, Marcus Cunha Jr, Jeffrey D Shulman. 2018. Strategic information transmission in peer-to-peer lending markets. *Journal of Marketing* **82**(2) 42–63.

Cao, Xinyu, Juanjuan Zhang. 2021. Preference learning and demand forecast. *Marketing Science* **40**(1) 62–79.

Chen, Fangruo. 2003. Information sharing and supply chain coordination. *Handbooks in operations research and management science* **11** 341–421.

Chen, Ming, Zhi-Long Chen. 2015. Recent developments in dynamic pricing research: multiple products, competition, and limited demand information. *Production and Operations Management* **24**(5) 704–731.

Chen, Ningyuan, Guillermo Gallego. 2018. A primal-dual learning algorithm for personalized dynamic pricing with an inventory constraint. *Available at SSRN 3301153* .

Chen, Yuxin, Ganesh Iyer. 2002. Research note - consumer addressability and customized pricing. *Marketing Science* **21**(2) 197–208.

Ching, Andrew T, Tülin Erdem, Michael P Keane. 2013. Learning models: An assessment of progress, challenges, and new developments. *Marketing Science* **32**(6) 913–938.

Commission, European. 2025. The digital markets act: ensuring fair and open digital markets URL `https://www.eu-digital-markets-act.com/Digital_Markets_Act_Articles.html`.

Dukes, Anthony, Esther Gal-Or, Tansev Geylani. 2017. *Bilateral information sharing and pricing incentives in a retail channel*. Springer.

Easley, David, Nicholas M Kiefer. 1988. Controlling a stochastic process with unknown parameters. *Econometrica: Journal of the Econometric Society* 1045–1064.

Fudenberg, Drew, Jean Tirole. 1991. Perfect bayesian equilibrium and sequential equilibrium. *journal of Economic Theory* **53**(2) 236–260.

Gal-Or, Esther, Mordechai Gal-Or, Anthony Dukes. 2007. Optimal information revelation in procurement schemes. *The RAND Journal of Economics* **38**(2) 400–418.

Gal-Or, Esther, Tansev Geylani, Anthony J Dukes. 2008. Information sharing in a channel with partially informed retailers. *Marketing Science* **27**(4) 642–658.

Gallego, Guillermo, Masoud Talebian. 2012. Demand learning and dynamic pricing for multi-version products. *Journal of Revenue and Pricing Management* **11**(3) 303–318.

Gümüş, Mehmet. 2014. With or without forecast sharing: Competition and credibility under information asymmetry. *Production and Operations Management* **23**(10) 1732–1747.

Guo, Liang, Tian Li, Hongtao Zhang. 2014. Strategic information sharing in competing channels. *Production and Operations Management* **23**(10) 1719–1731.

Guo, Liang, Ying Zhao. 2009. Voluntary quality disclosure and market interaction. *Marketing Science* **28**(3) 488–501.

Ha, Albert Y, Shilu Tong. 2008. Contracting and information sharing under supply chain competition. *Management science* **54**(4) 701–715.

Ha, Albert Y, Shilu Tong, Hongtao Zhang. 2011. Sharing demand information in competing supply chains with production diseconomies. *Management science* **57**(3) 566–581.

Hansen, Karsten T, Kanishka Misra, Mallesh M Pai. 2021. Frontiers: Algorithmic collusion: Supra-competitive prices via independent algorithms. *Marketing Science* **40**(1) 1–12.

Harrington, Joseph E, Jr. 1995. Experimentation and learning in a differentiated-products duopoly. *Journal of Economic Theory* **66**(1) 275–288.

He, Chuan, Johan Marklund, Thomas Vossen. 2008. Research note - vertical information sharing in a volatile market. *Marketing Science* **27**(3) 513–530.

Huang, Yufeng, Paul B Ellickson, Mitchell J Lovett. 2022. Learning to set prices. *Journal of Marketing Research* **59**(2) 411–434.

Jain, Aditya. 2022. Sharing demand information with retailer under upstream competition. *Management Science* **68**(7) 4983–5001.

Kamenica, Emir, Matthew Gentzkow. 2011. Bayesian persuasion. *American Economic Review* **101**(6) 2590–2615.

Keller, Godfrey, Sven Rady. 2003. Price dispersion and learning in a dynamic differentiated-goods duopoly. *RAND Journal of Economics* 138–165.

Lai, Guoming, Huihui Liu, Wenqiang Xiao, Xinyi Zhao. 2022. "fulfilled by amazon": A strategic perspective of competition at the e-commerce platform. *Manufacturing & Service Operations Management* **24**(3) 1406–1420.

Lin, Kyle Y. 2006. Dynamic pricing with real-time demand learning. *European Journal of Operational Research* **174**(1) 522–538.

Liu, Yunchuan, Z John Zhang. 2006. Research note - the benefits of personalized pricing in a channel. *Marketing Science* **25**(1) 97–105.

Liu, Zekun, Dennis J Zhang, Fuqiang Zhang. 2021. Information sharing on retail platforms. *Manufacturing & Service Operations Management* **23**(3) 606–619.

Long, Fei, Kinshuk Jerath, Miklos Sarvary. 2022. Designing an online retail marketplace: Leveraging information from sponsored advertising. *Marketing Science* **41**(1) 115–138.

Malachard, V. 2020. How to improve your amazon customer analytics in 2021 URL `https://web.archive.org/web/20220704112403/https://insights.nozzle.ai/improve-amazon-customer-analytics`.

McLennan, Andrew. 1984. Price dispersion and incomplete learning in the long run. *Journal of Economic dynamics and control* **7**(3) 331–347.

Misra, Kanishka, Eric M Schwartz, Jacob Abernethy. 2019. Dynamic online pricing with incomplete information using multiarmed bandit experiments. *Marketing Science* **38**(2) 226–252.

Mroz, B. 2018. For retailers, the key to bigger sales is data URL `https://www.digitalcommerce360.com/2018/04/02/for-retailers-the-key-to-bigger-sales-is-data/`.

Narayan, Vishal, Vithala R Rao, Carolyne Saunders. 2011. How peer influence affects attribute preferences: A bayesian updating mechanism. *Marketing Science* **30**(2) 368–384.

Perakis, Georgia, Divya Singhvi. 2019. Dynamic pricing with unknown non-parametric demand and limited price changes. *Available at SSRN 3336949* .

Perakis, Georgia, Anshul Sood. 2006. Competitive multi-period pricing for perishable products: A robust optimization approach. *Mathematical Programming* **107**(1) 295–335.

Petruzzi, Nicholas C, Maqbool Dada. 1999. Pricing and the newsvendor problem: A review with extensions. *Operations research* **47**(2) 183–194.

Pryhodko, G. 2020. Typology of marketplaces: with 4 examples listed to help your understanding URL `https://wiki.rademade.com/4-typology-of-marketplaces`.

Rothschild, Michael. 1974. A two-armed bandit theory of market pricing. *Journal of Economic Theory* **9**(2) 185–202.

Shamir, Noam, Hyoduk Shin. 2016. Public forecast information sharing in a market with competing supply chains. *Management Science* **62**(10) 2994–3022.

Shi, Zijun, Kannan Srinivasan, Kaifu Zhang. 2023. Design of platform reputation systems: Optimal information disclosure. *Marketing Science* **42**(3) 500–520.

Sokol, D. Daniel, Marshall Van Alstyne. 2021. The rising risk of platform regulation. *MIT Sloan Management Review* **62**(2) 6A–10A.

Statista. 2025. Revenue share of the e-commerce market worldwide from 2017 to 2030, by sales channel URL `https://www.statista.com/statistics/534123/e-commerce-share-of-retail-sales-worldwide`.

Stern, Matt, John R Birge. 2020. Dynamic learning in strategic pricing games. *Available at SSRN 3579123* .

Täuscher, Karl, Sven M Laudien. 2018. Understanding platform business models: A mixed methods study of marketplaces. *European Management Journal* **36**(3) 319–329.

TDA. 2019. 5 steps for creating a successful wechat marketing strategy URL `https://web.archive.org/web/20240618223811/https://topdigital.agency/5-steps-for-creating-a-successful-wechat-marketing-strategy/`.

Tsunoda, Yushi, Yusuke Zennyo. 2021. Platform information transparency and effects on third-party suppliers and offline retailers. *Production and Operations Management* **30**(11) 4219–4235.

Vives, Xavier. 1999. *Oligopoly pricing: old ideas and new tools*. MIT press.

Wang, Yunjie, Albert Y Ha, Shilu Tong. 2022. Sharing manufacturer's demand information in a supply chain with price and service effort competition. *Manufacturing & Service Operations Management* **24**(3) 1698–1713.

Yakkundi. 2016. Digital experience technology and delivery priorities URL `https://www.forrester.com/report/Digital+Experience+Technology+And+Delivery+Priorities+2016/-/E-RES132461`.

Zha, Yong, Quan Li, Tingliang Huang, Yugang Yu. 2023. Strategic information sharing of online platforms as resellers or marketplaces. *Marketing Science* **42**(4) 659–678.

E-Companion for the Impact of CIS on Online Marketplaces

EC.1. Notations and Demand Model

The list of notations used in the paper and electronic companion is provided in Table EC.1.1.

Table EC.1.1 List of notations

	Parameters and Random Variables
ϵ_t	Random noise in period $t \in \{1,2\}$ which is uniformly distributed between 0 and 1
θ	True demand state, where $\theta \in \{H, L\}$
$\overline{\theta}$	A-priori belief of platform and sellers on demand state
θ_O	Posterior belief of platform on demand state after observing the first-period sales
θ_i	Posterior belief of seller i on demand state after observing the first-period sales
$\alpha_i(\theta)$	Market size of seller i under demand state θ
$\overline{\alpha}_i$	A-priori average demand for seller i, $\overline{\alpha}_i = \frac{\alpha_i(H)+\alpha_i(L)}{2}$
γ	Demand substitution coefficient between seller A and seller B
β	Commission rate collected by the platform per unit sold
J	CIS subscription regime between platform and sellers, where $J \in \{AB, A, B, \emptyset\}$
$J = AB$	Platform provides CIS to both sellers
$J = \emptyset$	Platform does not provide CIS
$J = i$	Platform provides CIS to seller i only, where $i \in \{A, B\}$
$J = fb$	Platform centrally sets sellers' prices in each period (first-best solution)
$d_{i,t}$	Demand of seller i in period t, where $i \in \{A,B\}$ and $t \in \{1,2\}$
π^J_{Ci}	Channel profit generated by seller i under J-type information sharing arrangement
$\pi^J_{i,t}$	Payoff of seller i in period t under J-type information sharing arrangement
π^J_i	Total payoff of seller i under J-type information sharing arrangement
$\pi^J_{O,t}$	Payoff of platform in period t under J-type information sharing arrangement
$CS^J_{i,t}$	Consumer Surplus from seller i in period t under J-type information sharing arrangement
CS^J_t	Total Consumer Surplus in period t under J-type information sharing arrangement
CS^J	Total Consumer Surplus under J-type information sharing arrangement
	Decision variables
ϕ	CIS subscription fee
y_i	Seller-i subscription decision $y_i \in \{0,1\}$
$p_{i,t}$	Price set by seller i in period t, where $i \in \{A,B\}$ and $t \in \{1,2\}$
P^J_i	$P^J_i \equiv (p^J_{i,1}, p^J_{i,2})$ represents the price trajectory of seller i over two periods

Also, the demand model is introduced in this section. Demand in period $t \in \{1,2\}$ is

$$d_{A,t}(p_{A,t}, p_{B,t}) = \alpha_A(\theta) - p_{A,t} + 2\gamma\epsilon_t(p_{B,t} - p_{A,t}) \tag{EC.1.1}$$

$$d_{B,t}(p_{A,t}, p_{B,t}) = \alpha_B(\theta) - p_{B,t} + 2\gamma\epsilon_t(p_{A,t} - p_{B,t}) \tag{EC.1.2}$$

To simplify the notation, we assume that the total market size is normalized and equal to one under both demand states. Accordingly, we use $\alpha_A(\theta) = \alpha_\theta$ and $\alpha_B(\theta) = 1 - \alpha_\theta$. We assume $\theta = H$ with probability $\frac{1}{2}$ or $\theta = L$ with the same probability $\frac{1}{2}$ and that $\frac{1}{2} \leq \alpha_L \leq \alpha_H \leq 1$. We assume that the random noise over the two periods, i.e., ϵ_1 and ϵ_2 are independent and both are uniformly distributed over $[0,1]$. By taking the expectation of the demand function with respect to ϵ_t, and collecting terms in $p_{A,t}$ and $p_{B,t}$, we obtain the following linear form for the mean demand functions:

$$\mu_{A,t}(p_{A,t}, p_{B,t}) = \overline{\alpha}_A - (1+\gamma)p_{A,t} + \gamma p_{B,t} \tag{EC.1.3}$$

$$\mu_{B,t}(p_{A,t}, p_{B,t}) = \overline{\alpha}_B - (1+\gamma)p_{B,t} + \gamma p_{A,t} \tag{EC.1.4}$$

where $\overline{\alpha}_A = \frac{\alpha_H+\alpha_L}{2}$, and $\overline{\alpha}_B = \frac{2-\alpha_H-\alpha_L}{2}$.

EC.2. Proofs of all Propositions

EC.2.1. Proposition 1

In this section we analyze the first-best scenario, where the platform centrally sets prices. We begin by analyzing the demand model and then optimizing the profit. Since the platform is able to set prices, she will optimize in her own favor.

Proof of Proposition 1: After the first period, platform has observed both sellers' sales and can identify the true state of demand $\boldsymbol{\theta}$ and ϵ_1 by solving Equations (EC.1.1) and (EC.1.2) simultaneously. In other words, the platform will be informed about the true demand state at the end of the first period, regardless of the first period price decisions. Platform's objective function is to maximize its total profit over the two periods as follows:

$$\max_{P_i^{fb},P_{-i}^{fb}} \quad \pi_O^{fb} = \beta \sum_{i\in\{A,B\}} \pi_{C|i}^{fb}\left(P_i^{fb},P_{-i}^{fb}\right)$$

where,

$$\pi_{C|i}^{fb}\left(P_i^{fb},P_{-i}^{fb}\right) = E_{\boldsymbol{\theta},\epsilon_t}\left[\sum_{t=1}^{2} d_{i,t}^{fb}\left(p_{i,t}^{fb},p_{-i,t}^{fb}\right)p_{i,t}^{fb}\right].$$

In the above objective function, since the platform will definitely learn the demand state at the end of the first period, it will set prices based on the exact demand model in the second period. Therefore, to solve this optimization we can use backward induction and start from the last period.

In the second period, the platform will be choosing the prices to maximize its second period profit, where they will be state dependent, i.e., $\pi_{O,2}(p_{A,2},p_{B,2}|\boldsymbol{\theta}) = p_{A,2}(\alpha_{\boldsymbol{\theta}} - p_{A,2} + \gamma(p_{B,2}-p_{A,2})) + p_{B,2}(1-\alpha_{\boldsymbol{\theta}} - p_{B,2} + \gamma(p_{A,2}-p_{B,2}))$. It is straightforward to show that $\frac{\partial^2\pi_{O,2}}{\partial p_{A,2}^2} < 0$, $\frac{\partial^2\pi_{O,2}}{\partial p_{B,2}^2} < 0$, and $\left(\frac{\partial^2\pi_{O,2}}{\partial p_{A,2}^2}.\frac{\partial^2\pi_{O,2}}{\partial p_{B,2}^2} - \left[\frac{\partial^2\pi_{O,2}}{\partial p_{A,2}\partial p_{B,2}}\right]^2\right) > 0$. So, we can use first order condition to find optimal prices from $\left[\frac{\partial\pi_{O,2}}{\partial p_{A,2}}, \frac{\partial\pi_{O,2}}{\partial p_{B,2}}\right] = [0,0]$ as

$$p_{A,2}^{fb} = \frac{\alpha_{\boldsymbol{\theta}}+\gamma}{2(1+2\gamma)}, \quad \text{and} \quad p_{B,2}^{fb} = \frac{1-\alpha_{\boldsymbol{\theta}}+\gamma}{2(1+2\gamma)}.$$

Note that the average price is $\frac{p_{A,2}^{fb}+p_{B,2}^{fb}}{2} = \overline{p}^{fb} = \frac{1}{4}$ and the price difference is $p_{A,2}^{fb} - p_{B,2}^{fb} = \frac{2\alpha_{\boldsymbol{\theta}}-1}{2(1+2\gamma)} = 2\Delta_{\boldsymbol{\theta}}^{fb} > 0$.

Now to find the first period prices, platform needs to optimize its profit using the mean demand function since it does not have access to the demand information in the first period. Note that using backward induction logic, second period profits will be constant values in total profit at the beginning of the planning horizon. Therefore, the platform faces the following problem:

$\pi_{O,1}(p_{A,1},p_{B,1}) = p_{A,1}(\overline{\alpha}_A - p_{A,1} + \gamma(p_{B,1}-p_{A,1})) + p_{B,1}(\overline{\alpha}_B - p_{B,1} + \gamma(p_{A,1}-p_{B,1}))$, where the expectation is taken with respect to both ϵ_1 and $\boldsymbol{\theta}$. It is straightforward to show that $\frac{\partial^2\pi_{O,1}}{\partial p_{A,1}^2} < 0$, $\frac{\partial^2\pi_{O,1}}{\partial p_{B,1}^2} < 0$, and $\frac{\partial^2\pi_{O,1}}{\partial p_{A,1}^2}\frac{\partial^2\pi_{O,1}}{\partial p_{B,1}^2} - \left[\frac{\partial^2\pi_{O,1}}{\partial p_{A,1}\partial p_{B,1}}\right]^2 > 0$. So, we can use first order condition to find optimal prices from $\left[\frac{\partial\pi_{O,1}}{\partial p_{A,1}}, \frac{\partial\pi_{O,1}}{\partial p_{B,1}}\right] = [0,0]$ as

$$p_{A,1}^{fb} = \frac{\alpha_H+\alpha_L+2\gamma}{4(1+2\gamma)}, \quad \text{and} \quad p_{B,1}^{fb} = \frac{2-\alpha_H-\alpha_L+2\gamma}{4(1+2\gamma)}.$$

Note that the average price is $\frac{p_{A,1}^{fb}+p_{B,1}^{fb}}{2} = \overline{p}^{fb} = \frac{1}{4}$ and the price difference is $p_{A,1}^{fb} - p_{B,1}^{fb} = \frac{\alpha_H+\alpha_L-1}{2(1+2\gamma)} = \frac{2\gamma}{1+2\gamma}\frac{\alpha_H+\alpha_L-1}{\alpha_H-\alpha_L}\frac{\alpha_H-\alpha_L}{4\gamma} = 2z_p^{fb}\Delta > 0$.

EC.2.2. Proposition 2

In this section we provide the proof for the characterization of equilibrium under regime $J = AB$, where information is shared with both sellers. Before proving Proposition 2, we introduce Lemma 1.

Lemma 1: Before deriving equilibrium prices, we characterize the second-period pricing game between the sellers given their posterior beliefs $\boldsymbol{\theta}_A$ and $\boldsymbol{\theta}_B$. To simplify the flow of the analysis we drop the time index in this part, because this subsection focuses on second-period prices. Therefore, p denotes seller i's second period price and p_{-i} represents the other seller's second period price.

Proof of Lemma 1: For the most general case we let $\overline{\alpha}_{i,2} = E_{\boldsymbol{\theta}_i}[\alpha_i(\boldsymbol{\theta}_i)]$ and let ρ_i' denote the posterior probability that seller i believes that seller $-i$ learns the true state of demand. Accordingly, if seller i identifies the true state of the demand, then the they can find the optimal second period price as:

$$p_i(\boldsymbol{\theta}) = \arg\max_p \pi_i(\boldsymbol{\theta}) = \arg\max_p \quad p\Big[\alpha_i(\boldsymbol{\theta}) - (1+\gamma)p + \gamma\Big(\rho_i' p_{-i}(\boldsymbol{\theta}) + (1-\rho_i')p_{-i}(\overline{\boldsymbol{\theta}})\Big)\Big]$$

and if seller i cannot identify the true state of the demand, then we have

$$p_i(\overline{\boldsymbol{\theta}}) = \arg\max_p \pi_i(\overline{\boldsymbol{\theta}}) = \arg\max_p \quad p\Big[\overline{\alpha}_{i,2} - (1+\gamma)p + \gamma\Big(\rho_i'\big[\rho_i p_{-i}(H) + (1-\rho_i)p_{-i}(L)\big] + (1-\rho_i')p_{-i}(\overline{\boldsymbol{\theta}})\Big)\Big]$$

Note that all profit functions are concave and solving for the first order conditions simultaneously gives us the following general expressions:

$$\begin{aligned}
p_{i,2}(\boldsymbol{\theta}) = \; & \frac{\gamma + (2+\gamma)\alpha_i(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)} + \rho_i'(1-\rho_{-i}')\frac{\gamma^2[\overline{\alpha}_{i,2} - \alpha_i(\boldsymbol{\theta})]}{(2+3\gamma)[4+8\gamma+\gamma^2(4-\rho_i'\rho_{-i}')]} \\
& + (1-\rho_i')\rho_{-i}'\frac{2\gamma(1+\gamma)[\overline{\alpha}_{-i,2} - \alpha_{-i}(\boldsymbol{\theta})]}{(2+3\gamma)[4+8\gamma+\gamma^2(4-\rho_i'\rho_{-i}')]} \\
& + (1-\rho_i')(1-\rho_{-i}')\frac{2\gamma(1+\gamma)[\gamma(\overline{\alpha}_{i,2} - \alpha_i(\boldsymbol{\theta})) + 2(1+\gamma)(\overline{\alpha}_{-i,2} - \alpha_{-i}(\boldsymbol{\theta})]}{(2+\gamma)(2+3\gamma)[4+8\gamma+\gamma^2(4-\rho_i'\rho_{-i}')]} \\
p_{i,2}(\overline{\boldsymbol{\theta}}) = \; & \frac{\gamma + (2+\gamma)\overline{\alpha}_{i,2}}{(2+\gamma)(2+3\gamma)} + (1-\rho_i')\rho_{-i}'\frac{2\gamma(1+\gamma)(\overline{\alpha}_{i,2} + \overline{\alpha}_{-i,2} - 1)}{(2+3\gamma)[4+8\gamma+\gamma^2(4-\rho_i'\rho_{-i}')]} \\
& + (1-\rho_i')(1-\rho_{-i}')\frac{4\gamma(1+\gamma)^2(\overline{\alpha}_{i,2} + \overline{\alpha}_{-i,2} - 1)}{(2+\gamma)(2+3\gamma)[4+8\gamma+\gamma^2(4-\rho_i'\rho_{-i}')]}
\end{aligned}$$

Proof of Proposition 2: We first characterize the equilibrium prices under $J = AB$, and then analyze the subscription fee and consumer surplus.

Equilibrium Prices: After the first period and when sales have been realized, platform can identify the true state of demand $\boldsymbol{\theta}$ and ϵ_1 by solving Eq. (EC.1.1) and Eq. (EC.1.2) simultaneously. In other words, the platform will be informed about the true demand state at the end of the first period, and since we are in regime $J = AB$, information will be shared with sellers too.

Both sellers want to maximize their total profit. Because both of them have access to information in the second period, they will set prices based on exact demand function in that period. Therefore, we can use backward induction logic to find the optimal prices.

In the second period, sellers will choose prices to maximize their profit, which will be state dependent, $\boldsymbol{\theta}_A = \boldsymbol{\theta}_B = \boldsymbol{\theta}$ and $\rho'_A = \rho'_B = 1$ as given in §EC.2.2. Through a similar analysis as in §EC.2.1, we derive the second period prices as follows:

$$p_{A,2}^{AB} = \frac{\gamma + (2+\gamma)\alpha_A(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)}, \quad \text{and} \quad p_{B,2}^{AB} = \frac{\gamma + (2+\gamma)\alpha_B(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)}. \tag{EC.2.1}$$

Now in the first period, sellers do not know the demand state and hence, they will use mean demand function. Using backward induction, second period profits will be constant values in total profit at the beginning of the planning horizon. Therefore, they face the following problem in the first period: $\pi_{A,1}(p_{A,1}, p_{B,1}) = p_{A,1}(\overline{\alpha}_A - p_{A,1} + \gamma(p_{B,1} - p_{A,1}))$ and $\pi_{B,1}(p_{A,1}, p_{B,1}) = p_{B,1}(\overline{\alpha}_B - p_{B,1} + \gamma(p_{A,1} - p_{B,1}))$. It is straightforward to show that equilibrium prices can be found from first order conditions $\left[\frac{\partial \pi_{A,1}}{\partial p_{A,1}}, \frac{\partial \pi_{B,1}}{\partial p_{B,1}}\right] = [0,0]$ as

$$p_{A,1}^{AB} = \frac{2\gamma + (2+\gamma)(\alpha_H + \alpha_L)}{2(2+\gamma)(2+3\gamma)}, \quad \text{and} \quad p_{B,1}^{AB} = \frac{4(1+\gamma) - (2+\gamma)(\alpha_H + \alpha_L)}{2(2+\gamma)(2+3\gamma)}. \tag{EC.2.2}$$

Note that we can rewrite the above expression as:

$$p_{A,1}^{AB} = \overline{p}^{AB} + z_p^{AB}\Delta, \quad \text{and} \quad p_{B,1}^{AB} = \overline{p}^{AB} - z_p^{AB}\Delta,$$

where $\overline{p}^{AB} = \frac{1}{2(2+\gamma)}$, $z_p^{AB} = \frac{2\gamma(\alpha_H+\alpha_L-1)}{(2+3\gamma)(\alpha_H-\alpha_L)}$, and $\Delta = \frac{\alpha_H-\alpha_L}{4\gamma}$. Therefore, the average price is $\overline{p}^{AB} = \frac{p_{A,1}^{AB}+p_{B,1}^{AB}}{2} = \frac{1}{2(2+\gamma)}$ and the price difference is $p_{A,1}^{AB} - p_{B,1}^{AB} = \frac{\alpha_H+\alpha_L-1}{2+3\gamma} = 2z_p^{AB}\Delta > 0$.

Designing subscription fee: According to the sequence of events (Figure 1 in the paper), the platform decides on the subscription fee ϕ to induce a specific subscription regime $J \in \{AB, A, B, \emptyset\}$ on the sellers. Moreover, from Eq. (1) in the paper, the payoff of seller i can be written as $\pi_i^J\left(P_i^J, P_{-i}^J\right) = (1-\beta)\pi_{C|i}^J\left(P_i^J, P_{-i}^J\right) - y_i\phi$, where P_i^J represents the price trajectory of seller i over two periods; i.e., $P_i^J \equiv (p_{i,1}^J, p_{i,2}^J)$. For ease of explanation, given the subscription decisions (y_A, y_B), we show sellers' profits in the following Table EC.2.1.

Table EC.2.1 Seller's profits under different subscription regimes

	$y_A = 0$	$y_A = 1$
	$J = \emptyset$	$J = A$
$y_B = 0$	$\pi_A^{\emptyset}\left(P_A^{\emptyset}, P_B^{\emptyset}\right) = (1-\beta)\pi_{C\mid A}^{\emptyset}\left(P_A^{\emptyset}, P_B^{\emptyset}\right)$	$\pi_A^{A}\left(P_A^{A}, P_B^{A}\right) = (1-\beta)\pi_{C\mid A}^{A}\left(P_A^{A}, P_B^{A}\right) - \phi$
	$\pi_B^{\emptyset}\left(P_B^{\emptyset}, P_A^{\emptyset}\right) = (1-\beta)\pi_{C\mid B}^{\emptyset}\left(P_B^{\emptyset}, P_A^{\emptyset}\right)$	$\pi_B^{A}\left(P_B^{A}, P_A^{A}\right) = (1-\beta)\pi_{C\mid B}^{A}\left(P_B^{A}, P_A^{A}\right)$
	$J = B$	$J = AB$
$y_B = 1$	$\pi_A^{B}\left(P_A^{B}, P_B^{B}\right) = (1-\beta)\pi_{C\mid A}^{B}\left(P_A^{B}, P_B^{B}\right)$	$\pi_A^{AB}\left(P_A^{AB}, P_B^{AB}\right) = (1-\beta)\pi_{C\mid A}^{AB}\left(P_A^{AB}, P_B^{AB}\right) - \phi$
	$\pi_B^{B}\left(P_B^{B}, P_A^{B}\right) = (1-\beta)\pi_{C\mid B}^{B}\left(P_B^{B}, P_A^{B}\right) - \phi$	$\pi_B^{AB}\left(P_B^{AB}, P_A^{AB}\right) = (1-\beta)\pi_{C\mid B}^{AB}\left(P_B^{AB}, P_A^{AB}\right) - \phi$

In what follows, we characterize the optimal level of subscription fee that the platform needs to post to induce subscription regime $J = AB$. From Table EC.2.1, the platform can induce both sellers to subscribe to CIS by satisfying two incentive compatibility constraints.

First, given seller A subscribes to CIS, seller B has no incentive not to subscribe to CIS:

$$(1-\beta)\pi_{C\mid B}^{AB}\left(P_B^{AB}, P_A^{AB}\right) - \phi^{AB} \geq (1-\beta)\pi_{C\mid B}^{A}\left(P_B^{A}, P_A^{A}\right)$$

Or, equivalently

$$\phi^{AB} + S_B^{AB} = (1-\beta)\left[\pi_{C\mid B}^{AB}\left(P_B^{AB}, P_A^{AB}\right) - \pi_{C\mid B}^{A}\left(P_B^{A}, P_A^{A}\right)\right] \tag{EC.2.3}$$

where S_B^{AB} is the slack variable to transfer the inequality to equality, and the optimal value of prices $P_i^{AB} \equiv (p_{i,1}^{AB}, p_{i,2}^{AB})$ and $P_i^{A} \equiv (p_{i,1}^{A}, p_{i,2}^{A})$ are characterized in §EC.2.2 and §EC.2.8, respectively. Note that, the right-hand side of Eq. (EC.2.3) represents the additional profit the channel gains from seller B subscribing to the CIS service. Meanwhile, the left-hand side illustrates the distribution of this surplus: the seller B's share, S_B^{AB}, and the platform's share, ϕ.

Second, given seller B subscribes to CIS, seller A has no incentive not to subscribe to CIS:

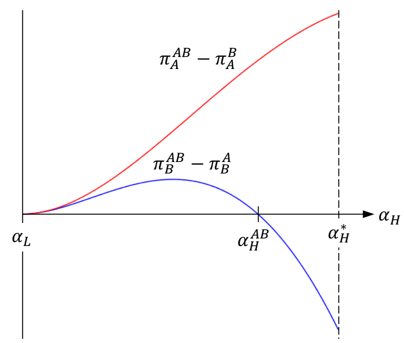
$$(1-\beta)\pi_{C|A}^{AB}\left(P_A^{AB}, P_B^{AB}\right) - \phi^{AB} \geq (1-\beta)\pi_{C|A}^{B}\left(P_A^{B}, P_B^{B}\right)$$

Or, equivalently

$$\phi^{AB} + S_A^{AB} = (1-\beta)\left[\pi_{C|A}^{AB}\left(P_A^{AB}, P_B^{AB}\right) - \pi_{C|A}^{B}\left(P_A^{B}, P_B^{B}\right)\right] \tag{EC.2.4}$$

with optimal prices $P_i^{B} \equiv (p_{i,1}^{B}, p_{i,2}^{B})$ under regime $J = B$.

Figure EC.2.1 Feasibility condition to induce subscription regime $J = AB$ ($\gamma = 1$, $\alpha_L = 0.6$)



From Figure EC.2.1, one can verify that both incentive compatibility constraints (EC.2.3) and (EC.2.4) are met (i.e., the right-hand side of constraints (EC.2.3) and (EC.2.4) are both positive) only if $\alpha_H \in [\alpha_L, \alpha_H^{AB}]$, where α_H^{AB} is the solution of constraint (EC.2.3) under binding condition. Moreover, for a given ϕ^{AB}, one can show that the right-hand-side of Eq. (EC.2.3) is less than the right-hand-side of Eq. (EC.2.4). It means that any subscription fee $\phi^{AB} \in [0, (1-\beta)\left[\pi_{C|B}^{AB}\left(P_B^{AB}, P_A^{AB}\right) - \pi_{C|B}^{A}\left(P_B^{A}, P_A^{A}\right)\right])$ can induce both sellers to subscribe to CIS. However, given the platform objective function that maximizes the subscription fee, the optimal solution is as follows:

$$\phi^{AB} = (1-\beta)\left[\pi_{C|B}^{AB}\left(P_B^{AB}, P_A^{AB}\right) - \pi_{C|B}^{A}\left(P_B^{A}, P_A^{A}\right)\right]$$
$$S_B^{AB} = 0$$
$$S_A^{AB} = (1-\beta)\left[\left(\pi_{C|A}^{AB}\left(P_A^{AB}, P_B^{AB}\right) - \pi_{C|A}^{B}\left(P_A^{B}, P_B^{B}\right)\right) - \left(\pi_{C|B}^{AB}\left(P_B^{AB}, P_A^{AB}\right) - \pi_{C|B}^{A}\left(P_B^{A}, P_A^{A}\right)\right)\right]$$

Consumer Surplus: We now compute the consumer surplus under regime $J = AB$. To do this, we consider each period separately.

In the first period, consumer surplus is the expected surplus generated by each seller's demand across possible demand states. For each demand state and seller, it is calculated as follows:

$$CS_{i,1}^{AB}(\boldsymbol{\theta}) = \int_0^{d_{i,1}(\boldsymbol{\theta})} p_{i,1}(q)dq - p_{i,1}d_{i,1}$$

$$= \frac{d^*_{i,1}(\alpha_i(\boldsymbol{\theta}) + \gamma p^*_{-i,1} - (1+\gamma)p^*_{i,1})}{2(1+\gamma)}$$
$$= \frac{(d^*_{i,1}(\boldsymbol{\theta}))^2}{2(1+\gamma)},$$

where $(d^*_{i,1}(\boldsymbol{\theta}))^2$ is derived with optimal first period prices $(p^{AB}_{A,1}, p^{AB}_{B,1})$ characterized in Eq. (EC.2.2). Therefore, aggregating across sellers, for the first period consumer surplus under regime $J = AB$ we have:

$$\begin{aligned}
CS_1^{AB} &= E_{\boldsymbol{\theta}}\left[\sum_{i\in\{A,B\}} CS^{AB}_{i,1}(\boldsymbol{\theta})\right] \\
&= \frac{1}{2}\Big(\frac{(d^*_{A,1}(H))^2}{2(1+\gamma)} + \frac{(d^*_{A,1}(L))^2}{2(1+\gamma)}\Big) + \frac{1}{2}\Big(\frac{(d^*_{B,1}(H))^2}{2(1+\gamma)} + \frac{(d^*_{B,1}(L))^2}{2(1+\gamma)}\Big) \\
&= \frac{\begin{array}{l}\gamma^4\left(10a_H^2 - 16a_Ha_L - 2a_H + 10a_L^2 - 2a_L + 10\right)+\\ \gamma^3\left(54a_H^2 - 84a_Ha_L - 12a_H + 54a_L^2 - 12a_L + 36\right)+\\ \gamma^2\left(101a_H^2 - 150a_Ha_L - 26a_H + 101a_L^2 - 26a_L + 50\right)+\\ \gamma\left(76a_H^2 - 104a_Ha_L - 24a_H + 76a_L^2 - 24a_L + 32\right)+\\ \left(20a_H^2 - 24a_Ha_L - 8a_H + 20a_L^2 - 8a_L + 8\right)\end{array}}{4\,(\gamma+1)\,(\gamma+2)^2\,(3\gamma+2)^2}.
\end{aligned}$$

Since CS_1^{AB} is derived based on the optimal prices with mean demand function, we will refer to it as CS_{mean}.

Similarly for the second period, consumer surplus will be acquired with optimal second period prices with true demand states derived in Eq. (EC.2.1). Aggregating across sellers, for the second period consumer surplus under regime $J = AB$ we have:

$$\begin{aligned}
CS_2^{AB} &= E_{\boldsymbol{\theta}}\left[\sum_{i\in\{A,B\}} CS^{AB}_{i,2}(\boldsymbol{\theta})\right] \\
&= \frac{1}{2}\Big(\frac{(d^*_{A,2}(H))^2}{2(1+\gamma)} + \frac{(d^*_{A,2}(L))^2}{2(1+\gamma)}\Big) + \frac{1}{2}\Big(\frac{(d^*_{B,2}(H))^2}{2(1+\gamma)} + \frac{(d^*_{B,2}(L))^2}{2(1+\gamma)}\Big) \\
&= \frac{(1+\gamma)\left[(a_H^2 + a_L^2 - a_H - a_L + 5)\,\gamma^2 \;+\; 4\,(a_H^2 + a_L^2 - a_H - a_L + 2)\,\gamma \;+\; 4\,(a_H^2 + a_L^2 - a_H - a_L + 1)\right]}{2\,(\gamma+2)^2\,(3\gamma+2)^2} \\
&= \frac{(1+\gamma)\left[(a_H^2 + a_L^2 - a_H - a_L + 1)(2+\gamma)^2 + 4\gamma(1+\gamma)\right]}{2\,(\gamma+2)^2\,(3\gamma+2)^2}.
\end{aligned}$$

Since CS_2^{AB} is derived based on optimal prices with exact demand function, we will refer to it as CS_{exact}. Hence, total consumer surplus under regime $J = AB$ is

$$CS^{AB} = CS_1^{AB} + CS_2^{AB} = CS_{mean} + CS_{exact}.$$

EC.2.3. Proposition 3

Suppose now that the platform does not share information with sellers, so each seller observes only her own sales. We study how first-period decisions affect sellers' second-period information states.

Proof of Proposition 3: For expositional brevity, we drop the time index and use (p_A, p_B) be the first period prices of seller A and seller B, respectively. There will be no need to use the second period prices for belief updating. We analyze the belief updating for each seller separately.

Belief updating by seller A: Let S_A be the sales observed by seller A at the end of the first period. seller A knows that her sales is coming from $\alpha_H - p_A + 2\gamma\epsilon_1(p_B - p_A)$ or $\alpha_L - p_A + 2\gamma\epsilon_1(p_B - p_A)$ but does not know which one. However, in some situations seller A can deduce the real value of α and ϵ_1. When seller A only observes its own sales, there are three possibilities: seller A identifies $\boldsymbol{\theta} = H$, seller A identifies $\boldsymbol{\theta} = L$, seller A cannot identify $\boldsymbol{\theta}$. In what follows, we derive the probability of each outcome.

The main logic to derive the probability of learning is that S_A can be compared against two thresholds: the highest possible sale under the Low state and the lowest possible sale under the High state. Note that since $\alpha_H \geq \alpha_L$, then with the same value for ϵ_1, seller A's sale under High demand state is higher than her sale under Low demand state. Depending on how the realized first-period demand compares against the two thresholds, we can identify three cases:

- If realized demand lies above the Low-state maximum, the state must be High.
- If realized demand lies below the High-state minimum, the state must be Low.
- If realized demand falls between the two thresholds, the seller cannot identify the state, since the value of ϵ_1 is unknown.

Using these cases, we derive the probability of correctly identifying the true state for seller A. Because the sign of $(p_B - p_A)$ will affect the realized sale, we will break down our analysis into two cases as follows.

Case I ($p_B \leq p_A$): Figure EC.2.2 facilitates our discussion for the case where $p_B \leq p_A$. We derive the probability of each state $\boldsymbol{\theta} = H$, $\boldsymbol{\theta} = L$, and $\boldsymbol{\theta} = \overline{\boldsymbol{\theta}}$ in the following.

Case I.A (True demand state $\boldsymbol{\theta} = H$): If the true demand state is high, then seller A can identify this if her sales S_A is sufficiently high. In other words, seller A can identify that S_A is coming through the High demand state, if the realized sale is higher than her highest possible sale in Low demand state. In this case, since $p_B - p_A \leq 0$ the highest possible sale in Low demand state for seller A happens at $\epsilon_1 = 0$ and is equal to $\alpha_L - p_A$. We can write this statement as follows:

$$
\begin{aligned}
Pr(\boldsymbol{\theta}_A = H \big| \boldsymbol{\theta} = H) &= Pr(S_A \geq \alpha_L - p_A \big| \boldsymbol{\theta} = H) \\
&= Pr(\alpha_H - p_A + 2\gamma\epsilon_1(p_B - p_A) \geq \alpha_L - p_A) \\
&= Pr(\epsilon_1 \leq \frac{\alpha_H - \alpha_L}{2\gamma(p_A - p_B)}) \\
&= Pr(\epsilon_1 \leq \frac{2\Delta}{p_A - p_B}) \\
&= \frac{2\Delta}{p_A - p_B}.
\end{aligned}
$$

On the other hand, if the true demand state is High, seller A can never mistakenly infer the state to be Low. This is because if seller A wants to learn the Low demand state, the realized sale must be lower than the lowest possible sale in High state, which is impossible. Therefore, we can show that

$$Pr(\boldsymbol{\theta}_A = L | \boldsymbol{\theta} = H) = 0$$

Finally, the probability of remaining uninformed is

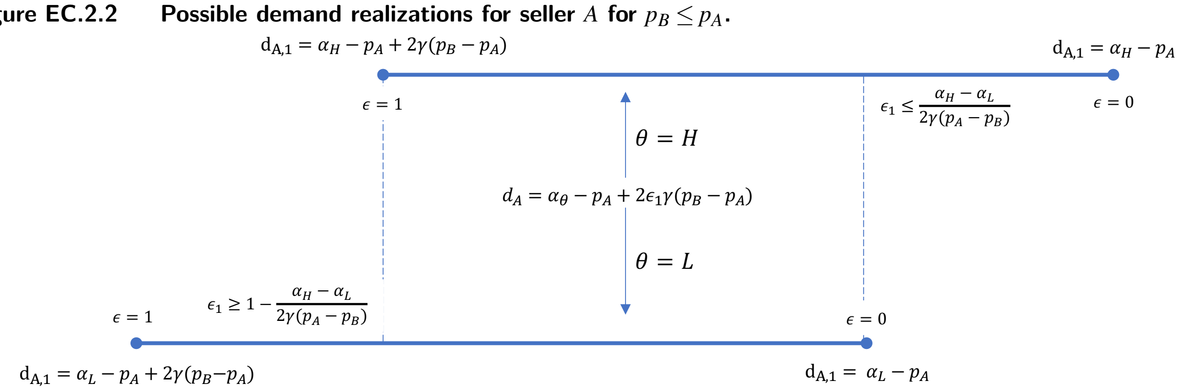
$$Pr(\boldsymbol{\theta}_A = \bar{\boldsymbol{\theta}} | \boldsymbol{\theta} = H) = 1 - Pr(\boldsymbol{\theta}_A = H | \boldsymbol{\theta} = H) - Pr(\boldsymbol{\theta}_A = L | \boldsymbol{\theta} = H) = 1 - \frac{2\Delta}{p_A - p_B}.$$

Case I.B (True demand state $\boldsymbol{\theta} = L$): By a similar argument, if the true demand state is Low, seller A can identify this if S_A is sufficiently low. In other words, seller A can identify that S_A is coming through the Low demand state, if the realized sale is lower than the lowest possible sale in High demand state. In this case, since $p_B - p_A \leq 0$ the lowest possible sale in High demand state for seller A happens at $\boldsymbol{\epsilon}_1 = 1$, and is equal $\alpha_H - p_A + 2\gamma(p_B - p_A)$. We can write this statement as follows:

$$\begin{aligned}
Pr(\boldsymbol{\theta}_A = L \big| \boldsymbol{\theta} = L) &= Pr(S_A \leq \alpha_H - p_A + 2\gamma(p_B - p_A) \big| \boldsymbol{\theta} = L) \\
&= Pr(\alpha_L - p_A + 2\gamma\boldsymbol{\epsilon}_1(p_B - p_A) \leq \alpha_H - p_A + 2\gamma(p_B - p_A)) \\
&= Pr(1 - \frac{\alpha_H - \alpha_L}{2\gamma(p_A - p_B)} \leq \boldsymbol{\epsilon}_1) \\
&= Pr(1 - \frac{2\Delta}{p_A - p_B} \leq \boldsymbol{\epsilon}_1) \\
&= \frac{2\Delta}{p_A - p_B},
\end{aligned}$$

Again, if the true demand state is Low, seller A can never mistakenly infer the state to be High. This is because if seller A wants to learn the High demand state, the realized sale must be higher than the highest possible sale in Low state, which is impossible. Therefore, we can show that $Pr(\theta_A = H | \theta = L) = 0$. Hence, the probability of remaining uninformed is again $Pr(\boldsymbol{\theta}_A = \overline{\boldsymbol{\theta}} \big| \boldsymbol{\theta} = L) = Pr(\boldsymbol{\epsilon}_1 \leq 1 - \frac{2\Delta}{p_A - p_B}) = 1 - \frac{2\Delta}{p_A - p_B}$.

Figure EC.2.2 Possible demand realizations for seller A for $p_B \leq p_A$.



Finally, to complete the proof, when seller A cannot learn the demand state and stays uninformed, her posterior beliefs remain $\frac{1}{2}$ on both states. First, by applying Bayes' rule:

$$\begin{aligned}
Pr(\boldsymbol{\theta}_A = \overline{\boldsymbol{\theta}}) &= Pr(\boldsymbol{\theta} = L) Pr(\boldsymbol{\theta}_A = \overline{\boldsymbol{\theta}} \big| \boldsymbol{\theta} = L) + Pr(\boldsymbol{\theta} = H) Pr(\boldsymbol{\theta}_A = \overline{\boldsymbol{\theta}} \big| \boldsymbol{\theta} = H) \\
&= \frac{1}{2}(1 - \frac{2\Delta}{p_A - p_B}) + \frac{1}{2}(1 - \frac{2\Delta}{p_A - p_B}) \\
&= 1 - \frac{2\Delta}{p_A - p_B}
\end{aligned}$$

and accordingly, $Pr(\theta_A = \theta) = \frac{2\Delta}{p_A - p_B}$. Then, by substituting this back to Bayes' rule yields the posterior probability:

$$\begin{aligned} Pr(\theta = H | \theta_A = \overline{\theta}) &= \frac{Pr(\theta = H \; and \; \theta_A = \overline{\theta})}{Pr(\theta_A = \overline{\theta})} \\ &= \frac{\frac{1}{2}(1 - \frac{2\Delta}{p_A - p_B})}{1 - \frac{2\Delta}{p_A - p_B}} \\ &= \frac{1}{2} \end{aligned}$$

and similarly, $Pr(\theta = L | \theta_A = \overline{\theta}) = \frac{1}{2}$.

Case II ($p_B \geq p_A$)**:** Figure EC.2.3 facilitates our discussion for the case where $p_B \geq p_A$. We derive the probability of each state $\theta = H$, $\theta = L$, and $\theta = \overline{\theta}$ in the following. The analysis in this case is similar to Case I. The only difference is that because $p_B - p_A \geq 0$, the highest possible sale at Low demand state will happen at $\epsilon_1 = 1$, and will be equal to $\alpha_L - p_A + 2\gamma(p_B - p_A)$, and the lowest possible sale at High demand state will happen at $\epsilon_1 = 0$, and will be equal to $\alpha_H - p_A$.

Case II.A (True demand state $\theta = H$): If the true demand state is High, then seller A can identify this if her sales S_A is sufficiently high. In other words, seller A can identify that S_A is coming through the High demand state, if the realized sale is higher than her highest possible sale in Low demand state, i.e.,

$$\begin{aligned} Pr(\theta_A = H \big| \theta = H) &= Pr(S_A \geq \alpha_L - p_A + 2\gamma(p_B - p_A) \big| \theta = H) \\ &= Pr(1 - \frac{2\Delta}{p_B - p_A} \leq \epsilon_1) \\ &= \frac{2\Delta}{p_B - p_A}. \end{aligned}$$

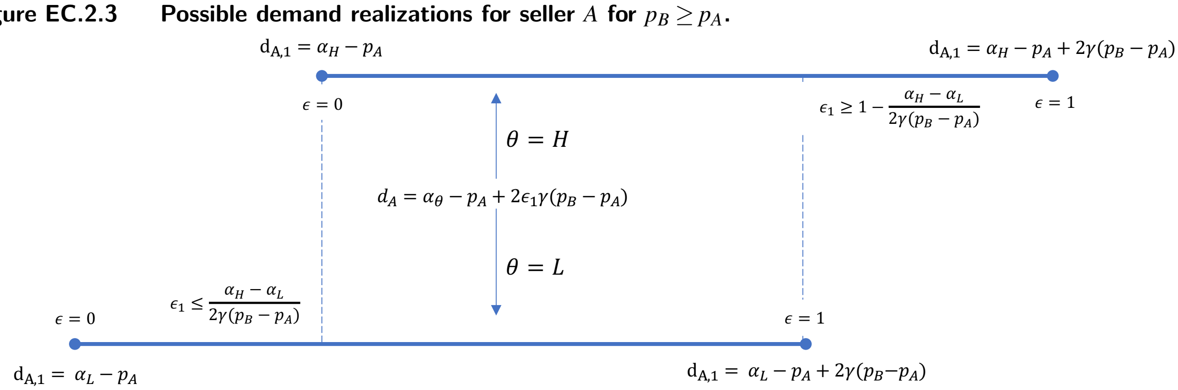
Accordingly, $Pr(\theta_A = \overline{\theta} \big| \theta = H) = Pr(\epsilon_1 \leq 1 - \frac{2\Delta}{p_B - p_A}) = 1 - \frac{2\Delta}{p_B - p_A}$.

Case II.B (True demand state $\theta = L$): If the true demand state is Low, then seller A can identify this if her sales S_A is sufficiently low, i.e.,

$$\begin{aligned} Pr(\theta_A = L \big| \theta = L) &= Pr(S_A \leq \alpha_H - p_A \big| \theta = L) \\ &= Pr(\epsilon_1 \leq \frac{2\Delta}{p_B - p_A}) \\ &= \frac{2\Delta}{p_B - p_A}. \end{aligned}$$

Accordingly, $Pr(\theta_A = \overline{\theta} \big| \theta = L) = Pr(\frac{2\Delta}{p_B - p_A} \leq \epsilon_1) = 1 - \frac{2\Delta}{p_B - p_A}$.

Figure EC.2.3 Possible demand realizations for seller A for $p_B \geq p_A$.



Finally, to complete the proof, when seller A cannot learn the demand state and stays uninformed, her posterior beliefs remain $\frac{1}{2}$ on both states. First, by applying Bayes' rule:

$$\begin{aligned}
Pr(\theta_A = \overline{\theta}) &= Pr(\theta = L)Pr(\theta_A = \overline{\theta}\big|\theta = L) + Pr(\theta = H)Pr(\theta_A = \overline{\theta}\big|\theta = H)\\
&= \frac{1}{2}(1 - \frac{2\Delta}{p_B - p_A}) + \frac{1}{2}(1 - \frac{2\Delta}{p_B - p_A})\\
&= 1 - \frac{2\Delta}{p_B - p_A}
\end{aligned}$$

and accordingly, $Pr(\theta_A = \theta) = \frac{2\Delta}{p_B - p_A}$. Then, by substituting this back to Bayes' rule yields the posterior probability:

$$\begin{aligned}
Pr(\theta = H|\theta_A = \overline{\theta}) &= \frac{Pr(\theta = H \text{ and } \theta_A = \overline{\theta})}{Pr(\theta_A = \overline{\theta})}\\
&= \frac{\frac{1}{2}(1 - \frac{2\Delta}{p_B - p_A})}{1 - \frac{2\Delta}{p_B - p_A}}\\
&= \frac{1}{2}
\end{aligned}$$

And similarly, $Pr(\theta = L|\theta_A = \overline{\theta}) = \frac{1}{2}$. In other words, if seller A cannot identify the true demand state θ, then the posterior distribution is identical to the prior.

Belief updating by seller B: The same reasoning applies symmetrically to seller B, which in turn leads to identical probabilities of correct learning and remaining uninformed. Since the total market available to seller B in High and Low states are $1 - \alpha_H$ and $1 - \alpha_L$, and $\alpha_L \leq \alpha_H$, under the same scenario with other parameters and random variables, seller B's sale under Low demand state is higher than her sale under High demand state. Therefore, the two thresholds that we need to compare sales with are highest possible sale under High demand state and lowest possible sale under Low demand state. Since the sign of $p_A - p_B$ will affect the realized sales, we will break down our analysis into the following two cases based on the sign of $p_A - p_B$.

Case I ($p_B \leq p_A$): We derive the probability of each state $\theta = H$, $\theta = L$, and $\theta = \overline{\theta}$ in the following. Because $p_B - p_A \leq 0$, the highest possible sale at High demand state will happen at $\epsilon_1 = 1$, and will be equal to $1 - \alpha_H - p_B + 2\gamma(p_A - p_B)$, and the lowest possible sale at Low demand state will happen at $\epsilon_1 = 0$, and will be equal to $1 - \alpha_L - p_B$.

Case I.A (True demand state $\theta = H$): If the true demand state is High, then seller B can identify this if her sales S_B is sufficiently low. In other words, seller B can identify that S_B is coming through the High demand state, if the realized sale is lower than her lowest possible sale in Low demand state, i.e.,

$$Pr(\theta_B = H\big|\theta = H) = Pr(S_B \leq 1 - \alpha_L - p_B\big|\theta = H) = Pr(\epsilon_1 \leq \frac{2\Delta}{p_A - p_B}) = \frac{2\Delta}{p_A - p_B}.$$

Accordingly, $Pr(\theta_B = \overline{\theta}\big|\theta = H) = Pr(\epsilon_1 \leq 1 - \frac{2\Delta}{p_A - p_B}) = 1 - \frac{2\Delta}{p_A - p_B}$.

Case I.B (True demand state $\theta = L$): If the true demand state is Low, then seller B can identify this if her sales S_B is significantly High, i.e.,

$$Pr(\theta_B = L\big|\theta = L) = Pr(S_B \geq 1 - \alpha_H - p_B + 2\gamma(p_A - p_B)\big|\theta = L) = Pr(\epsilon_1 \geq 1 - \frac{2\Delta}{p_A - p_B}) = \frac{2\Delta}{p_A - p_B}.$$

Accordingly, $Pr(\theta_B = \overline{\theta}\big|\theta = L) = 1 - \frac{2\Delta}{p_A - p_B}$.

Case II ($p_B \geq p_A$): We derive the probability of each state $\theta = H$, $\theta = L$, and $\theta = \overline{\theta}$ in the following. Because $p_B - p_A \geq 0$, the highest possible sale at High demand state will happen at $\epsilon_1 = 0$, and will be equal to $1 - \alpha_H - p_B$, and the lowest possible sale under Low demand state will happen at $\epsilon_1 = 1$, and will be equal to $1 - \alpha_L - p_B + 2\gamma(p_A - p_B)$.

Case II.A (True demand state $\theta = H$): If the true demand state is High, then seller B can identify this if her sales S_B is sufficiently low. In other words, seller B can identify that S_B is coming through the High demand state, if the realized sale is lower than her lowest possible sale in Low demand state, i.e.,

$$Pr(\theta_B = H \big| \theta = H) = Pr(S_B \leq 1 - \alpha_L - p_B + 2\gamma(p_A - p_B) \big| \theta = H) = Pr(\epsilon_1 \geq 1 - \frac{2\Delta}{p_B - p_A}) = \frac{2\Delta}{p_B - p_A}.$$

Accordingly, $Pr(\theta_B = \overline{\theta} \big| \theta = H) = Pr(\epsilon_1 \leq 1 - \frac{2\Delta}{p_B - p_A}) = 1 - \frac{2\Delta}{p_B - p_A}$.

Case II.B (True demand state $\theta = L$): If the true demand state is Low, then seller B can identify this if her sales S_B is significantly High, i.e.,

$$Pr(\theta_B = L \big| \theta = L) = Pr(S_B \geq 1 - \alpha_H - p_B \big| \theta = L) = Pr(\epsilon_1 \leq \frac{2\Delta}{p_B - p_A}) = \frac{2\Delta}{p_B - p_A}.$$

Accordingly, $Pr(\theta_B = \overline{\theta} \big| \theta = L) = Pr(\frac{2\Delta}{p_B - p_A} \leq \epsilon_1) = 1 - \frac{2\Delta}{p_B - p_A}$.

Finally, to complete the proof, when seller B cannot learn the demand state and stays uninformed, her posterior beliefs remain $\frac{1}{2}$ on both states. By applying Bayes' rule:

$$\begin{aligned} Pr(\theta_B = \overline{\theta}) &= Pr(\theta = L) Pr(\theta_B = \overline{\theta} \big| \theta = L) + Pr(\theta = H) Pr(\theta_B = \overline{\theta} \big| \theta = H) \\ &= \frac{1}{2}(1 - \frac{2\Delta}{p_B - p_A}) + \frac{1}{2}(1 - \frac{2\Delta}{p_B - p_A}) \\ &= 1 - \frac{2\Delta}{p_B - p_A} \end{aligned}$$

and accordingly, $Pr(\theta_B = \theta) = \frac{2\Delta}{p_B - p_A}$. Then, by substituting this back to Bayes' rule yields the posterior probability:

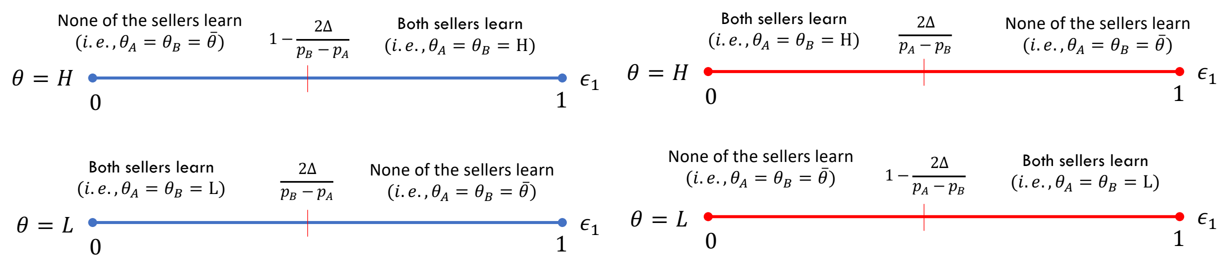
$$\begin{aligned} Pr(\theta = H | \theta_B = \overline{\theta}) &= \frac{Pr(\theta = H \ and \ \theta_B = \overline{\theta})}{Pr(\theta_B = \overline{\theta})} \\ &= \frac{\frac{1}{2}(1 - \frac{2\Delta}{p_B - p_A})}{1 - \frac{2\Delta}{p_B - p_A}} \\ &= \frac{1}{2} \end{aligned}$$

And similarly, $Pr(\theta = L | \theta_B = \overline{\theta}) = \frac{1}{2}$. In other words, if seller B cannot identify the true demand state θ, then the posterior distribution is identical to the prior.

Posterior beliefs: In this section we first summarize the demand learning opportunities for sellers, then discuss about the link between learning and normalized demand. Figure EC.2.4 provides an illustration of the posterior distribution for all possible realizations of α and ϵ_1 for the given first period prices (p_A, p_B). Based on the findings of the belief updating analysis for sellers, we have two possibilities: either both sellers identify the true state of the demand at the same time or neither seller can identify the true state of the demand (and their posteriors are identical to their priors). We can summarize the posterior beliefs as follows:

$$\begin{cases} \text{if } p_A \geq p_B: \quad \theta_i = \begin{cases} H & \text{if } \epsilon_1 \leq \frac{2\Delta}{p^{\emptyset}_{A,1} - p^{\emptyset}_{B,1}} \text{ and } \theta = H \\ L & \text{if } \epsilon_1 \geq 1 - \frac{2\Delta}{p^{\emptyset}_{A,1} - p^{\emptyset}_{B,1}} \text{ and } \theta = L \\ \bar{\theta} & \text{otherwise,} \end{cases} \\ \text{if } p_A \leq p_B: \quad \theta_i = \begin{cases} H & \text{if } \epsilon_1 \geq 1 - \frac{2\Delta}{p^{\emptyset}_{A,1} - p^{\emptyset}_{B,1}} \text{ and } \theta = H \\ L & \text{if } \epsilon_1 \leq \frac{2\Delta}{p^{\emptyset}_{A,1} - p^{\emptyset}_{B,1}} \text{ and } \theta = L \\ \bar{\theta} & \text{otherwise,} \end{cases} \end{cases}$$

Figure EC.2.4 Demand learning for each seller based on the possible demand realization.



(a) $p_A \leq p_B$ (b) $p_A \geq p_B$

Accordingly, the second period profit of the sellers in the two symmetric cases, given the information state they are in, can be simplified and written as:

$$\begin{aligned} \pi_{i,2}(\boldsymbol{\theta}, \boldsymbol{\theta}) &= p_{i,2}(\boldsymbol{\theta})\Big(\alpha_i(\boldsymbol{\theta}) - p_{i,2}(\boldsymbol{\theta}) + \gamma\big(p_{-i,2}(\boldsymbol{\theta}) - p_{i,2}(\boldsymbol{\theta})\big)\Big) \\ \pi_{i,2}(\bar{\boldsymbol{\theta}}) &= p_{i,2}(\bar{\boldsymbol{\theta}})\Big(\alpha_i(\boldsymbol{\theta}) - p_{i,2}(\bar{\boldsymbol{\theta}}) + \gamma\big(p_{-i,2}(\bar{\boldsymbol{\theta}}) - p_{i,2}(\bar{\boldsymbol{\theta}})\big)\Big) \end{aligned} \tag{EC.2.5}$$

However, in cases that sellers do not learn the demand state, if only seller i is provided with information via the platform through subscription, then the second period profit of the seller will be:

$$\pi_{i,2}(\boldsymbol{\theta}, \bar{\boldsymbol{\theta}}) = p_{i,2}(\boldsymbol{\theta}, \bar{\boldsymbol{\theta}})\Big(\alpha_i(\boldsymbol{\theta}) - p_{i,2}(\boldsymbol{\theta}, \bar{\boldsymbol{\theta}}) + \gamma\big(p_{-i,2}(\bar{\boldsymbol{\theta}}) - p_{i,2}(\boldsymbol{\theta}, \bar{\boldsymbol{\theta}})\big)\Big) \tag{EC.2.6}$$

Now, we can combine demand learning with normalized demand. Recall that we defined the normalized price difference as $z^d_p = \frac{|p_A - p_B|}{2\Delta}$ which gives us $f(z^d_p) = \frac{(z^d_p - 1)}{2(z^d_p + 1)} = \frac{|p_A - p_B| - 2\Delta}{2|p_A - p_B| + 4\Delta}$. In addition, we defined the normalized demand as:

$$z^{d|J}_i(\boldsymbol{\theta}) = \frac{|d_{i,1}(p_{i,1}, p_{-i,1}) - E_{\boldsymbol{\theta},\boldsymbol{\epsilon}}[d_{i,1}(p_{i,1}, p_{-i,1})|\boldsymbol{\theta}, \boldsymbol{\epsilon}]|}{\max_{\boldsymbol{\theta},\boldsymbol{\epsilon}}\{d_{i,1}(p_{i,1}, p_{-i,1})\} - \min_{\boldsymbol{\theta},\boldsymbol{\epsilon}}\{d_{i,1}(p_{i,1}, p_{-i,1})\}}\} = \frac{\big|\alpha_i(\boldsymbol{\theta}) - \bar{\alpha}_i + (2\boldsymbol{\epsilon} - 1)\gamma(p_{-i} - p_i)\big|}{\alpha_H - \alpha_L + 2\gamma|p_i - p_{-i}|}.$$

In fact, with some algebraic manipulations, we can show that:

$$z^{d|J}_i(H) = z^{d|J}_{-i}(H) = \frac{\big|2\Delta + (2\boldsymbol{\epsilon} - 1)(p_{-i} - p_i)\big|}{4\Delta + 2|p_i - p_{-i}|} \quad \text{and} \quad z^{d|J}_i(L) = z^{d|J}_{-i}(L) = \frac{\big|-2\Delta + (2\boldsymbol{\epsilon} - 1)(p_i - p_{-i})\big|}{4\Delta + 2|p_{-i} - p_i|}.$$

Again, since the sign of $p_A - p_B$ matters in the above expressions we will consider two cases based on the sign of $p_A - p_B$.

$$\begin{cases} \text{if } p_A \leq p_B: & \begin{cases} z^d_A(H) = z^d_B(H) \geq f(z_p) & \text{if and only if } \boldsymbol{\epsilon} \geq 1 - \frac{2\Delta}{(p_B - p_A)}, \\ z^d_A(L) = z^d_B(L) \geq f(z_p) & \text{if and only if } \boldsymbol{\epsilon} \leq \frac{2\Delta}{(p_B - p_A)}. \end{cases} \\ \text{if } p_A \geq p_B: & \begin{cases} z^d_A(H) = z^d_B(H) \geq f(z_p) & \text{if and only if } \boldsymbol{\epsilon} \leq \frac{2\Delta}{(p_A - p_B)}, \\ z^d_A(L) = z^d_B(L) \geq f(z_p) & \text{if and only if } \boldsymbol{\epsilon} \geq 1 - \frac{2\Delta}{(p_A - p_B)}. \end{cases} \end{cases}$$

EC.2.4. Proposition 4

In this section we characterize the equilibrium under $J=\emptyset$, where platform does not share information with sellers at the end of the first period. The objective function that sellers face is to maximize their total profit across two periods. We start by exploring the second period mechanism and then we derive the first period equilibrium prices.

Proof of Proposition 4: In the second period, either they have learned the true demand or not. Also, from §EC.2.3 we know that they will both learn the demand or both stay in the dark. It will never be the case that just one of them learns the demand under $J=\emptyset$. Therefore, if they learn the demand, which will happen with $\frac{2\Delta}{|p_A-p_B|}$ probability, they will set prices based on exact demand function and corresponding optimal prices (Eq. EC.2.1). If they do not learn the demand, with $1-\frac{2\Delta}{|p_A-p_B|}$ probability, they will set prices based on mean demand function and its optimal prices (Eq. EC.2.2).

Therefore, we need to derive first period equilibrium prices for sellers. For expositional brevity, we drop the time index for the first period prices and denote them as (p_A,p_B). We follow the sequence of events in the setting of $J=\emptyset$ regime. In the first period, sellers set prices as (p_A,p_B) and their expected profit will be $\pi_{A,1}(p_A,p_B)=p_A[\overline{\alpha}_A-p_A+\gamma(p_B-p_A)]$ and $\pi_{B,1}(p_A,p_B)=p_B[\overline{\alpha}_B-p_B+\gamma(p_A-p_B)]$. Each seller will observe its own sales at the end of the first period and form a belief about the status of the demand as well as the information obtained by the opponent as described in §EC.2.3, and they set prices in the second period based on the information set they are at which will be one of the two following possibilities: $(\boldsymbol{\theta}_A,\boldsymbol{\theta}_B)\in\{(\boldsymbol{\theta},\boldsymbol{\theta}),(\overline{\boldsymbol{\theta}},\overline{\boldsymbol{\theta}})\}$.

Since the threshold for learning is either $\frac{2\Delta}{|p_A-p_B|}$ or $1-\frac{2\Delta}{|p_A-p_B|}$, the relation between p_A, p_B, and 2Δ affects learning opportunities. Namely, if $\frac{2\Delta}{|p_A-p_B|}\geq 1$ which corresponds to $|p_A-p_B|\leq 2\Delta$, sellers will always learn the true demand regardless of the realization of ϵ_1. Otherwise, if $p_B+2\Delta\leq p_A$ or $p_B-2\Delta\geq p_A$, they will learn in some cases and remain uninformed in others. Therefore, we break down our analysis into the following three zones based on their learning opportunities: Zone 1 in which $p_B+2\Delta\leq p_A$, Zone 2 in which $p_B-2\Delta\leq p_A\leq p_B+2\Delta$, and Zone 3 in which $p_A\leq p_B-2\Delta$. In the following we will discuss sellers' expected total profit in each zone separately.

Zone 1 - $p_B+2\Delta\leq p_A$: Since $\frac{2\Delta}{p_A-p_B}\leq 1$, sellers may or may not learn the demand in the second period. Therefore, they will be in one of the following information sets in the second period: (H,H), (L,L), and $(\overline{\boldsymbol{\theta}},\overline{\boldsymbol{\theta}})$. Based on their learning probability, the expected total profit function of each seller then is as follows:

$$
\begin{aligned}
\pi_A^{Z1} &= \pi_{A,1}(p_A,p_B)+\frac{2\Delta}{p_A-p_B}\left[\frac{1}{2}\pi_{A,2}(H,H)+\frac{1}{2}\pi_{A,2}(L,L)\right]+\left(1-\frac{2\Delta}{p_A-p_B}\right)\pi_{A,2}(\overline{\boldsymbol{\theta}},\overline{\boldsymbol{\theta}})\\
&= \pi_{A,1}(p_A,p_B)+\frac{2\Delta}{p_A-p_B}\frac{(\alpha_H-\alpha_L)^2(1+\gamma)}{4(2+3\gamma)^2}+\frac{(1+\gamma)[\gamma+(2+\gamma)\overline{\alpha}_A]^2}{(2+\gamma)^2(2+3\gamma)^2}\\
\pi_B^{Z1} &= \pi_{B,1}(p_A,p_B)+\frac{2\Delta}{p_A-p_B}\left[\frac{1}{2}\pi_{B,2}(H,H)+\frac{1}{2}\pi_{B,2}(L,L)\right]+\left(1-\frac{2\Delta}{p_A-p_B}\right)\pi_{B,2}(\overline{\boldsymbol{\theta}},\overline{\boldsymbol{\theta}})\\
&= \pi_{B,1}(p_A,p_B)+\frac{2\Delta}{p_A-p_B}\frac{(\alpha_H-\alpha_L)^2(1+\gamma)}{4(2+3\gamma)^2}+\frac{(1+\gamma)[\gamma+(2+\gamma)\overline{\alpha}_B]^2}{(2+\gamma)^2(2+3\gamma)^2}
\end{aligned}
$$

Zone 2 - $|p_A-p_B|\leq 2\Delta$: In this case, each seller will observe its own sales at the end of the first period and automatically identify the true state of the demand since $\frac{2\Delta}{|p_A-p_B|}\geq 1$. Therefore, the expected total profit function of each seller is:

$$
\begin{aligned}
\pi_A^{Z2} &= \pi_{A,1}(p_A,p_B)+\frac{1}{2}\pi_{A,2}(H,H)+\frac{1}{2}\pi_{A,2}(L,L)\\
&= \pi_{A,1}(p_A,p_B)+\frac{(1+\gamma)\left[\gamma+(2+\gamma)\alpha_H\right]^2}{2(2+\gamma)^2(2+3\gamma)^2}+\frac{(1+\gamma)[\gamma+(2+\gamma)\alpha_L]^2}{2(2+\gamma)^2(2+3\gamma)^2}\\
\pi_B^{Z2} &= \pi_{B,1}(p_A,p_B)+\frac{1}{2}\pi_{B,2}(H,H)+\frac{1}{2}\pi_{B,2}(L,L)\\
&= \pi_{B,1}(p_A,p_B)+\frac{(1+\gamma)[\gamma+(2+\gamma)(1-\alpha_H)]^2}{2(2+\gamma)^2(2+3\gamma)^2}+\frac{(1+\gamma)[\gamma+(2+\gamma)(1-\alpha_L)]^2}{2(2+\gamma)^2(2+3\gamma)^2}
\end{aligned}
$$

Zone 3 - $p_A \leq p_B - 2\Delta$: This case is similar to zone one. Since $\frac{2\Delta}{p_B-p_A} \leq 1$, sellers may or may not learn the demand in the second period. Therefore, they will be in one of the following information sets in the second period: (H,H), (L,L), and $(\overline{\theta},\overline{\theta})$. Based on their learning probability, the expected total profit function of each seller then is as follows:

$$\begin{aligned}
\pi_A^{Z3} &= \pi_{A,1}(p_A,p_B) + \tfrac{2\Delta}{p_B-p_A}\left[\tfrac{1}{2}\pi_{A,2}(H,H) + \tfrac{1}{2}\pi_{A,2}(L,L)\right] + \left(1 - \tfrac{2\Delta}{p_B-p_A}\right)\pi_{A,2}(\overline{\theta},\overline{\theta}) \\
&= \pi_{A,1}(p_A,p_B) + \tfrac{2\Delta}{p_B-p_A}\tfrac{(\alpha_H-\alpha_L)^2(1+\gamma)}{4(2+3\gamma)^2} + \tfrac{(1+\gamma)[\gamma+(2+\gamma)\overline{\alpha}_A]^2}{(2+\gamma)^2(2+3\gamma)^2} \\
\pi_B^{Z3} &= \pi_{B,1}(p_A,p_B) + \tfrac{2\Delta}{p_B-p_A}\left[\tfrac{1}{2}\pi_{B,2}(H,H) + \tfrac{1}{2}\pi_{B,2}(L,L)\right] + \left(1 - \tfrac{2\Delta}{p_B-p_A}\right)\pi_{B,2}(\overline{\theta},\overline{\theta}) \\
&= \pi_{B,1}(p_A,p_B) + \tfrac{2\Delta}{p_B-p_A}\tfrac{(\alpha_H-\alpha_L)^2(1+\gamma)}{4(2+3\gamma)^2} + \tfrac{(1+\gamma)[\gamma+(2+\gamma)\overline{\alpha}_B]^2}{(2+\gamma)^2(2+3\gamma)^2}
\end{aligned}$$

Now, to derive optimal first period prices, we need to understand the best response of each seller, and then investigate the existence and uniqueness of a pure strategy equilibrium for the sellers in the first period. Therefore, in the following we examine the best response function of each seller separately, and then we combine them to get the equilibrium structure.

Best response function of seller A: We derive the best response function of seller A. Based on the three demand learning possibilities, we first investigate the local optimum in each zone in isolation and then combine them to find the best response. For each zone, we examine the behavior of seller A's expected profit, and understand whether it is decreasing or increasing in p_A given a p_B.

Zone 1 - $p_B + 2\Delta \leq p_A$: to understand the seller A's expected profit behavior, we examine her first derivative with respect to p_A:

$$\frac{\partial \pi_A^{Z1}}{\partial p_A} = \overline{\alpha}_A + \gamma p_B - 2(1+\gamma)p_A - \frac{(\alpha_H - \alpha_L)^3(1+\gamma)}{8\gamma(2+3\gamma)^2(p_A-p_B)^2}$$

meaning that $\frac{\partial \pi_A^{Z1}}{\partial p_A}\Big|_{p_A=p+2\Delta, p_B=p} \leq 0$ if and only if:

$$p_B \geq p_B^{A1} \equiv \frac{(28\gamma^3+55\gamma^2+36\gamma+8)\alpha_L - (10\gamma^3+31\gamma^2+28\gamma+8)\alpha_H}{2\gamma(2+\gamma)(2+3\gamma)^2}.$$

In other words, seller A's expected profit is decreasing in p_A where $p_B \geq p_B^{A1}$ and increasing otherwise. Therefore, if $p_B \geq p_B^{A1}$, seller A will set the lowest possible price which is $p_A = p_B + 2\Delta$ in zone 1. Otherwise, if $p_B \leq p_B^{A1}$, then seller A would set a higher price $p_A \geq p_B + 2\Delta$ in zone 1.

Zone 2 - $|p_A - p_B| \leq 2\Delta$: analysis of this case depends on the sign of $p_A - p_B$. Therefore, we break it down as follows:

Zone 2.1. $p_A - p_B \leq 0$, $p_A \leq p_B + 2\Delta$: by investigating the first derivative of seller A's expected profit in this zone we see that:

$$\frac{\partial \pi_A^{Z2}}{\partial p_A} = \overline{\alpha}_A + \gamma p_B - 2(1+\gamma)p_A$$

meaning that $\frac{\partial \pi_A^{Z2}}{\partial p_A}\Big|_{p_A=p+2\Delta, p_B=p} \leq 0$ if and only if $p_B \geq p_B^{A2} \equiv \frac{(2+3\gamma)\alpha_L-(2+\gamma)\alpha_H}{2\gamma(2+\gamma)}$. In other words, seller A's expected profit is decreasing in p_A where $p_B \geq p_B^{A2}$ and increasing otherwise. Therefore, if $p_B \geq p_B^{A2}$, then seller A would set a price $p_A \leq p_B + 2\Delta$ in zone 2, otherwise, if $p_B \leq p_B^{A2}$ then seller A would set the highest possible price $p_A = p_B + 2\Delta$ in zone 2.

Zone 2.2. $p_A - p_B \geq 0$, $p_A \geq p_B - 2\Delta$: by investigating the first derivative of seller A's expected profit in this zone we see that $\frac{\partial \pi_A^{Z2}}{\partial p_A}\Big|_{p_A=p-2\Delta, p_B=p} \leq 0$ if and only if $p \geq p_B^{A3} \equiv \frac{(2+3\gamma)\alpha_H-(2+\gamma)\alpha_L}{2\gamma(2+\gamma)}$. In other words, seller A's expected profit is decreasing in p_A where $p_B \geq p_B^{A3}$ and increasing otherwise. Therefore, if $p_B \leq p_B^{A3}$, then seller A would set a price $p_A \geq p_B - 2\Delta$ in zone 2, otherwise, if $p_B \geq p_B^{A3}$ then seller A would set the lowest possible price $p_A = p_B - 2\Delta$ in zone 2.

Zone 3 - $p_A \leq p_B - 2\Delta$**:** this case is similar to zone 1. Again by investigating the first derivative of seller A's expected profit in this zone we see that:

$$\frac{\partial \pi_A^{Z3}}{\partial p_A} = \overline{\alpha}_A + \gamma p_B - 2(1+\gamma)p_A + \frac{(\alpha_H - \alpha_L)^3(1+\gamma)}{8\gamma(2+3\gamma)^2(p_B - p_A)^2}$$

meaning that $\frac{\partial \pi_A^{Z3}}{\partial p_A}\Big|_{p_A=p-2\Delta, p_B=p} \leq 0$ if and only if:

$$p_B \geq p_B^{A4} \equiv \frac{(28\gamma^3+55\gamma^2+36\gamma+8)\alpha_H - (10\gamma^3+31\gamma^2+28\gamma+8)\alpha_L}{2\gamma(2+\gamma)(2+3\gamma)^2}.$$

In other words, seller A's expected profit is decreasing in p_A where $p_B \geq p_B^{A4}$ and increasing otherwise. Therefore, if $p_B \geq p_B^{A4}$, then seller A would set a price $p_A \leq p_B - 2\Delta$ in zone 3, otherwise, if $p_B \leq p_B^{A4}$ then seller A would set the highest possible price $p_A = p_B - 2\Delta$ in zone 3.

Now that we have analyzed the pricing behavior of seller A, we investigate the relation between p_B's thresholds to get the structure of seller A's best response function. We see that $p_B^{A4} - p_B^{A3} = \frac{\gamma(1+\gamma)(\alpha_H-\alpha_L)}{2(2+\gamma)(2+3\gamma)^2}$ and $p_B^{A3} - p_B^{A2} = \frac{2(1+\gamma)(\alpha_H-\alpha_L)}{\gamma(2+\gamma)}$ and $p_B^{A2} - p_B^{A1} = \frac{\gamma(1+\gamma)(\alpha_H-\alpha_L)}{2(2+\gamma)(2+3\gamma)^2}$ so $p_B^{A1} \leq p_B^{A2} \leq p_B^{A3} \leq p_B^{A4}$ for $\alpha \leq \alpha_H$.Therefore, seller A's best responses can be summarized as in Table EC.2.2.

Table EC.2.2 Seller A's best response based on p_B range

p_B range	Seller A's expected profit's behavior	Seller A's best response
$p_B \leq p_B^{A1}$	• π_A^{Z3} increasing for $p_A \leq p_B - 2\Delta$, • π_A^{Z2} increasing for $p_B - 2\Delta \leq p_A \leq p_B + 2\Delta$, • π_A^{Z1} increasing-decreasing for $p_A \leq p_B - 2\Delta$.	$\frac{\partial \pi_A^{Z1}}{\partial p_A} = 0$ in zone 1
$p_B \in [p_B^{A1}, p_B^{A2}]$	• π_A^{Z3} increasing for $p_A \leq p_B - 2\Delta$, • π_A^{Z2} increasing for $p_B - 2\Delta \leq p_A \leq p_B + 2\Delta$, • π_A^{Z1} decreasing for $p_A \leq p_B - 2\Delta$.	$p_A = p_B + 2\Delta$
$p_B \in [p_B^{A2}, p_B^{A3}]$	• π_A^{Z3} increasing for $p_A \leq p_B - 2\Delta$, • π_A^{Z2} increasing-decreasing for $p_B - 2\Delta \leq p_A \leq p_B + 2\Delta$, • π_A^{Z1} decreasing for $p_A \leq p_B - 2\Delta$.	$\frac{\partial \pi_A^{Z2}}{\partial p_A} = 0$ in zone 2
$p_B \in [p_B^{A3}, p_B^{A4}]$	• π_A^{Z3} increasing for $p_A \leq p_B - 2\Delta$, • π_A^{Z2} decreasing for $p_B - 2\Delta \leq p_A \leq p_B + 2\Delta$, • π_A^{Z1} decreasing for $p_A \leq p_B - 2\Delta$.	$p_A = p_B - 2\Delta$
$p_B^{A4} \leq p_B$	• π_A^{Z3} increasing-decreasing for $p_A \leq p_B - 2\Delta$, • π_A^{Z2} decreasing for $p_B - 2\Delta \leq p_A \leq p_B + 2\Delta$, • π_A^{Z1} decreasing for $p_A \leq p_B - 2\Delta$.	$\frac{\partial \pi_A^{Z3}}{\partial p_A} = 0$ in zone 3

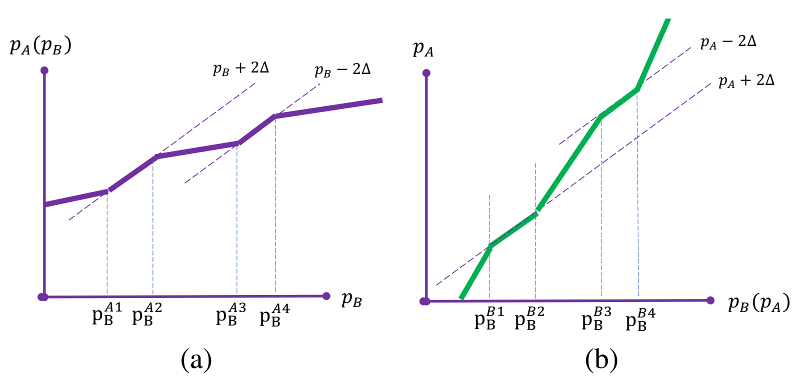
Best response function of seller B: The analysis for seller B will be similar to the one for seller A: in each zone we will examine the behavior of the expected total function. Therefore, we skip the algebra (it is available from authors upon request) and present seller B's best responses in Table EC.2.3.

Table EC.2.3 Seller B's best response based on p_A range

p_A **range**	**Seller B's expected profit's behavior**	**Seller B's best response**
$p_B^{B4} - 2\Delta \leq p_A$	• π_B^{Z1} increasing-decreasing for $p_B \leq p_A - 2\Delta$, • π_B^{Z2} decreasing for $p_A - 2\Delta \leq p_B \leq p_A + 2\Delta$, • π_B^{Z3} decreasing for $p_A + 2\Delta \leq p_B$.	$\frac{\partial \pi_B^{Z1}}{\partial p_B} = 0$ in zone 1
$p_B^{B3} - 2\Delta \leq p_A \leq p_B^{B4} - 2\Delta$	• π_B^{Z1} increasing for $p_B \leq p_A - 2\Delta$, • π_A^{Z2} decreasing for $p_B - 2\Delta \leq p_A \leq p_B + 2\Delta$, • π_A^{Z1} decreasing for $p_A \leq p_B - 2\Delta$.	$p_B = p_A - 2\Delta$
$p_B^{B2} + 2\Delta \leq p_A \leq p_B^{B3} - 2\Delta$	• π_B^{Z1} increasing for $p_B \leq p_A - 2\Delta$, • π_B^{Z2} increasing-decreasing for $p_A - 2\Delta \leq p_B \leq p_A + 2\Delta$, • π_B^{Z3} decreasing for $p_A + 2\Delta \leq p_B$.	$\frac{\partial \pi_B^{Z2}}{\partial p_B} = 0$ in zone 2
$p_B^{B1} + 2\Delta \leq p_A \leq p_B^{B2} + 2\Delta$	• π_B^{Z1} increasing for $p_B \leq p_A - 2\Delta$, • π_B^{Z2} increasing for $p_A - 2\Delta \leq p_B \leq p_A + 2\Delta$, • π_B^{Z3} decreasing for $p_A + 2\Delta \leq p_B$.	$p_B = p_A + 2\Delta$
$p_A \leq p_B^{B1} + 2\Delta$	• π_B^{Z1} increasing for $p_B \leq p_A - 2\Delta$, • π_B^{Z2} increasing for $p_A - 2\Delta \leq p_B \leq p_A + 2\Delta$, • π_B^{Z3} increasing-decreasing for $p_A + 2\Delta \leq p_B$,	$\frac{\partial \pi_B^{Z3}}{\partial p_B} = 0$ in zone 3

Figure EC.2.5 illustrates the best response functions of the two sellers.

Figure EC.2.5 Best response functions for seller A and seller B

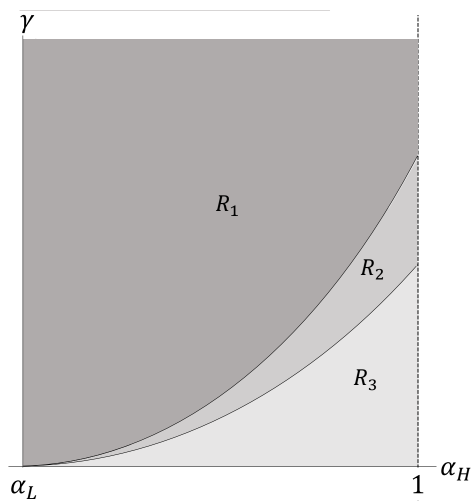


Equilibrium structure: Now that we have both sellers' best response functions, we are able to derive the equilibrium structure. To do so, we superimpose the two best response functions based on the values of exogenous problem parameters; namely α_H, α_L, and γ. Note that $p_B^{A3} - p_B^{B2} = \frac{(2+5\gamma)\alpha_H - (2+\gamma)\alpha_L - 2\gamma}{2\gamma(2+\gamma)} \geq 0$ for $\frac{1}{2} \leq \alpha_L \leq \alpha_H$. We first divide the space of possible parameters combinations to understand the characteristics of equilibrium in each particular set of parameters, then we examine the equilibrium prices in each region.

1. **Region 1,** $\alpha_H \leq \alpha_{H1} \equiv \frac{(47\gamma^3+80\gamma^2+44\gamma+8)\alpha_L-2\gamma(2+3\gamma)^2}{11\gamma^3+32\gamma^2+28\gamma+8}$: in this region, $p_B^{B4} \leq p_B^{A1}$. It satisfies the necessary condition in zone 1 for seller A and B (first row in Table EC.2.2 and EC.2.3). Hence, there exists a unique pure strategy equilibrium which satisfies $\frac{\partial \pi_B^{Z1}}{\partial p_B} = 0$ and $\frac{\partial \pi_A^{Z1}}{\partial p_A} = 0$.
2. **Region 2.1,** $\alpha_{H1} \leq \alpha_H \leq \alpha_{H2} \equiv \frac{(46\gamma^3+79\gamma^2+44\gamma+8)\alpha_L-2\gamma(2+3\gamma)^2}{10\gamma^3+31\gamma^2+28\gamma+8}$: in this region $p_B^{B3} \leq p_B^{A1} \leq p_B^{B4} \leq p_B^{A2}$ meaning that there exists a continuum of equilibrium points satisfying $p_B = p, p_A = p + 2\Delta$ for $p \in [p_B^{A1}, p_B^{B4}]$
3. **Region 2.2,** $\alpha_{H2} \leq \alpha_H \leq \alpha_{H3} \equiv \frac{(2+5\gamma)\alpha_L-2\gamma}{2+\gamma}$: in this region $p_B^{A1} \leq p_B^{B3} \leq p_B^{A2} \leq p_B^{B4}$. meaning that there exists a continuum of equilibrium points satisfying $p_B = p, p_A = p + 2\Delta$ for $p \in [p_B^{B3}, p_B^{A2}]$
4. **Region 3,** $\alpha_{H3} \leq \alpha_H$: in this region $p_B^{A2} \leq p_B^{B3}$ meaning that there exists a unique pure strategy equilibrium which satisfies $\frac{\partial \pi_B^{Z2}}{\partial p_B} = 0$ & $\frac{\partial \pi_A^{Z2}}{\partial p_A} = 0$.

Figure EC.2.6 shows the corresponding regions above. Now we characterize the equilibrium in each region separately.

Figure EC.2.6 Regions based on the value of α_H and γ



Characterizing equilibrium in Region 1 ($\alpha_L \le \alpha_H \le \alpha_{H1}$): As we discussed, the equilibrium in this region will satisfy $\frac{\partial \pi_B^{Z1}}{\partial p_B} = 0$ and $\frac{\partial \pi_A^{Z1}}{\partial p_A} = 0$. It is the same as to say that the first order conditions must be satisfied. These constraints which must be satisfied by the equilibrium solution are as follows:

$$\overline{\alpha}_A + \gamma p_B - 2(1+\gamma)p_A - \frac{(\alpha_H - \alpha_L)^3(1+\gamma)}{8\gamma(2+3\gamma)^2(p_A - p_B)^2} = 0 \tag{EC.2.7}$$

$$\overline{\alpha}_B + \gamma p_A - 2(1+\gamma)p_B + \frac{(\alpha_H - \alpha_L)^3(1+\gamma)}{8\gamma(2+3\gamma)^2(p_A - p_B)^2} = 0 \tag{EC.2.8}$$

To solve this system of equation we add and subtract them.

- Adding Equations (EC.2.7) and (EC.2.8): $\overline{\alpha}_A + \overline{\alpha}_B - (2+\gamma)(p_A + p_B) = 0$ and therefore, $p_A + p_B = \frac{\overline{\alpha}_A + \overline{\alpha}_B}{2+\gamma} = \frac{1}{2+\gamma}$.
- Taking the difference of Equations (EC.2.7) and (EC.2.8): $\overline{\alpha}_A - \overline{\alpha}_B - (2+3\gamma)(p_A - p_B) - \frac{(\alpha_H - \alpha_L)^3(1+\gamma)}{4\gamma(2+3\gamma)^2(p_A - p_B)^2} = 0$.

Now, our problem is reduced to find p_A and p_B which satisfy the following constraint.

$$(p_A^{\emptyset} - p_B^{\emptyset})^3 - \frac{\alpha_H + \alpha_L - 1}{2+3\gamma}(p_A^{\emptyset} - p_B^{\emptyset})^2 + \frac{(\alpha_H - \alpha_L)^3(1+\gamma)}{4\gamma(2+3\gamma)^2} = 0 \tag{EC.2.9}$$

To do so, we can rewrite the equation as $x^3 - \frac{\alpha_H + \alpha_L - 1}{2+3\gamma}x^2 + \frac{(\alpha_H - \alpha_L)^3(1+\gamma)}{4\gamma(2+3\gamma)^3} = 0$ where $x \equiv p_A - p_B$. In other words, we are looking for the root of this function, which gives us the price difference, instead of searching for the two price decisions separately. However, it is not possible to write the roots of this cubic function algebraically in closed form, but possible in trigonometric form as following:

$$\begin{aligned} x_j &= \frac{\alpha_H + \alpha_L - 1}{3(2+3\gamma)}\left[1 - 2\cos\left(\frac{\arccos\left(-1 + \frac{27(\alpha_H - \alpha_L)^3(1+\gamma)}{8\gamma(\alpha_H + \alpha_L - 1)^3}\right) - 2\pi j}{3}\right)\right] \\ &= \frac{\alpha_H + \alpha_L - 1}{3(2+3\gamma)}\kappa(n,\gamma,j) \text{ for } j = 0,1,2 \end{aligned}$$

where $n \equiv \frac{\alpha_H - \alpha_L}{\alpha_H + \alpha_L - 1}$ and

$$\kappa(n,\gamma,j) \equiv 1 - 2\cos\left(\frac{\arccos\left(-1 + \frac{27(1+\gamma)}{8\gamma}n^3\right) - 2\pi j}{3}\right). \tag{EC.2.10}$$

Up to this point, we have found that $p_A + p_B = \frac{1}{2+\gamma}$ and $p_A - p_B = x_j$. However, the analysis is not done yet. We need to check whether x_j roots are real or not, and ensure that it satisfies the condition in zone 1 and region 1. To do so we need to check if x_j is real and if it is compatible with Region 1 and Zone 1 conditions.

1. Checking x_j to be real: To do so, we need to examine the sign of the discriminant of the cubic equation which is $\frac{n^3(1+\gamma)}{\gamma}(1 - \frac{27n^3(1+\gamma)}{16\gamma})$. If the discriminant is negative, then there is 1 real root ($j = 0$) and 2 complex conjugate roots ($j = 1,2$). If the discriminant is positive, $n \le \left(\frac{16\gamma}{27(1+\gamma)}\right)^{\frac{1}{3}}$, then there are three distinct real roots.

2. Check if these roots are compatible with Region 1 conditions: In the analysis of Region 1, we have $\alpha_H \le \alpha_{H1}$ which transforms to $n \le \frac{2\gamma(2+3\gamma)^2}{8+36\gamma+56\gamma^2+29\gamma^3}$. Moreover, this is also the tighter bound, i.e., $\frac{2\gamma(2+3\gamma)^2}{8+36\gamma+56\gamma^2+29\gamma^3} \le \left(\frac{16\gamma}{27(1+\gamma)}\right)^{\frac{1}{3}}$. So, when we are analyzing Region 1, parameters of interest are: $\gamma > 0$ and $\alpha_L \le \alpha_H \le \alpha_{H1}$ or equivalently $0 \le n \le \frac{2\gamma(2+3\gamma)^2}{8+36\gamma+56\gamma^2+29\gamma^3}$. So, all three roots are real for parameters that are in Region 1.

3. Check if these roots are in Zone 1 , i.e., $p_A > p_B + 2\Delta$: we show that the first two roots ($j = 0, 1$) fail to satisfy this condition and that the third root ($j = 2$) satisfies this condition only if $\alpha_H \le \alpha_{H1}$.

   (a) The first root ($j = 0$) fails to satisfy $x_0 \ge 0$ (i.e., $p_A \ge p_B$) since $\kappa(n,\gamma,j)$ is negative for any $n \le \left(\frac{16\gamma}{27(1+\gamma)}\right)^{1/3}$.

   (b) We now show that $x_1 \le 2\Delta$ (i.e., $p_A - p_B \le 2\Delta$). Note that $x_1 - 2\Delta = \frac{\alpha_H+\alpha_L-1}{3(2+3\gamma)}\kappa(n,\gamma,1) - \frac{\alpha_H-\alpha_L}{2\gamma} = \frac{\alpha_H+\alpha_L-1}{3(2+3\gamma)}\left[\kappa(n,\gamma,1) - \frac{3(2+3\gamma)n}{2\gamma}\right]$

   The function $\kappa(n,\gamma,1) - \frac{3(2+3\gamma)n}{2\gamma}$ can be complicated to analyze, but we can actually plot it (via a computer software) for all values of γ and n that satisfy our constraints, i.e., $\gamma > 0$ and $n \in [0, \left(\frac{16\gamma}{27(1+\gamma)}\right)^{\frac{1}{3}}]$ and observe that it is negative meaning that $x_1 = p_A - p_B \le 2\Delta$.

   (c) Last claim is that $x_2 \ge 2\Delta$ for $\alpha_H \le \alpha_{H1}$. Note that $x_2 - 2\Delta = \frac{\alpha_H+\alpha_L-1}{3(2+3\gamma)}\kappa(n,\gamma,2) - \frac{\alpha_H-\alpha_L}{2\gamma} = \frac{\alpha_H+\alpha_L-1}{3(2+3\gamma)}\left[\kappa(n,\gamma,2) - \frac{3(2+3\gamma)n}{2\gamma}\right]$.

   Our approach is to investigate $K_1(\gamma,n) \equiv \kappa(n,\gamma,2) - \frac{3(2+3\gamma)n}{2\gamma}$ for all values of γ, n that satisfy our constraints, i.e., $\gamma > 0$ and $n \in [0, \left(\frac{16\gamma}{27(1+\gamma)}\right)^{\frac{1}{3}}]$ and observe that it is positive only for $\alpha_H \le \alpha_{H1}$.

   So, this proves that out of the three real roots, i.e., three potential equilibrium solutions in Zone 1, only one (x_2) satisfies the boundary conditions. Therefore, we showed:

$$\begin{cases} \frac{p_A+p_B}{2} = \frac{1}{2(2+\gamma)} = \overline{p}^{\emptyset} \\ \frac{p_A-p_B}{2} = \frac{\alpha_H+\alpha_L-1}{3(2+3\gamma)}\kappa(n,\gamma,2) = 2z_p^{\emptyset}\Delta \end{cases}$$

Hence, the first period prices in Region 1 can be written as $p_A = \overline{p}^{\emptyset} + z_p^{\emptyset}\Delta$ and $p_B = \overline{p}^{\emptyset} - z_p^{\emptyset}\Delta$.

Characterizing equilibrium in Region 2 ($\alpha_{H1} \le \alpha_H \le \alpha_{H3}$): In region 2, if $\alpha_{H1} \le \alpha_H \le \alpha_{H2}$ we have a continuum of equilibrium solutions $(p + 2\Delta, p)$ for $p \in [p_B^{A1}, p_B^{B4}]$ and for $\alpha_{H2} \le \alpha_H \le \alpha_{H3}$ we have a continuum of equilibrium solutions $(p + 2\Delta, p)$ for $p \in [p_B^{B3}, p_B^{A2}]$. We can show that there exists a unique Pareto dominant equilibrium in each of these cases. The discussion is omitted due to space limitations but is available from the authors upon request.

Characterizing equilibrium in Region 3 ($\alpha_{H3} \le \alpha_H$): In region 3, we know that the first period prices are such that, $|p_A - p_B| \le 2\Delta$ and both sellers can identify the true state of the demand. Therefore, in the second period they will set prices with exact demand function as shown in Eq. (EC.2.1). This means that the second period profits are constant with respect to the first period prices. Consequently, we can use backward induction to find the first period prices. Solving first order conditions $\frac{\partial \pi_A^{Z2}(p_A,p_B)}{\partial p_A} = 0$, $\frac{\partial \pi_B^{Z2}(p_A,p_B)}{\partial p_A} = 0$ gives us $p_A = \frac{\gamma+(2+\gamma)[\alpha_H+\alpha_L]}{2(2+\gamma)(2+3\gamma)}$ and $p_B = \frac{\gamma+(2+\gamma)[2-\alpha_H-\alpha_L]}{2(2+\gamma)(2+3\gamma)}$. However, prices should also satisfy zone 2 condition: $|p_A - p_B| \le 2\Delta$. Note that $|p_A - p_B| \le 2\Delta$ if and only if $\alpha_H \ge \alpha_{H3}$.

Summary of the equilibrium under no information sharing:

Now that we have characterized equilibrium prices in each region, we summarize them in this section. First period prices in each region under no information sharing are written in the following format:

$$p_{A,1}^{\emptyset} = \overline{p}^{\emptyset} + z_p^{\emptyset}\Delta \quad \text{and} \quad p_{B,1}^{\emptyset} = \overline{p}^{\emptyset} - z_p^{\emptyset}\Delta \tag{EC.2.11}$$

where $\overline{p}^{\emptyset}$ is the price average, $2\Delta z_p^{\emptyset}$ is the price difference between sellers in each region as follows.

$$\overline{p}^{\emptyset} = \begin{cases} \frac{1}{2(2+\gamma)} & \text{in Region 1} \\ min\{\frac{\alpha_L(2+5\gamma)-\alpha_H(2+\gamma)}{4\gamma(2+\gamma)}, \\ \frac{4(2+3\gamma)^2+(8+30\gamma+32\gamma^2+9\gamma^3)\alpha_H-(24+78\gamma+68\gamma^2+9\gamma^3)\alpha_L}{4\gamma(2+\gamma)(2+3\gamma)^2}\} & \text{in Region 2} \\ \frac{1}{2(2+\gamma)} & \text{in Region 3} \end{cases} \tag{EC.2.12}$$

$$z_p^{\emptyset} = \begin{cases} \frac{1}{3}\left[1-2cos\left(\frac{\arccos\left(-1+\frac{27(1+\gamma)}{8\gamma}\left(\frac{\alpha_H-\alpha_L}{\alpha_H+\alpha_L-1}\right)^3\right)-4\pi}{3}\right)\right]\frac{2\gamma}{2+3\gamma}\frac{\alpha_H+\alpha_L-1}{\alpha_H-\alpha_L} & \text{in Region 1} \\ 1 & \text{in Region 2} \\ \frac{2\gamma}{2+3\gamma}\frac{\alpha_H+\alpha_L-1}{\alpha_H-\alpha_L} & \text{in Region 3} \end{cases} \tag{EC.2.13}$$

$$\begin{cases} \text{Region 1} & \alpha_H \leq \alpha_H^* \equiv \frac{(47\gamma^3+80\gamma^2+44\gamma+8)\alpha_L-2\gamma(2+3\gamma)^2}{11\gamma^3+32\gamma^2+28\gamma+8} \\ \text{Region 2} & \alpha_H^* \leq \alpha_H \leq \alpha_H^{**} \equiv \frac{(2+5\gamma)\alpha_L-2\gamma}{2+\gamma} \\ \text{Region 3} & \alpha_H^{**} \leq \alpha_H \end{cases} \tag{EC.2.14}$$

Note that $z_p^{\emptyset} \geq 1$ in Region 1, meaning that the price difference $p_{A,1}^{\emptyset} - p_{B,1}^{\emptyset} = 2z_p^{\emptyset} \geq 2\Delta$ and therefore, sellers may or may not identify the true state of the demand depending on their demand observation. At the end of the first period, sellers update their beliefs as described in Figure EC.2.4(b). In region 2, we have $z_p^{\emptyset} = 1$, and in region 3, we have $z_p^{\emptyset} \leq 1$, meaning that the price difference $p_{A,1}^{\emptyset} - p_{B,1}^{\emptyset} = 2z_p^{\emptyset} \leq 2\Delta$ and therefore, sellers identify the true state of the demand independent of their demand observation.

Designing Subscription Fee to Induce $J = \emptyset$: To design a subscription fee to induce both sellers not to opt CIS service, the two following incentive compatibility constraints (based on Table EC.2.1) should be met: First, given seller A's decision not to subscribe to CIS, seller B has also incentive not to subscribe, and second, given seller B's decision not to subscribe to CIS, then seller A has also no incentive to subscribe. We can write the two constraints as follows:

$$(1-\beta)\pi_{C|B}^{\emptyset}\left(P_B^{\emptyset}, P_A^{\emptyset}\right) \geq (1-\beta)\pi_{C|B}^{B}\left(P_B^{B}, P_A^{B}\right) - \phi^{\emptyset} \tag{EC.2.15}$$

$$(1-\beta)\pi_{C|A}^{\emptyset}\left(P_A^{\emptyset}, P_B^{\emptyset}\right) \geq (1-\beta)\pi_{C|A}^{A}\left(P_A^{A}, P_B^{A}\right) - \phi^{\emptyset} \tag{EC.2.16}$$

Note that, constraints (EC.2.15) and (EC.2.16) define two thresholds for the subscription fee $\phi^{\emptyset}$;

$$\phi^{\emptyset} \geq S_B^{\emptyset} = (1-\beta)[\pi_{C|B}^{B} - \pi_{C|B}^{\emptyset}], \quad \text{and} \quad \phi^{\emptyset} \geq S_A^{\emptyset} = (1-\beta)[\pi_{C|A}^{A} - \pi_{C|A}^{\emptyset}].$$

Therefore, to satisfy both incentive compatibility constraints, any subscription fee $\phi \in \left[\max\{S_B^{\emptyset}, S_A^{\emptyset}\}, +\infty\right)$ can induce both sellers not to subscribe to CIS.

Consumer Surplus under No Information Sharing: To get the consumer surplus under no information sharing, we first break it down to the consumer surplus in the first and second period. In the first period, consumer surplus will be acquired through the following formulation with corresponding prices derived in Eq. (EC.2.12).

$$\begin{aligned} CS_1^{\emptyset} &= E_{\theta}\left[\sum_{i\in\{A,B\}} CS_{i,1}^{\emptyset}(\theta)\right] \\ &= \frac{1}{2}\left(\frac{(d_{A,1}^*(H))^2}{2(1+\gamma)} + \frac{(d_{A,1}^*(L))^2}{2(1+\gamma)}\right) + \frac{1}{2}\left(\frac{(d_{B,1}^*(H))^2}{2(1+\gamma)} + \frac{(d_{B,1}^*(L))^2}{2(1+\gamma)}\right) \end{aligned}$$

In the second period, sellers will either learn the demand with $\frac{2\Delta}{|p_A - p_B|}$ probability and the consumer surplus will be derived with exact demand function and optimal prices, or they do not learn the demand and hence the consumer surplus will follow the mean demand function and corresponding optimal prices. We have derived these two consumer surpluses in §EC.2.2. Therefore, in the second period, consumer surplus under $J = \emptyset$ will be $CS_2^{\emptyset} = \frac{2\Delta}{|p_A - p_B|} CS_{exact} + (1 - \frac{2\Delta}{|p_A - p_B|}) CS_{mean}$.

Finally, total consumer surplus under $J = \emptyset$ will be $CS^{\emptyset} = CS_1^{\emptyset} + \frac{2\Delta}{|p_A - p_B|} CS_{exact} + (1 - \frac{2\Delta}{|p_A - p_B|}) CS_{mean}$.

EC.2.5. Proposition 5

In this section, we compare platform profit under $J = AB$ and $J = \emptyset$, and prove that there exists a threshold $\alpha_H^{AB\to\emptyset}$ above which platform prefers $J = \emptyset$ over $J = AB$.

Proof of Proposition 5: First we consider the difference between platform profit under $J = AB$ and $J = \emptyset$. To simplify equations we use Eq. (EC.2.9).

$$\begin{aligned}\Delta_O^{AB-\emptyset} &\equiv \pi_O^{AB} - \pi_O^{\emptyset}\\ &= \frac{(1+2\gamma)(p_A^{\emptyset}-p_B^{\emptyset})^2}{2} - \frac{(\alpha_H+\alpha_L-1)(p_A^{\emptyset}-p_B^{\emptyset})}{2} + \frac{(1+\gamma)[(\alpha_H+\alpha_L-1)^2+(\alpha_H-\alpha_L)^2]}{2(2+3\gamma)^2}\\ &\quad -\frac{(1+\gamma)(\alpha_H-\alpha_L)^3}{4\gamma(2+3\gamma)^2(2z_p^{\emptyset}\Delta)}\\ &= \frac{(2\alpha_L-1)^2}{324\gamma(2+3\gamma)^2(1-n)^2\kappa(n,\gamma,2)}\Big[2\gamma(1+2\gamma)\kappa^3(n,\gamma,2) - 18\gamma(2+3\gamma)\kappa^2(n,\gamma,2)\\ &\quad +162\gamma(1+\gamma)(1+n^2)\kappa(n,\gamma,2) - 729n^3(1+\gamma)(2+3\gamma)\Big]\end{aligned}$$

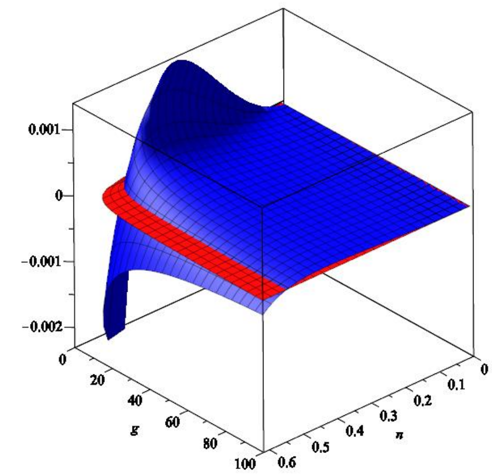
Note that $\Delta_O^{AB-\emptyset} \geq 0$ if and only if $\Omega_O(n,\gamma) > 0$, where:

$$\Omega_O(n,\gamma) \equiv 2\gamma(1+2\gamma)\kappa^3(n,\gamma,2) - 18\gamma(2+3\gamma)\kappa^2(n,\gamma,2) + 162\gamma(1+\gamma)(1+n^2)\kappa(n,\gamma,2) - 729n^3(1+\gamma)(2+3\gamma).$$

Therefore, we need to examine the behavior of $\Omega_O(n,\gamma)$. Since it is a cubic function of n, we start by $\frac{\partial^3\Omega_O(n,\gamma)}{\partial n^3}$. Observe that $\frac{\partial^3\Omega_O(n,\gamma)}{\partial n^3} < 0$ for all $0 < \gamma$ and $0 \leq n \leq \frac{2\gamma(2+3\gamma)^2}{8+36\gamma+56\gamma^2+29\gamma^3}$. Therefore, $\frac{\partial^2\Omega_O(n,\gamma)}{\partial n^2}$ is decreasing in Region 1. Therefore we need to check the endpoints of $\frac{\partial^2\Omega_O(n,\gamma)}{\partial n^2}$ to see whether it changes sign or not. We see that $\frac{\partial^2\Omega_O(n,\gamma)}{\partial n^2}\Big|_{n=0} = \frac{1+\gamma}{4+12\gamma+9\gamma^2} > 0$ and $\frac{\partial^2\Omega_O(n,\gamma)}{\partial n^2}\Big|_{n=n^{UB}} < 0$. Hence, $\frac{\partial^2\Omega_O(n,\gamma)}{\partial n^2}$ changes sign from positive to negative. Therefore, $\frac{\partial\Omega_O(n,\gamma)}{\partial n}$ is increasing-decreasing in n. To check whether $\frac{\partial\Omega_O(n,\gamma)}{\partial n}$ changes sign or not, we look at its end points. We see that $\frac{\partial\Omega_O(n,\gamma)}{\partial n}\Big|_{n=0} = 0$ and $\frac{\partial\Omega_O(n,\gamma)}{\partial n}\Big|_{n=n^{UB}} < 0$. Therefore, $\frac{\partial\Omega_O(n,\gamma)}{\partial n}$ changes sign from positive to negative.

Based on this, we understand that $\Omega_O(n,\gamma)$ is an increasing-decreasing function of n. So, we need to check the sign of its end points to see whether it is always positive or always negative, or it changes sign once or twice. Looking at the end points, we first see that $\Omega_O(n,\gamma)\Big|_{n=0} = 0$. Since we showed that $\Omega_O(n,\gamma)$ is an increasing-decreasing function of n, so there is at most one sign change point for $\Omega_O(n,\gamma)$. We observe that $\Omega_O(n,\gamma)\Big|_{n=n^{UB}} < 0$. This means that there exists a unique root $n^*(\gamma) \in [0, n^{UB}]$ such that for $n \leq n^*(\gamma)$ we have $\Omega_O(n,\gamma) > 0$ and vice versa. In addition, we know that n is monotone increasing in α_H hence we can conclude that $\Omega_O(n,\gamma)$ is increasing-decreasing in α_H. Consequently, there exists a single sign change (say at $\alpha_H^{AB\to\emptyset}$) for $\Omega_O(n,\gamma)$ as illustrated in Figure EC.2.7.

Figure EC.2.7 Sign of $\Omega_O(n,\gamma)$. The red coloured plate is where the vertical axis is 0.



Hence, we showed that there exists a threshold $\alpha_H^{AB\to\emptyset}$ above which platform prefers $J=\emptyset$ over $J=AB$ and the proof is complete. The corresponding equilibrium subscription fees, prices, and belief-updating rules are summarized in Table EC.2.4.

Table EC.2.4 Equilibrium Decisions Under AB and ϕ Regimes

Regions	$z_p>1$		$z_p=1$	$z_p<1$
	R_1	R_2	R_3	R_4
J^*	AB	$\emptyset$	$\emptyset$	$\emptyset$
ϕ	$(1-\beta)\left[\pi_{C\vert B}^{AB}\left(P_B^{AB},P_A^{AB}\right)-\pi_{C\vert B}^{A}\left(P_B^{A},P_A^{A}\right)\right]$	$\phi\in\left[\max\{S_B^{\emptyset},S_A^{\emptyset}\},+\infty\right)$		
First period prices	$\begin{cases}p_{A,1}=\frac{\gamma+(2+\gamma)(\alpha_H+\alpha_L)}{2(2+\gamma)(2+3\gamma)}\\ p_{B,1}=\frac{4(1+\gamma)-(2+\gamma)(\alpha_H+\alpha_L)}{2(2+\gamma)(2+3\gamma)}\end{cases}$	$\begin{cases}p_{A,1}=\overline{p}^{\emptyset}+z_p^{\emptyset}\Delta\\ p_{B,1}=\overline{p}^{\emptyset}-z_p^{\emptyset}\Delta\end{cases}$	$\begin{cases}p_{A,1}=\overline{p}^{\emptyset}+\Delta\\ p_{B,1}=\overline{p}^{\emptyset}-\Delta\end{cases}$	$\begin{cases}p_{A,1}=\frac{\gamma+(2+\gamma)(\alpha_H+\alpha_L)}{2(2+\gamma)(2+3\gamma)}\\ p_{B,1}=\frac{4(1+\gamma)-(2+\gamma)(\alpha_H+\alpha_L)}{2(2+\gamma)(2+3\gamma)}\end{cases}$
Belief updating	$\boldsymbol{\theta}_i=\boldsymbol{\theta}$ for both sellers	$\theta_i=\begin{cases}H & \text{if } \epsilon_1\le\frac{2\Delta}{p_{A,1}-p_{B,1}} \text{ and } \boldsymbol{\theta}=H\\ L & \text{if } \epsilon_1\ge 1-\frac{2\Delta}{p_{A,1}-p_{B,1}} \text{ and } \boldsymbol{\theta}=L\\ \bar{\boldsymbol{\theta}} & \text{otherwise,}\end{cases}$	$\boldsymbol{\theta}_i=\boldsymbol{\theta}$ for both sellers	
Second period prices	$\begin{cases}p_{A,2}=\frac{\gamma+(2+\gamma)\alpha_A(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)}\\ p_{B,2}=\frac{\gamma+(2+\gamma)\alpha_B(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)}\end{cases}$	$\begin{cases}\boldsymbol{\theta}_i=\boldsymbol{\theta} & :\begin{cases}p_{A,2}=\frac{\gamma+(2+\gamma)\alpha_A(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)}\\ p_{B,2}=\frac{\gamma+(2+\gamma)\alpha_B(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)}\end{cases}\\ \boldsymbol{\theta}_i=\bar{\boldsymbol{\theta}} & :\begin{cases}p_{A,2}=\frac{\gamma+(2+\gamma)(\alpha_H+\alpha_L)}{2(2+\gamma)(2+3\gamma)}\\ p_{B,2}=\frac{4(1+\gamma)-(2+\gamma)(\alpha_H+\alpha_L)}{2(2+\gamma)(2+3\gamma)}\end{cases}\end{cases}$	$\begin{cases}p_{A,2}=\frac{\gamma+(2+\gamma)\alpha_A(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)}\\ p_{B,2}=\frac{\gamma+(2+\gamma)\alpha_B(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)}\end{cases}$	

Note: $\overline{p}^{AB}=\overline{p}^{\emptyset}=\frac{1}{2(2+\gamma)}$, $z_p^{\emptyset}=\left(\frac{1}{3}-\frac{2}{3}\cos\left(\frac{1}{3}\arccos\left(-1+\frac{27(1+\gamma)}{8\gamma}\left(\frac{\alpha_H-\alpha_L}{\alpha_H+\alpha_L-1}\right)^3\right)-\frac{4}{3}\pi\right)\right)\frac{2\gamma}{2+3\gamma}\frac{\alpha_H+\alpha_L-1}{\alpha_H-\alpha_L}$, $z_p^{AB}=\frac{2\gamma(\alpha_H+\alpha_L-1)}{(2+3\gamma)(\alpha_H-\alpha_L)}$, $\Delta=\frac{\alpha_H-\alpha_L}{4\gamma}$.

EC.2.6. Proposition 6

We compare sellers' equilibrium characteristics and consumer surplus preferences under regimes AB and $\emptyset$.

Proof of Proposition 6: Since the two regimes are identical in Region 3, we do not consider Region 3 in this analysis, and mostly focus on Region 1 where $\frac{1}{2} \leq \alpha_L \leq \alpha_H \leq \alpha_{H1}$.

Price comparison of sellers under AB **and** $\emptyset$**:** In the second period, under regime $J = AB$ or under regime $J = \emptyset$ when sellers learn, their prices follow Eq. (EC.2.1). If they do not learn under $J = \emptyset$, they will set prices based on Eq. (EC.2.2) in the second period. Therefore, we focus on first period prices. Recall that:

$$p_{A,1}^{AB} = \overline{p}^{AB} + z_p^{AB}\Delta, \quad \text{and} \quad p_{B,1}^{AB} = \overline{p}^{AB} - z_p^{AB}\Delta,$$
$$p_{A,1}^{\emptyset} = \overline{p}^{\emptyset} + z_p^{\emptyset}\Delta, \quad \text{and} \quad p_{B,1}^{\emptyset} = \overline{p}^{\emptyset} - z_p^{\emptyset}\Delta$$

where $\overline{p}^{AB} = \overline{p}^{\emptyset} = \frac{1}{2(2+\gamma)}$, and $z_p^{AB} = \frac{2\gamma(\alpha_H+\alpha_L-1)}{(2+3\gamma)(\alpha_H-\alpha_L)}$, and $z_p^{\emptyset} = \frac{2\gamma}{3(2+3\gamma)}\frac{\alpha_H+\alpha_L-1}{\alpha_H-\alpha_L}\kappa(n,\gamma,2)$.

Now, to analyze prices, we need to understand the behavior of $\kappa(n,\gamma,2)$, because price average in both cases are similar. We check the end points and the derivative of $\kappa(n,\gamma,2)$ with respect to γ and n.

With Eq. (EC.2.10), observe that $\kappa(n,\gamma,2)$ is decreasing in n. In addition, $\kappa(0,\gamma,2) = 3$, and $\kappa(\frac{2\gamma(2+3\gamma)^2}{8+36\gamma+56\gamma^2+29\gamma^3},\gamma,2)$ is monotone decreasing in γ and converges to $1 + \sqrt{3}cos\left(\frac{arcsin(4706/24389)}{3}\right) + sin\left(\frac{arcsin(4706/24389)}{3}\right) \approx 2.7932$ as $\gamma \to \infty$. It means that in Region 1, $2.7932 \leq \kappa(n,\gamma,2) \leq 3$. It follows from there that:

$$p_{A,1}^{AB} - p_{A,1}^{\emptyset} = \frac{2\gamma\Delta}{3n(2+3\gamma)}[3-\kappa(n,\gamma,2)] \geq 0 \quad \text{and} \quad p_{B,1}^{AB} - p_{B,1}^{\emptyset} = \frac{2\gamma\Delta}{3n(2+3\gamma)}[\kappa(n,\gamma,2)-3] \leq 0.$$

Hence, we showed that seller A's first period price is higher under $J = AB$ than $J = \emptyset$ and it is the opposite for seller B.

Profit comparison of sellers under AB **and** $\emptyset$**:** In this section we compare sellers' profit regimes under AB and $\emptyset$ individually.

Seller A: Let us consider the profit difference for seller A under regimes AB and $\emptyset$.

$$\begin{aligned}\Delta_A^{AB-\emptyset} &= \pi_A^{AB} - \pi_A^{\emptyset} = \frac{(2\alpha_L-1)^2\left[\Omega_O(n,\gamma) - \frac{2\gamma^2(2+3\gamma)\kappa(n,\gamma,2)[81-\kappa^3(n,\gamma,2)](1-n)}{27(2\alpha_L-1)}\right]}{24\gamma(2+3\gamma)^2\kappa(n,\gamma,3)(1-n)^2} \\ &= \frac{(2\alpha_L-1)^2\left[\Omega_O(n(\alpha_H,\alpha_L),\gamma) - \Omega_{AB}(n(\alpha_H,\alpha_L),\gamma,\alpha_L)\right]}{24\gamma(2+3\gamma)^2\kappa(n,\gamma,3)(1-n)^2}\end{aligned}$$

Let $\Omega_A(\alpha_H,\alpha_L,\gamma) \equiv \Omega_O(n(\alpha_H,\alpha_L),\gamma) - \Omega_{AB}(n(\alpha_H,\alpha_L),\gamma,\alpha_L)$. Note that $\Delta_A^{AB-\emptyset} \geq 0$ if and only if $\Omega_A(\alpha_H,\alpha_L,\gamma) \geq 0$. Moreover, $\Omega_A(\alpha_H = \alpha_L,\alpha_L,\gamma)$ and $\Omega_A(\alpha_H = \alpha_{H1},\alpha_L,\gamma) < 0$ (which we illustrate in a 3D plot in Figure EC.2.8a). In order to analyze the curvature of $\Omega_A(\alpha_H,\alpha_L,\gamma)$ we first analyze $\Omega_O(n(\alpha_H,\alpha_L),\gamma)$ and $\Omega_{AB}(n(\alpha_H,\alpha_L),\gamma,\alpha_L)$ separately. Specifically, we look at the first and second order derivatives of $\Omega_O(n(\alpha_H,\alpha_L),\gamma)$ and $\Omega_{AB}(n(\alpha_H,\alpha_L),\gamma,\alpha_L)$ with respect to α_H. Even though these functions have three variables, namely $(\alpha_H,\alpha_L,\gamma)$, we are able to factor out an expression $(2\alpha_L-1)$ through algebraic manipulations and to re-structure in the form of $(2\alpha_L-1)f(n,\gamma)$ where the remaining part of the expression $f(n,\gamma)$ is a function of two variables n and γ. Since $2\alpha_L - 1 \geq 0$, we are able to analyze the sign of these functions via the sign of $f(n,\gamma)$ and we can do that via 3D plots or implicit-plots. In Figure EC.2.8b we present the implicit plot and summarize our findings in four regions.

Region 1: $\Omega_O(n(\alpha_H,\alpha_L),\gamma)$ is convex-increasing and $\Omega_{AB}(n(\alpha_H,\alpha_L),\gamma,\alpha_L)$ is convex-increasing w.r.t. α_H. Region 2: $\Omega_O(n(\alpha_H,\alpha_L),\gamma)$ is concave-increasing and $\Omega_{AB}(n(\alpha_H,\alpha_L),\gamma,\alpha_L)$ is convex-increasing w.r.t. α_H. Region 3:

$\Omega_O(n(\alpha_H,\alpha_L),\gamma)$ is concave-decreasing and $\Omega_{AB}(n(\alpha_H,\alpha_L),\gamma,\alpha_L)$ is convex-increasing w.r.t. α_H. Region 4: $\Omega_O(n(\alpha_H,\alpha_L),\gamma)$ is concave-decreasing and $\Omega_{AB}(n(\alpha_H,\alpha_L),\gamma,\alpha_L)$ is concave-increasing w.r.t. α_H.

As a result, we can conclude that $\Omega_A(\alpha_H,\alpha_L,\gamma)=\Omega_O(n(\alpha_H,\alpha_L),\gamma)-\Omega_{AB}(n(\alpha_H,\alpha_L),\gamma,\alpha_L)$ is concave in region 2, concave-decreasing in region 3, and decreasing in region 4 with respect to α_H. In addition, $\frac{\partial\Omega_A(\alpha_H,\alpha_L,\gamma)}{\partial\alpha_H}\Big|_{\alpha_H=\alpha_L}=0$ and $\frac{\partial^2\Omega_A(\alpha_H,\alpha_L,\gamma)}{\partial\alpha_H^2}\Big|_{\alpha_H=\alpha_L}=\frac{108\gamma(1+\gamma)}{(2\alpha_L-1)^2}>0$. So, Ω_A is convex-increasing as α_H approaches to α_L from right. Now, we are ready to analyze the sign of $\Omega_A(\alpha_H,\alpha_L,\gamma)$ in all the above four regions.

We know that $\Omega_A(\alpha_H,\alpha_L,\gamma)$ starts from zero at the lower bound $\alpha_H=\alpha_L$ and is increasing-convex in that neighborhood. In region 2 it is concave and in regions 3 and 4 it is monotone decreasing. At the upper bound it is negative $\Omega_A(\alpha_{H1},\alpha_L,\gamma)<0$. Our claim is that $\Omega_A(\alpha_H,\alpha_L,\gamma)$ changes sign from positive to negative once in regions 3 or 4.

Let us start with examining $\Omega_A(\alpha_H,\alpha_L,\gamma)$ in region 1 and 2 according to Figure EC.2.8b. We show that $\Omega_A(\alpha_H,\alpha_L,\gamma)$ has a lower bound which is always non-negative in these region. Observe that:

$$\Omega_O(n(\alpha_H,\alpha_L),\gamma)-\Omega_{AB}(n(\alpha_H,\alpha_L),\gamma,\alpha_L)\geq\Omega_O(n(\alpha_H,\alpha_L),\gamma)-2\gamma^2(2+3\gamma)\kappa(n,\gamma,2)[3-\kappa^3(n,\gamma,2)/27](1-n)\equiv\Omega_{LB}(n,\gamma)$$

we can plot this function of two variables and show that it is positive in region 1 and 2 (Figure EC.2.8c). Since the lower bound is non-negative, we can conclude that $\Omega_A(\alpha_H,\alpha_L,\gamma)\geq 0$ in these regions.

Therefore, since $\Omega_A(\alpha_H,\alpha_L,\gamma)$ is monotone decreasing in regions 3 and 4, and it is negative at the upper bound, it must be the case that $\Omega_A(\alpha_H,\alpha_L,\gamma)$ is an increasing-decreasing function of α_H and that there exists a unique threshold for α_H (namely α_H^A) such that $\Omega_A(\alpha_H,\alpha_L,\gamma)$ is positive for α_H below that threshold and negative for α_H above that threshold.

To summarize, we proved that there exists a unique threshold (namely α_H^A) above which, seller A prefers regime $\emptyset$ over AB and the proof is complete.

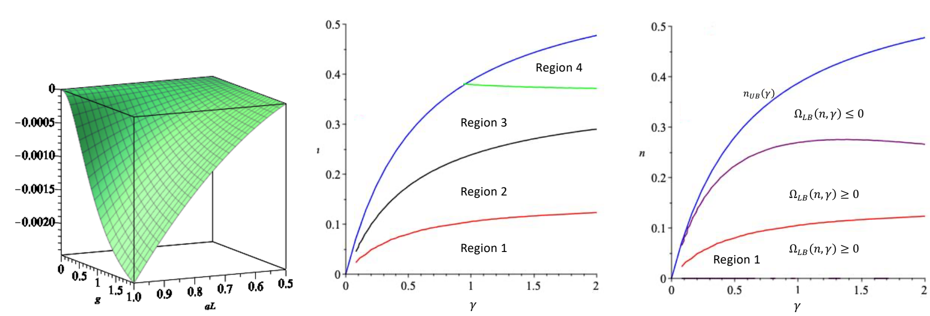
Seller B: Let us consider the profit difference for seller B.

$$\Delta_B^{AB-\emptyset}=\pi_B^{AB}-\pi_B^{\emptyset}=\frac{(2\alpha_L-1)^2\left[\Omega_O(n(\alpha_H,\alpha_L),\gamma)+\Omega_{AB}(n(\alpha_H,\alpha_L),\gamma,\alpha_L)\right]}{24\gamma(2+3\gamma)^2\kappa(n,\gamma,3)(1-n)^2}$$

Note that $\Delta_B^{AB-\emptyset}\geq 0$ if and only if $\Omega_O(n(\alpha_H,\alpha_L),\gamma)+\Omega_{AB}(n(\alpha_H,\alpha_L),\gamma,\alpha_L)\geq 0$. We can show that there exists a lower bound for this expression for all $\frac{1}{2}\leq\alpha_L\leq\alpha_H\leq 1$. Since $\frac{(1-n)}{2\alpha_L-1}=\frac{1}{\alpha_L}\geq 1$, the lower bound is a function of two variables n and γ as following:

$$\Omega_O(n(\alpha_H,\alpha_L),\gamma)+\Omega_{AB}(n(\alpha_H,\alpha_L),\gamma,\alpha_L)\geq\Omega_O(n(\alpha_H,\alpha_L),\gamma)+2\gamma^2(2+3\gamma)\kappa(n,\gamma,2)[3-\kappa^3(n,\gamma,2)/27].$$

Figure EC.2.8 Analysis of seller A's information sharing preference.



(a) Sign of $\Delta\pi_A(\alpha_{H1},\alpha_L,\gamma)$ (b) Curvature of $\Omega_A(\alpha_H,\alpha_L,\gamma)$ (c) Sign of $\Omega_{LB}(n,\gamma)$

So, if we show that this lower bound is always positive, we have proved that $\Delta_B^{AB-\emptyset} \geq 0$. Even though the expression is quite complicated with trigonometric functions involved, we can plot it in the region of interest and identify its behavior since it is a function of only two variables. In fact, we can see that the above expression is increasing in γ and positive for all the parameters that we are interested. Therefore, $\Omega_O\left(n(\alpha_H,\alpha_L),\gamma\right)+\Omega_{AB}\left(n(\alpha_H,\alpha_L),\gamma,\alpha_L\right) \geq 0$ for all parameters, and hence, seller B always prefers regime AB to $\emptyset$ in Region 1.

Consumer Surplus comparison under AB and $\emptyset$: We first show that consumer surplus under regime $J = AB$ and regime $\emptyset$ can be decomposed as follows:

$$\begin{aligned} CS^{AB} &= CS_{mean} + CS_{exact} \\ CS^{\emptyset} &= CS_1^{\emptyset} + \frac{\Delta}{|p_A - p_B|} CS_{exact} + (1 - \frac{\Delta}{|p_A - p_B|}) CS_{mean} \end{aligned}$$

Using this decomposition, we can express the difference between two consumer surpluses as follows:

$$CS^{\emptyset} - CS^{AB} = CS_1^{\emptyset} - CS_{mean} + (1 - \frac{\Delta}{|p_A - p_B|})(CS_{mean} - CS_{exact}) \tag{EC.2.17}$$

As calculated in §EC.2.2,

$$CS_{mean} - CS_{exact} = \frac{(2\gamma+1)\,(a_H - a_L)^2\,(4\gamma+3)}{4\,(1+\gamma)\,(2+3\gamma)^2} \geq 0$$

So, $(1 - \frac{\Delta}{|p_A - p_B|})(CS_{mean} - CS_{exact}) \geq 0$. Now, we need to compare $CS_1^{\emptyset}$ and CS_{mean} (which is the first period consumer surplus under regime AB):

$$CS_1^{\emptyset} - CS_{mean} = E_{\boldsymbol{\theta}}\Bigg[\sum_{i\in\{A,B\}} \frac{(d_{i,1}^{\emptyset})^2 - (d_{i,1}^{AB})^2}{4(1+\gamma)}\Bigg].$$

where $d_{i,1}^{\emptyset}$ and $d_{i,1}^{AB}$ are derived with corresponding optimal prices in Eq. (EC.2.11) and (EC.2.2).

There are two demand states. In order to do this comparison, we consider each demand state individually and show that $CS_1^{\emptyset}(\theta) \geq CS_{mean}(\theta)$ for each demand state θ, where $\theta \in \{H,L\}$. Also, we use the results in §here we showed $p_{A,1}^{AB} - p_{A,1}^{\emptyset} = x \geq 0$ and $p_{B,1}^{AB} - p_{B,1} = -x \leq 0$. In what follows, we show that $CS_1^{\emptyset}(\boldsymbol{\theta}) \geq CS_{mean}(\boldsymbol{\theta})$ for $\theta \in \{H,L\}$.

$\boldsymbol{\theta = H}$: We can write the consumer surplus difference as following under High demand state.

$$CS_1^{\emptyset}(H) - CS_{mean}(H) = -\frac{2(1+2\gamma)x}{2(1+\gamma)}(1 - 2\alpha_H - (1+2\gamma)x + 2(1+2\gamma)\Delta z_p^{AB})$$

Note that $CS_1^{\emptyset}(H) - CS_{mean}(H) \geq 0$ if $\Gamma_H \equiv 1 - 2\alpha_H - (1+2\gamma)x + 2(1+2\gamma)\Delta z_p^{AB} \leq 0$. With some rearranging and substituting Δz_p^{AB}, we can show that:

$$\begin{aligned} \Gamma_H &= 1 - 2\alpha_H - \frac{(1+2\gamma)(1-\alpha_H)}{2+3\gamma} - (1+2\gamma)(x - \frac{\alpha_L}{2+3\gamma}) \\ &= \frac{1+\gamma}{2+3\gamma} - \frac{2\alpha_H(1+\gamma)}{2+3\gamma} - \frac{(1+2\gamma)(\alpha_H - \alpha_L)}{2+3\gamma} - (1+2\gamma)x \\ &= \frac{(1+\gamma)(1-2\alpha_H)}{2+3\gamma} - \frac{(1+2\gamma)(\alpha_H - \alpha_L)}{2+3\gamma} - (1+2\gamma)x \end{aligned}$$

Since $\alpha_H \geq \alpha_L \geq 0.5$, $\gamma \geq 0$, and $x \geq 0$ all parts in the above expression are non-positive. Therefore, we showed that $\Gamma_H \leq 0$ and hence, $CS_1^{\emptyset}(H) \geq CS_{mean}(H)$ for all parameters of interest.

$\theta = L$: Through a similar analysis we can show that $CS_1^{\emptyset}(L) \geq CS_{mean}(L)$. We can write the consumer surplus difference as following under Low demand state.

$$CS_1^{\emptyset}(L) - CS_{mean}(L) = -\frac{2(1+2\gamma)x}{4(1+\gamma)}(1 - 2\alpha_L - (1+2\gamma)x + 2(1+2\gamma)\Delta z_p^{AB})$$

Note that $CS_1^{\emptyset}(L) - CS_{mean}(L) \geq 0$ if $\Gamma_L = 1 - 2\alpha_L - (1+2\gamma)x + 2(1+2\gamma)\Delta z_p^{AB} \leq 0$. With some rearranging and substituting Δz_p^{AB}, we can show that:

$$\begin{aligned}\Gamma_L &= 1 - 2\alpha_L - \frac{(1+2\gamma)(1-\alpha_L)}{2+3\gamma} - (1+2\gamma)(x - \frac{\alpha_H}{2+3\gamma}) \\ &= (\frac{1+\gamma}{2+3\gamma} - \alpha_L) - \frac{(1+\gamma)(2\alpha_L - \alpha_H)}{2+3\gamma} - (1+2\gamma)x\end{aligned}$$

We will show in the following that each part of Γ_L is non-positive. Since $\frac{1+\gamma}{2+3\gamma}$ is decreasing on γ in $\gamma \geq 0$ with maximum of $\frac{1}{2}$ and $\alpha_L \geq \frac{1}{2}$, then $(\frac{1+\gamma}{2+3\gamma} - \alpha_L) \leq 0$. Also, from $\frac{1}{2} \leq \alpha_L \leq \alpha_H \leq 1$, $\gamma \geq 0$ and $x \geq 0$, the rest of the expression is also non-positive. Therefore, we showed that $\Gamma_L \leq 0$ and hence $CS_1^{\emptyset}(L) - CS_{mean}(L) \geq 0$.

Since $CS_1^{\emptyset}(L) - CS_{mean}(L) \geq 0$ and $CS_1^{\emptyset}(H) \geq CS_{mean}(H)$, the expectation over them will also be positive. Therefore, we showed that $CS_1^{\emptyset} - CS_{mean} \geq 0$

To conclude, we proved that $(1 - \frac{\Delta}{|p_A - p_B|})(CS_{mean} - CS_{exact}) \geq 0$ and $CS_1^{\emptyset} - CS_{mean} \geq 0$. Hence, their summation is also non-negative: and $CS^{\emptyset} \geq CS^{AB}$. We showed that consumer surplus under no information sharing regime is always greater than or equal to consumer surplus under regime AB.

EC.2.7. Proposition 7

In this section, we study the mechanisms involved in information sharing with only seller B. This analysis is almost identical to the analysis of the case in which the platform shares information with seller A (which we will cover in the next section) only except that the two sellers are now swapped. We have characterized the equilibrium prices under regime B. However, since we will show that there is no feasible subscription fee to induce only seller B to subscribe, the proof for the equilibrium prices under regime $J = B$ is omitted due to space limitations but is available from the authors upon request.

Proof of Proposition 7: Based on Table EC.2.1, the following two incentive compatibility constraints should be met to only induce seller B to subscribe. First, given seller A's decision not to subscribe to CIS, seller B has an incentive to subscribe:

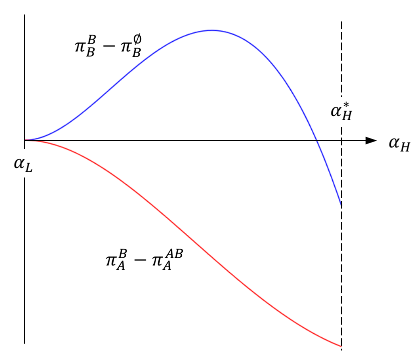
$$(1-\beta)\pi^{B}_{C|B}\left(P^{B}_{B}, P^{B}_{A}\right) - \phi^{B} \geq (1-\beta)\pi^{\emptyset}_{C|B}\left(P^{\emptyset}_{B}, P^{\emptyset}_{A}\right) \tag{EC.2.18}$$

Second, given seller B subscribes to CIS, seller A has no incentive to subscribe:

$$(1-\beta)\pi^{B}_{C|A}\left(P^{B}_{A}, P^{B}_{B}\right) \geq (1-\beta)\pi^{AB}_{C|A}\left(P^{AB}_{A}, P^{AB}_{B}\right) - \phi^{B} \tag{EC.2.19}$$

Note that constraint (EC.2.19) represents a lower bound for the subscription fee, while constraint (EC.2.18) signifies an upper bound, specifically, $\pi^{AB}_{C|A} - \pi^{B}_{C|A} < \phi^{B} < \pi^{B}_{C|B} - \pi^{\emptyset}_{C|B}$. However, it can be shown that the upper bound is always less than or equal to the lower bound, indicating that there is no feasible subscription fee that satisfies both incentive compatibility constraints. Also, Figure EC.2.9 shows that platform cannot induce regime $J = B$.

Figure EC.2.9 Feasibility conditions to induce regime $J = B$ ($\gamma = 1$, $\alpha_L = 0.6$)



EC.2.8. Proposition 8

In this section we analyze the equilibrium characteristics when platform shares information with seller A only. The flow of analysis is similar to what we have done in §EC.2.4: First, based on second period posterior beliefs of sellers about demand state, we break down our analysis into three cases; then in each case we derive the best response function of sellers; and finally, we superimpose their best responses to get the equilibrium in each case.

Proof of Proposition 8: In regime $J = A$, seller A will learn the true demand state before the second period regardless of the realization of ϵ_1. However, seller B still relies only on its own sales data and may or may not learn the demand. The learning opportunities for seller B is provided in §EC.2.3. Second period profits are related to their demand state. If seller B learns the demand, which will happen with $\frac{2\Delta}{|p_A - p_B|}$ probability, they will set prices based on exact demand function and corresponding optimal prices (Eq. EC.2.1). However, if seller B does not learn the demand, in the second period sellers are in one of the following information sets: $(H, \overline{\boldsymbol{\theta}})$ in which Only seller A identifies that $\boldsymbol{\theta} = H$, or $(L, \overline{\boldsymbol{\theta}})$ in which only seller A identifies that $\boldsymbol{\theta} = L$. In these two cases, seller B cannot identify the true value of α but knows that seller A is informed.

If $|p_A - p_B| \le 2\Delta$, then seller B can identify the value of α regardless of the realization ϵ_1. Otherwise, seller B may or may not learn the demand based on the realization of ϵ_1. Seller A has perfect information by the end of the first period and that seller B is aware of this. Note that the second period profits - based on the conjectures - are available in §EC.2.2. For expositional brevity, we drop the time index for the first period prices and denote them as (p_A, p_B).

Previously, we have shown that if both sellers learn the demand in the second period, their corresponding prices will be characterized as in Eq. (EC.2.1). In case that only seller A learns the demand, she will use the exact demand function while seller B will use mean demand function. Therefore, with first order conditions, their corresponding second period prices will be as following:

$$p^A_{A,2} = \frac{\gamma + (2+\gamma)\alpha_A(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)} + \frac{\gamma[\alpha_H + \alpha_L - 2\alpha_A(\boldsymbol{\theta})]}{4(1+\gamma)(2+3\gamma)}, \quad \text{and} \quad p^A_{B,2} = \frac{4(1+\gamma) - (2+\gamma)(\alpha_H + \alpha_L)}{2(2+\gamma)(2+3\gamma)} \tag{EC.2.20}$$

Therefore, based on possible demand learning opportunities for seller B we have the following three cases. The profit of each seller can then be written as follows.

Zone 1 - $p_B + 2\Delta \le p_A$: since $\frac{2\Delta}{p_A - p_B} \le 1$, seller B may or may not learn the demand in she second period. Therefore, they will be in one of the following information sets in the second period: (H,H), (L,L), $(H, \overline{\boldsymbol{\theta}})$, and $(L, \overline{\boldsymbol{\theta}})$. Based on their learning possibilities, the expected profit function for each seller is:

$$\begin{aligned}
\pi^{Z1}_A &= \pi_{A,1}(p_A,p_B) + \tfrac{2\Delta}{p_A - p_B}\left[\tfrac{1}{2}\pi_{A,2}(H,H) + \tfrac{1}{2}\pi_{A,2}(L,L)\right] \\
&+ \left(1 - \tfrac{2\Delta}{p_A - p_B}\right)\left[\tfrac{1}{2}\pi_{A,2}(H,\overline{\boldsymbol{\theta}}) + \tfrac{1}{2}\pi_{A,2}(L,\overline{\boldsymbol{\theta}})\right] \\
&= \pi_{A,1}(p_A,p_B) - \tfrac{(\alpha_H - \alpha_L)^3(4+5\gamma)}{32(1+\gamma)(2+3\gamma)^2(p_A - p_B)} + \textstyle\sum_{k=H,L} \tfrac{\left[2\gamma(1+\gamma)+(2+\gamma)\left[(2+3\gamma)\alpha_K - \gamma\overline{\alpha}_A\right]\right]^2}{8(1+\gamma)(2+\gamma)^2(2+3\gamma)^2} \\
\pi^{Z1}_B &= \pi_{B,1}(p_A,p_B) + \tfrac{2\Delta}{p_A - p_B}\left[\tfrac{1}{2}\pi_{B,2}(H,H) + \tfrac{1}{2}\pi_{B,2}(L,L)\right] + \left(1 - \tfrac{2\Delta}{p_A - p_B}\right)\pi_{B,2}(\overline{\boldsymbol{\theta}}) \\
&= \pi_{B,1}(p_A,p_B) + \tfrac{(\alpha_H - \alpha_L)^3(1+\gamma)\frac{1}{2}\frac{1}{2}}{2\gamma(2+3\gamma)^2(p_A - p_B)} + \tfrac{(1+\gamma)\left[\gamma + (2+\gamma)\overline{\alpha}_B\right]^2}{(2+\gamma)^2(2+3\gamma)^2}
\end{aligned}$$

Zone 2 - $|p_A - p_B| \le 2\Delta$: In this case, seller B will observe its own sales at the end of the first period and automatically identifies the true demand since $\frac{2\Delta}{|p_A-p_B|} \ge 1$. The expected profit function for each seller is therefore:

$$\begin{aligned}
\pi_A^{Z2} &= \pi_{A,1}(p_A,p_B) + \tfrac{1}{2}\pi_{A,2}(H,H) + \tfrac{1}{2}\pi_{A,2}(L,L)\\
&= \pi_{A,1}(p_A,p_B) + \tfrac{(1+\gamma)\left[\gamma+(2+\gamma)\alpha_H\right]^2}{2(2+\gamma)^2(2+3\gamma)^2} + \tfrac{(1+\gamma)[\gamma+(2+\gamma)\alpha_L]^2}{2(2+\gamma)^2(2+3\gamma)^2}\\
\pi_B^{Z2} &= \pi_{B,1}(p_A,p_B) + \tfrac{1}{2}\pi_{B,2}(H,H) + \tfrac{1}{2}\pi_{B,2}(L,L)\\
&= \pi_{B,1}(p_A,p_B) + \tfrac{(1+\gamma)[\gamma+(2+\gamma)(1-\alpha_H)]^2}{2(2+\gamma)^2(2+3\gamma)^2} + \tfrac{(1+\gamma)[\gamma+(2+\gamma)(1-\alpha_L)]^2}{2(2+\gamma)^2(2+3\gamma)^2}
\end{aligned}$$

Zone 3 - $p_A \le p_B - 2\Delta$:This case is similar to zone one. Since $\frac{2\Delta}{p_B-p_A} \le 1$, seller B may or may not learn the demand in the second period. Therefore, they will be in one of the following information sets in the second period: (H,H), (L,L), $(H,\overline{\theta})$, and $(L,\overline{\theta})$. The expected profit function of each seller is then:

$$\begin{aligned}
\pi_A^{Z3} &= \pi_{A,1}(p_A,p_B) + \tfrac{2\Delta}{p_B-p_A}\left[\tfrac{1}{2}\pi_{A,2}(H,H) + \tfrac{1}{2}\pi_{A,2}(L,L)\right]\\
&+ \left(1-\tfrac{2\Delta}{p_B-p_A}\right)\left[\tfrac{1}{2}\pi_{A,2}(H,\overline{\theta}) + \tfrac{1}{2}\pi_{A,2}(L,\overline{\theta})\right]\\
&= \pi_{A,1}(p_B,p_A) - \tfrac{(\alpha_H-\alpha_L)^3(4+5\gamma)}{32(1+\gamma)(2+3\gamma)^2(p_B-p_A)} + \textstyle\sum_{k=H,L} \tfrac{\left[2\gamma(1+\gamma)+(2+\gamma)\left[(2+3\gamma)\alpha_k-\gamma\overline{\alpha}_A\right]\right]^2}{8(1+\gamma)(2+\gamma)^2(2+3\gamma)^2} \qquad \text{(EC.2.21)}\\
\pi_B^{Z3} &= \pi_{B,1}(p_A,p_B) + \tfrac{2\Delta}{p_B-p_A}\left[\tfrac{1}{2}\pi_{B,2}(H,H) + \tfrac{1}{2}\pi_{B,2}(L,L)\right] + \left(1-\tfrac{2\Delta}{p_B-p_A}\right)\pi_{B,2}(\overline{\theta})\\
&= \pi_{B,1}(p_A,p_B) + \tfrac{(\alpha_H-\alpha_L)^3(1+\gamma)\frac{1}{2}\frac{1}{2}}{2\gamma(2+3\gamma)^2(p_B-p_A)} + \tfrac{(1+\gamma)\left[\gamma+(2+\gamma)\overline{\alpha}_B\right]^2}{(2+\gamma)^2(2+3\gamma)^2}
\end{aligned}$$

Best response function of seller A: We derive the best response function of seller A in this section. Based on the three demand learning possibilities, we first investigate the local optimum in each zone in isolation and then combine them to find the best response. For each zone, we examine the behavior of seller A's expected profit, and understand whether it is decreasing or increasing in p_A given a p_B.

Zone 1 - $p_B + 2\Delta \le p_A$**:** to understand the seller A's expected profit behavior, we examine her first derivative with respect to p_A:

$$\frac{\partial \pi_A^{Z1}}{\partial p_A} = \overline{\alpha}_A + \gamma p_B - 2(1+\gamma)p_A + \frac{(\alpha_H-\alpha_L)^3(4+5\gamma)}{32(1+\gamma)(2+3\gamma)^2(p_A-p_B)^2}$$

meaning that $\frac{\partial \pi_A^{Z1}}{\partial p_A}\Big|_{p_A=p+2\Delta, p_B=p} \le 0$ if and only if:

$$p_B \ge p_B^{A1} \equiv \frac{(103g^4+320g^3+360g^2+176g+32)\alpha_L - (31g^4+152g^3+232g^2+144g-32)\alpha_H}{8\gamma(1+\gamma)(2+\gamma)(2+3\gamma)^2}.$$

In other words, if $p_B \le p_B^{A1}$, then seller A would set a price $p_A \ge p_B + 2\Delta$ in zone 1, otherwise, if $p_B \ge p_B^{A1}$ then seller A would set a price $p_A = p_B + 2\Delta$.

Zone 2 - $|p_A - p_B| \le 2\Delta$**:** we now investigate the local optimum in zone 2. Note that:

$$\frac{\partial \pi_A^{Z2}}{\partial p_A} = \overline{\alpha}_A + \gamma p_B - 2(1+\gamma)p_A$$

meaning that $\frac{\partial \pi_A^{Z2}}{\partial p_A}\Big|_{p_A=p+2\Delta, p_B=p} \le 0$ if and only if

$$p_B > p_B^{A2} \equiv \frac{\alpha_L(2+3\gamma) - \alpha_H(2+\gamma)}{2\gamma(2+\gamma)}.$$

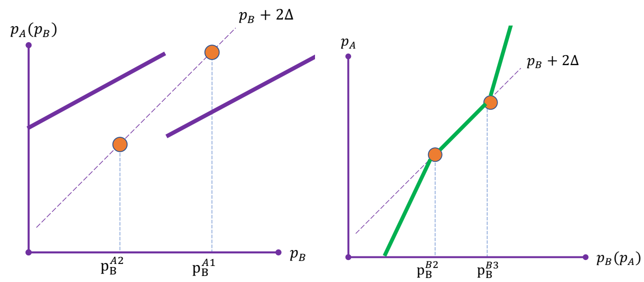
In other words, if $p_B \leq p_B^{A2}$, then seller A would set a price $p_A = p_B + 2\Delta$ in zone 2, otherwise, if $p_B \geq p_B^{A2}$ then seller A would set a price $p_A < p_B + 2\Delta$ in zone 2. Note that this is a local optimization in zone 2.

Also $p_B^{A1} - p_B^{A2} = \frac{\gamma^2(4+5\gamma)(\alpha_H-\alpha_L)}{8(1+\gamma)(2+\gamma)(2+3\gamma)^2} > 0$. This means the following:

- $p_B \leq p_B^{A2}$: π_A^{Z2} increasing for $p_A \leq p_B + 2\Delta$ and $\pi_{A,1} + \pi_A^{Z1}$ increasing at $p_A = p_B + 2\Delta$ so the best response is achieved by $\frac{\partial \pi_A^{Z1}}{\partial p_A} = 0$ in zone 1.
- For $p_B^{A2} \leq p_B \leq p_B^{A1}$ we have π_A^{Z2} increasing-decreasing for $p_A \leq p_B + 2\Delta$ and π_A^{Z1} increasing-decreasing at $p_A \geq p_B + 2\Delta$ meaning that the best response is achieved by comparing the two local optimums in zone 2 and zone 1.
- $p_B^{A1} \leq p_B$: π_A^{Z2} increasing-decreasing for $p_A \leq p_B + 2\Delta$ and π_A^{Z1} decreasing at $p_A \geq p_B + 2\Delta$ so the best response is achieved by $\frac{\partial \pi_A^{Z2}}{\partial p_A} =$ in zone 2.

In other words, there is a discontinuity and a jump down in the best response function of seller A as depicted in Figure EC.2.10(a).

Figure EC.2.10 **Best response functions for seller A and Seller when the platform shares information with seller A only.**



Best response function of seller B: Similar to the analysis for seller A we investigate the best response function of seller B in this section for each zone individually. The process is the same: in each zone we examine the behavior of the derivative of the expected profit function with respect to p_B and based on that, find the best response p_B for a given p_A.

Zone 1 - $p_B + 2\Delta \leq p_A$: observe that in this case:

$$\frac{\partial \pi_B^{Z1}}{\partial p_B}\Big|_{p_A=p+2\Delta, p_B=p} \leq 0$$

if and only if

$$p_B \geq p_B^{B1} \equiv \frac{2(2+3\gamma)^2 + \alpha_H\gamma(1+\gamma) - \alpha_L(8+25\gamma+19\gamma^2)}{2(2+\gamma)(2+3\gamma)^2}$$

In other words, if $p_A \leq p_B^{B1} + 2\Delta$, then seller B would set a price $p_B = p_A - 2\Delta$ in zone 1, otherwise, if $p_A \geq p_B^{B1} + 2\Delta$ then seller A would set a price $p_B \leq p_A + 2\Delta$ in zone 1.

Zone 2 - $|p_A - p_B| \leq 2\Delta$: we now investigate the local optimum in zone 2 by taking the derivative of the expected profit function as follows:

$$\frac{\partial \pi_B^{Z2}}{\partial p_B}\Big|_{p_A=p+2\Delta, p_B=p} \leq 0$$

if and only if $p > p_B^{B2} \equiv \frac{1-\alpha_L}{2+\gamma}$. In other words, if $p_A \le p_B^{B2} + 2\Delta$, then seller B would set a price $p_B > p_A - 2\Delta$ in zone 2, otherwise, if $p_A \ge p_B^{B2} + 2\Delta$ then seller B would set a price $p_B = p_A - 2\Delta$ in zone 2. Note that this is a local optimization in zone 2. Note that:

$$p_B^{B1} - p_B^{B2} = \frac{\gamma(1+\gamma)(\alpha_H - \alpha_L)}{2(2+\gamma)(2+3\gamma)^2} > 0.$$

This means the following:

- For $p_A \le p_B^{B2} + 2\Delta$ we have π_A^{Z1} increasing for $p_B \le p_A - 2\Delta$ and π_A^{Z2} increasing at $p_B = p_A - 2\Delta$ meaning that the best response is achieved by $\frac{\partial \pi_B^{Z2}}{\partial p_B} = 0$ in zone 2.
- For $p_B^{B2} + 2\Delta \le p_A \le p_B^{B1} + 2\Delta$ we have π_B^{Z1} increasing for $p_B \le p_A - 2\Delta$ and π_A^{Z2} decreasing at $p_B \ge p_A - 2\Delta$ meaning that the best response is achieved by $p_B = p_A - 2\Delta$.
- For $p_B^{B1} + 2\Delta \le p_A$ we have π_B^{Z1} increasing-decreasing for $p_B \le p_A - 2\Delta$ and π_B^{Z2} decreasing at $p_B \ge p_A - 2\Delta$ meaning that the best response is achieved by $\frac{\partial \pi_B^{Z1}}{\partial p_B} = 0$ in zone 1.

In other words, there is no discontinuity in the best response function of seller B, as depicted in Figure EC.2.10(b).

Equilibrium structure: Now that we have both sellers' best response functions, we are able to derive the equilibrium structure. To do so, we superimpose the two best response functions based on the values of exogenous problem parameters; namely α_H, α_L, and γ. Note that we have the following relation between parameters and price thresholds.

1. $\frac{\partial p_B^{A2} - p_B^{B1}}{\partial \alpha_H} < 0$ and $p_B^{A2} - p_B^{B1} > 0$ if and only if $\alpha_H \le \alpha_{H4} \equiv \frac{(1+2\gamma)(8+28\gamma+23\gamma^2)\alpha_L - 2\gamma(2+3\gamma)^2}{8+28\gamma+31\gamma^2+10\gamma^3}$
2. $\frac{\partial p_B^{A2} - p_B^{B2}}{\partial \alpha_H} < 0$ and $p_B^{A2} - p_B^{B2} > 0$ if and only if $\alpha_H \le \alpha_{H6} \equiv \frac{\alpha_L(2+5\gamma) - 2\gamma}{2+\gamma}$
3. $\frac{\partial p_B^{A1} - p_B^{B1}}{\partial \alpha_H} < 0$ and $p_B^{A1} - p_B^{B1} > 0$ if and only if $\alpha_H \le \alpha_{H5} \equiv \frac{(179\gamma^4+496\gamma^3+492\gamma^2+208\gamma+32)\alpha_L - 8(1+\gamma)(2+3\gamma)^2\gamma}{35\gamma^4+160\gamma^3+236\gamma^2+144\gamma+32}$
4. $\frac{\partial p_B^{A1} - p_B^{B2}}{\partial \alpha_H} < 0$ and $p_B^{A1} - p_B^{B2} > 0$ if and only if $\alpha_H \le \alpha_{H7} \equiv \frac{(175\gamma^4+488\gamma^3+488\gamma^2+208\gamma+32)\alpha_L - 8\gamma(1+\gamma)(2+3\gamma)^2}{31\gamma^4+152\gamma^3+232\gamma^2+144\gamma+32}$.

With comparing bounds we see that $\frac{1}{2} \le \alpha_L \le \alpha_{H4} \le \alpha_{H5} \le \alpha_{H6} \le \alpha_{H7}$. Therefore, based on the value of α_H one of the following possibilities will happen.

1. If $\alpha_L \le \alpha_H \le \alpha_{H4} \le \alpha_{H5} \le \alpha_{H6} \le \alpha_{H7}$, then $p_B^{B2} \le p_B^{B1} \le p_B^{A2} \le p_B^{A1}$ meaning that there exists a unique pure strategy equilibrium which satisfies $\frac{\partial \pi_B^{Z1}}{\partial p_B} = 0$ and $\frac{\partial \pi_A^{Z1}}{\partial p_A} = 0$.
2. If $\alpha_L \le \alpha_{H4} \le \alpha_H \le \alpha_{H5} \le \alpha_{H6} \le \alpha_{H7}$, then $p_B^{B2} \le p_B^{A2} \le p_B^{B1} \le p_B^{A1}$ meaning that there may exists a unique pure strategy equilibrium which satisfies $\frac{\partial \pi_B^{Z1}}{\partial p_B} = 0$ and $\frac{\partial \pi_A^{Z1}}{\partial p_A} = 0$ or there may not exists a pure strategy equilibrium.
3. If $\alpha_L \le \alpha_{H4} \le \alpha_{H5} \le \alpha_H \le \alpha_{H6} \le \alpha_{H7}$, then $p_B^{B2} \le p_B^{A2} \le p_B^{A1} \le p_B^{B1}$ meaning that there does not exist a pure strategy equilibrium.
4. If $\alpha_L \le \alpha_{H4} \le \alpha_{H5} \le \alpha_{H6} \le \alpha_H \le \alpha_{H7}$, then $p_B^{A2} \le p_B^{B2} \le p_B^{A1} \le p_B^{B1}$ meaning that there may exists a unique pure strategy equilibrium which satisfies $\frac{\partial \pi_B^{Z2}}{\partial p_B} = 0$ and $\frac{\partial \pi_A^{Z2}}{\partial p_A} = 0$ or there may not exists a pure strategy equilibrium.
5. If $\alpha_L \le \alpha_{H4} \le \alpha_{H5} \le \alpha_{H6} \le \alpha_{H7} \le \alpha_H$, then $p_B^{A2} \le p_B^{A1} \le p_B^{B2} \le p_B^{B1}$ meaning that there exists a unique pure strategy equilibrium which satisfies $\frac{\partial \pi_B^{Z2}}{\partial p_B} = 0$ and $\frac{\partial \pi_A^{Z2}}{\partial p_A} = 0$.

Based on the above examination, we take $\alpha_H \in [\alpha_4, \alpha_7]$ as the areas in which we do not have a pure strategy equilibrium. Note that this is an overestimation of the set of parameters over which we do not have an equilibrium. However, as we show below, this is still a very small percentage of the parameter values of interest $\{\alpha_L, \alpha_H, \gamma\}$. Indeed, let $\boldsymbol{\theta}_{noEQ}(\gamma)$ denote the size of the area $\{\alpha_H, \alpha_L | \frac{1}{2} \le \alpha_L \le 1, \alpha_{H4} \le \alpha_H \le \alpha_{H7}\}$. We can show that $\boldsymbol{\theta}_{noEQ}(\gamma) = \frac{(2\gamma+1)^2(2+3\gamma)^4\gamma^3}{2(184\gamma^4+380\gamma^3+279\gamma^2+84\gamma+8)(700\gamma^5+2199\gamma^4+2608\gamma^3+1448\gamma^2+368\gamma+32)}$. Now, let $\boldsymbol{\theta}_{Total}(\gamma)$ denote the size of the area $\{\alpha_H, \alpha_L | \frac{1}{2} \leq \alpha_L \leq 1, \alpha_L \leq \alpha_H \leq 1\}$. We can show that $\boldsymbol{\theta}_{Total}(\gamma) = \frac{(2\gamma+1)(1+3\gamma)}{2(1+4\gamma)^2}$. We can see that $\frac{\boldsymbol{\theta}_{noEQ}(\gamma)}{\boldsymbol{\theta}_{Total}(\gamma)}$ is increasing in γ and converges to $\frac{27}{4025} \approx 0.0067081 = 0.6781\%$ as $\gamma \to \infty$. Given that the area in which we may not have an equilibrium is significantly small, we focus our attention on the $\alpha \leq \alpha_{H4}$. In the next section, we identify the structure of the equilibrium in that region.

Equilibrium prices in $\alpha_L \leq \alpha_H \leq \alpha_{H4}$: Since the equilibrium in this region happens at $\frac{\partial \pi_B^{Z1}}{\partial p_B} = 0$ and $\frac{\partial \pi_A^{Z1}}{\partial p_A} = 0$, we look at the first order conditions in this region as follows:

$$\overline{\alpha}_A + \gamma p_B - 2(1+\gamma)p_A + \frac{(\alpha_H - \alpha_L)^3(4+5\gamma)}{32(1+\gamma)(2+3\gamma)^2(p_A - p_B)^2} = 0 \tag{EC.2.22}$$

$$\overline{\alpha}_B + \gamma p_A - 2(1+\gamma)p_B + \frac{(\alpha_H - \alpha_L)^3(1+\gamma)}{8\gamma(2+3\gamma)^2(p_A - p_B)^2} = 0 \tag{EC.2.23}$$

To solve this system of equations we get the summation and take the difference of them. The summation of Equations EC.2.22 and EC.2.23 gives us:

$$\frac{p_A + p_B}{2} = \frac{1}{2(2+\gamma)} + \frac{(a_H - a_L)^3}{64\gamma(1+\gamma)(2+\gamma)(p_A - p_B)^2} = \frac{1}{2(2+\gamma)} + \frac{\gamma(a_H - a_L)}{16(1+\gamma)(2+\gamma)(z_p^A)^2}$$

Taking the difference of Equation EC.2.22 and EC.2.23 gives us

$$\alpha_H + \alpha_L - 1 - (2+3\gamma)(p_A - p_B) - \frac{(\alpha_H - \alpha_L)^3(4+4\gamma-\gamma^2)}{32\gamma(1+\gamma)(2+3\gamma)^2(p_A - p_B)^2} = 0.$$

We can write this equation as $x^3 - \frac{\alpha_H+\alpha_L-1}{2+3\gamma}x^2 + \frac{(\alpha_H-\alpha_L)^3(4+4\gamma-\gamma^2)}{32\gamma(1+\gamma)(2+3\gamma)^3} = 0$ where $x \equiv p_A - p_B$. In other words, we are looking for the root of this function, which gives us the price difference, instead of searching for the two price decisions separately. It is not possible to write the roots of this cubic function algebraically in closed form, but possible in trigonometric form:

$$\begin{aligned} x_j &= \frac{\alpha_H + \alpha_L - 1}{3(2+3\gamma)}\left[1 - 2\cos\left(\frac{1}{3}\arccos\Big(-1 + \frac{27}{2}\frac{n^3(4+4\gamma-\gamma^2)}{32\gamma(1+\gamma)}\Big) - \frac{2\pi j}{3}\right)\right] \text{ for } j = 0, 1, 2 \\ &= \frac{4\gamma n}{3(2+3\gamma)}\kappa_A(n,\gamma,j)\Delta \text{ for } j = 0, 1, 2 \end{aligned}$$

where $n \equiv \frac{\alpha_H - \alpha_L}{\alpha_H + \alpha_L - 1}$ and

$$\kappa_A(n,\gamma,j) \equiv 1 - 2\cos\left(\frac{1}{3}\arccos\Big(-1 + \frac{27\frac{(4+4\gamma-\gamma^2)n^3}{32\gamma(1+\gamma)}}{2}\Big) - \frac{2j\pi}{3}\right) \tag{EC.2.24}$$

Up to this point, we have found that $p_A + p_B = \frac{1}{(2+\gamma)} + \frac{\gamma(a_H-a_L)}{8(1+\gamma)(2+\gamma)(z_p^A)^2}$ and $p_A - p_B = x_j$. However, the analysis is not done yet. We need to check whether x_j roots are real or not, and ensure that it satisfies the condition in zone 1.

First, we start by checking x_j to be real. To do so, we need to examine the sign of the discriminant of the cubic equation which is $\frac{n^3(4+4\gamma-\gamma^2)}{32\gamma(1+\gamma)}\big(4 - \frac{27n^3(4+4\gamma-\gamma^2)}{32\gamma(1+\gamma)}\big)$. If the discriminant is negative, then there is 1 real root ($j = 0$) and 2 complex conjugate roots ($j = 1, 2$). If the discriminant is positive, $n \leq \big(\frac{128\gamma(1+\gamma)}{27(4+4\gamma-\gamma^2)}\big)^{\frac{1}{3}}$, then there are three distinct real roots.

Second, we check if these roots are compatible with Region 1 conditions. In the analysis of Region 1. So, when we are analyzing Region 1, parameters of interest are: $\gamma > 0$ and $\alpha_L \leq \alpha_H \leq \alpha_{H1}$ or equivalently $0 \leq n \leq \frac{2\gamma(2+3\gamma)^2}{8+36\gamma+56\gamma^2+29\gamma^3}$. So, all three roots are real for parameters that are in Region 1.

Finally, we need to ensure that the roots are in Zone 1, i.e., $p_A > p_B + 2\Delta$. Below, we show that the first two roots ($j = 0, 1$) fail to satisfy this condition and that the third root ($j = 2$) satisfies this condition only if $\alpha_H \leq \alpha_{H1}$.

Putting all the analysis of x_j together, we will see that only x_2 satisfies all the conditions, and therefore we have:

$$\begin{aligned} p_A - p_B &= 2z_p^A\Delta \\ &= 2\frac{2\gamma(\alpha_H+\alpha_L-1)}{(2+3\gamma)(\alpha_H-\alpha_L)}\left[1-2\cos\left(\frac{1}{3}\arccos\left(-1+\frac{27}{2}\frac{(\alpha_H-\alpha_L)^3(4+4\gamma-\gamma^2)}{32\gamma(1+\gamma)(\alpha_H+\alpha_L-1)^3}\right)-\frac{4\pi}{3}\right)\right]\Delta \\ \frac{p_A+p_B}{2} &= \frac{1}{2(2+\gamma)}+\frac{(a_H-a_L)^3}{64\gamma(1+\gamma)(2+\gamma)(p_A-p_B)^2} = \frac{1}{2(2+\gamma)}+\frac{\gamma(a_H-a_L)}{16(1+\gamma)(2+\gamma)(z_p^A)^2} \end{aligned}$$

Equilibrium prices in $\alpha_{H7} \le \alpha_H \le 1$: In this case, we are in zone 2, meaning that sellers can identify the true demand state for any realization of ϵ_1. Accordingly, they set their first period prices to maximize their first period profits only, i.e., equal to the prices in which the platform shares information, giving us $p_A = \frac{2\gamma+(2+\gamma)(\alpha_H+\alpha_L)}{2(2+\gamma)(2+3\gamma)}$ and $p_B = \frac{2\gamma+(2+\gamma)(2-\alpha_H-\alpha_L)}{2(2+\gamma)(2+3\gamma)}$.

Summary of the equilibrium (information sharing with seller A only): Now that we have characterized equilibrium prices in each region, we summarize them in this section.

For $\alpha_L \le \alpha_H \le \alpha_{H4}$ we have a unique pure strategy equilibrium (p_A, p_B) given by:

$$p_{A,1}^A = \overline{p}_1^A + z_p^A\Delta, \quad p_{B,1}^A = \overline{p}_1^A - z_p^A\Delta$$

where:

$$\begin{aligned} \overline{p}_1^A &= \frac{1}{2(2+\gamma)}+\frac{\gamma(a_H-a_L)}{16(1+\gamma)(2+\gamma)(z_p^A)^2} \\ z_p^A &= \frac{2\gamma(\alpha_H+\alpha_L-1)}{(2+3\gamma)(\alpha_H-\alpha_L)}\left[1-2\cos\left(\frac{1}{3}\arccos\left(-1+\frac{27}{2}\frac{(\alpha_H-\alpha_L)^3(4+4\gamma-\gamma^2)}{32\gamma(1+\gamma)(\alpha_H+\alpha_L-1)^3}\right)-\frac{4\pi}{3}\right)\right] \end{aligned}$$

For $\alpha_{H4} \le \alpha_H \le \alpha_{H7}$ we have no pure strategy equilibrium. In fact, for $\alpha_H \in [\alpha_{H5}, \alpha_{H6}]$ we are guaranteed that there exists no pure strategy equilibrium. For $\alpha_H \in [\alpha_{H4}, \alpha_{H7}] - [\alpha_{H5}, \alpha_{H6}]$ we may have an equilibrium.

For $1 \ge \alpha_H \ge \alpha_{H7}$ we have a unique pure strategy equilibrium:

$$p_{A,1}^A = \frac{2\gamma+(2+\gamma)(\alpha_H+\alpha_L)}{2(2+\gamma)(2+3\gamma)}, \quad p_{B,1}^A = \frac{2\gamma+(2+\gamma)(2-\alpha_H-\alpha_L)}{2(2+\gamma)(2+3\gamma)}$$

Note that $z_p^A \ge 1$ for $\alpha_L \le \alpha_H \le \alpha_{H4}$, meaning that the price difference $p_{A,1}^A - p_{B,1}^A = 2z_p^A \ge 2\Delta$, and therefore, sellers may or may not identify the true state of the demand depending on their demand observation. In the remaining regions we have $z_p^A \le 1$ meaning that the price difference $p_{A,1}^A - p_{B,1}^A = 2z_p^A \le 2\Delta$ and therefore, sellers easily identify the true state of the demand independent of their demand observation. Based on their demand learning as described, they will set their second period based on Eq. (EC.2.1) or (EC.2.20).

Designing subscription fee under regime A: Based on Table EC.2.1, the following two incentive compatibility constraints should be met. First, given seller A subscribes to CIS, seller B has no incentive to subscribe:

$$(1-\beta)\pi_{C|B}^A\big(P_B^A, P_A^A\big) \ge (1-\beta)\pi_{C|B}^{AB}\big(P_B^{AB}, P_A^{AB}\big) - \phi^A$$

Or, equivalently;

$$\phi^A - S_B^{\emptyset} = (1-\beta)\big[\pi_{C|B}^{AB}\big(P_B^{AB}, P_A^{AB}\big) - \pi_{C|B}^A\big(P_B^A, P_A^A\big)\big] \tag{EC.2.25}$$

where $S_B^{\emptyset}$ indicates seller-B's surplus by not subscribing to CIS service. Second, given seller B's decision not to subscribe, seller A has an incentive to subscribe:

$$(1-\beta)\pi_{C|A}^A\big(P_A^A, P_B^A\big) - \phi^A \ge (1-\beta)\pi_{C|A}^{\emptyset}\big(P_A^{\emptyset}, P_B^{\emptyset}\big)$$

Or, equivalently;

$$\phi^A + S_A^A = (1-\beta)\left[\pi_{C|A}^{A}\left(P_A^A, P_B^A\right) - \pi_{C|A}^{\emptyset}\left(P_A^{\emptyset}, P_B^{\emptyset}\right)\right] \tag{EC.2.26}$$

From Figure EC.2.11, two possibilities emerge:

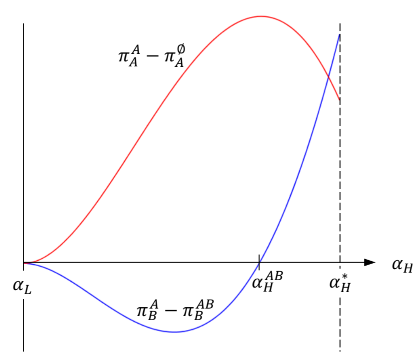
- $\alpha_H \in [\alpha_H^{AB}, \alpha_H^*]$: In this case, we have $\pi_{C|B}^{AB}\left(P_B^{AB}, P_A^{AB}\right) \leq \pi_{C|B}^{A}\left(P_B^A, P_A^A\right)$. First of all, constraint (EC.2.25) can be met for any $\phi^A \geq 0$ while $S_B^{\emptyset} = (1-\beta)\left[\pi_{C|B}^{A}\left(P_B^A, P_A^A\right) - \pi_{C|B}^{AB}\left(P_B^{AB}, P_A^{AB}\right)\right] \geq 0$. This means that seller B has no incentives to subscribe, even if $\phi^A = 0$, since her profit without subscription is higher than that if she subscribes. However, constraints (EC.2.26) can be met only if $\phi^A \leq (1-\beta)[\pi_{C|A}^{A}\left(P_A^A, P_B^A\right) - \pi_{C|A}^{\emptyset}\left(P_A^{\emptyset}, P_B^{\emptyset}\right)]$. Therefore, the feasible set of subscription fee includes $\phi^A \in \left[0, (1-\beta)[\pi_{C|A}^{A}\left(P_A^A, P_B^A\right) - \pi_{C|A}^{\emptyset}\left(P_A^{\emptyset}, P_B^{\emptyset}\right)]\right]$, while at optimality we have

$$\begin{aligned}
&\phi^A = (1-\beta)\left[\pi_{C|A}^{A}\left(P_A^A, P_B^A\right) - \pi_{C|A}^{\emptyset}\left(P_A^{\emptyset}, P_B^{\emptyset}\right)\right]\\
&S_A^A = 0\\
&S_B^{\emptyset} = (1-\beta)\left[\left(\pi_{C|A}^{A}\left(P_A^A, P_B^A\right) - \pi_{C|A}^{\emptyset}\left(P_A^{\emptyset}, P_B^{\emptyset}\right)\right) + \left(\pi_{C|B}^{A}\left(P_B^A, P_A^A\right) - \pi_{C|B}^{AB}\left(P_B^{AB}, P_A^{AB}\right)\right)\right]
\end{aligned}$$

- $\alpha_H \leq \alpha_H^{AB}$: From constraint (EC.2.25) any subscription fee $\phi^A > 0$ is feasible, while constraint (EC.2.26) puts an upper limit for the subscription fee ϕ^A. Given the fact that the platform aims to maximize the subscription fee, at optimality we have:

$$\begin{aligned}
&\phi^A = (1-\beta)\left[\pi_{C|A}^{A}\left(P_A^A, P_B^A\right) - \pi_{C|A}^{\emptyset}\left(P_A^{\emptyset}, P_B^{\emptyset}\right)\right]\\
&S_A^A = 0\\
&S_B^{\emptyset} = (1-\beta)\left[\left(\pi_{C|A}^{A}\left(P_A^A, P_B^A\right) - \pi_{C|A}^{\emptyset}\left(P_A^{\emptyset}, P_B^{\emptyset}\right)\right) - \left(\pi_{C|B}^{AB}\left(P_B^{AB}, P_A^{AB}\right) - \pi_{C|B}^{A}\left(P_B^A, P_A^A\right)\right)\right]
\end{aligned}$$

Figure EC.2.11 **Feasibility conditions to induce regime $J = A$ ($\gamma = 1$, $\alpha_L = 0.6$)**



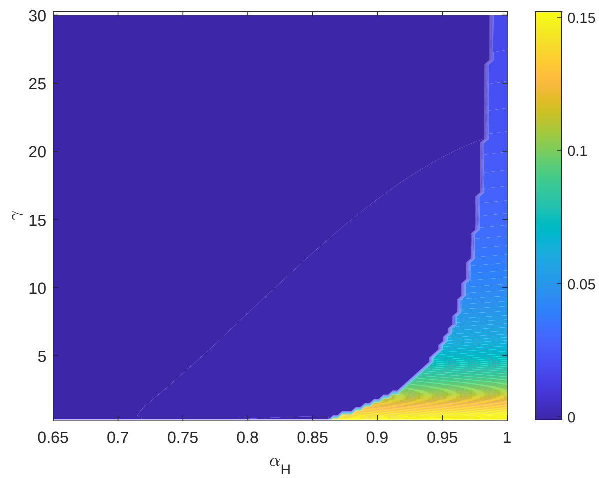
Consumer Surplus under $J = A$: Under regime $J = A$, deriving consumer surplus follows the same logic in $J = AB$ and $J = \emptyset$: it is the expectation of consumer surplus from seller A and B in two periods under different demand states. Therefore, we can write it as follows:

$$CS^A = E_{\theta}\left[\sum_{i\in\{A,B\}}\sum_{t\in\{1,2\}} CS_{i,t}^A\right] = \left[\sum_{i\in\{A,B\}}\sum_{t\in\{1,2\}}\sum_{\theta\in\{H,L\}} \frac{(d_{i,t}^A(\boldsymbol{\theta}))^2}{4(1+\gamma)}\right]$$

where $d_{i,t}^A(\boldsymbol{\theta})$ is derived with optimal prices under regime $J = A$ in each period $t \in \{1,2\}$ for seller A and B as characterized in §EC.2.8.

While the consumer surplus can still be expressed for each regime, comparing $CS^{\emptyset}$, CS^{AB}, and CS^{A} analytically becomes intractable. Accordingly, we numerically evaluated these quantities for a benchmark case ($\alpha_L = 0.65$) as shown in Figure EC.2.12. This figure shows the contour of $CS^{\emptyset} - CS^{A}$. Since in §EC.2.6 we showed that $CS^{\emptyset} \geq CS^{AB}$, we only need to compare $CS^{\emptyset}$ and CS^{A} to get the consumer surplus choice. We created 10000 test cases by varying parameters in the following ranges: $\alpha_L = 0.65$, $\alpha_L \leq \alpha_H \leq 1$ with 0.0035-length steps, and $0 \leq \gamma \leq 30$ with 0.3-length steps. The computational part has been done using a device with following specifications: 11th Gen Intel(R) Core(TM) i7-11370H, 3.30GHz (3.30 GHz), and MATLAB Software, version R2023b.

Figure EC.2.12 Consumer Surplus Comparison under $J \in \{A, \emptyset\}$**,** $\alpha_L = 0.65$



EC.2.9. Proposition 9

Proof of Proposition 9: Since we showed that regime $J = B$ is not implementable in Proposition 7 (§EC.2.7), in this section we compare prices in regimes $J \in \{AB, A, \emptyset\}$. Previously, in §EC.2.6 we showed that:

$$p^{AB}_{A,1} - p^{\emptyset}_{A,1} = \frac{2\gamma\Delta}{3n(2+3\gamma)}[3-\kappa(n,\gamma,2)] \geq 0 \quad \text{and} \quad p^{AB}_{B,1} - p^{\emptyset}_{B,1} = \frac{2\gamma\Delta}{3n(2+3\gamma)}[\kappa(n,\gamma,2)-3] \leq 0.$$

Now, recall that we defined

$$\kappa(n,\gamma,2) \equiv 1 - 2\cos\left(\frac{\arccos\left(-1+\frac{27(1+\gamma)n^3}{8\gamma}\right)-4\pi}{3}\right)$$
$$\kappa_A(n,\gamma,2) \equiv 1 - 2\cos\left(\frac{\arccos\left(-1+\frac{27(4+4\gamma-\gamma^2)n^3}{64\gamma(1+\gamma)}\right)-4\pi}{3}\right)$$

We can see that $\kappa_A(n,\gamma,2) > \kappa(n,\gamma,2)$ and recall that we have already established $2.7932 \leq \kappa(n,\gamma,2) \leq 3$ in Region 1, i.e., $\alpha_L \leq \alpha_H \leq \alpha_{H1}$. Accordingly, we have the following comparison across average first period prices in different subscription regimes.

$$\begin{aligned}
\frac{p^{AB}_{A,1}+p^{AB}_{B,1}}{2} = \frac{p^{\emptyset}_{A,1}+p^{\emptyset}_{B,1}}{2} &= \frac{1}{2(2+\gamma)} > 0 \\
\frac{p^{A}_{A,1}+p^{A}_{B,1}}{2} - \frac{p^{\emptyset}_{A,1}+p^{\emptyset}_{B,1}}{2} &= \frac{9n^3(2+3\gamma)^2(2\alpha_L-1)}{64\kappa_A^2(n,\gamma,2)(1+\gamma)\gamma(2+\gamma)(1-n)} \geq 0 \\
\frac{p^{\emptyset}_{A,1}+p^{\emptyset}_{B,1}}{2} - \frac{p^{B}_{A,1}+p^{B}_{B,1}}{2} &= \frac{9(6+7\gamma)(\alpha_H-\alpha_L)^3}{32\gamma(1+\gamma)(2+\gamma)\kappa_A^2(n,\gamma,2)} > 0
\end{aligned}$$

Let $\overline{p}^J = \frac{p^J_{A,1}+p^J_{B,1}}{2}$ denote the average first-period price in regime J. Then, we get:

$$\overline{p}^A > \overline{p}^{AB} = \overline{p}^{\emptyset}.$$

Also, for first period price differences, we have the following comparison across different subscription regimes.

$$\begin{aligned}
(p^{A}_{A,1} - p^{A}_{B,1}) - (p^{\emptyset}_{A,1} - p^{\emptyset}_{B,1}) &= \frac{\alpha_H+\alpha_L-1}{3(2+3\gamma)}\Big[\kappa_A(n,\gamma,2)-\kappa(n,\gamma,2)\Big] > 0 \\
(p^{AB}_{A,1} - p^{AB}_{B,1}) - (p^{A}_{A,1} - p^{A}_{B,1}) &= \frac{(\alpha_H+\alpha_L-1)[3-\kappa(n,\gamma,2)]}{6+9\gamma} > 0
\end{aligned}$$

which implies the following ranking of first-period price differences between the sellers:

$$z^{AB}_p > z^{A}_p > z^{\emptyset}_p.$$

EC.2.10. Proposition 10

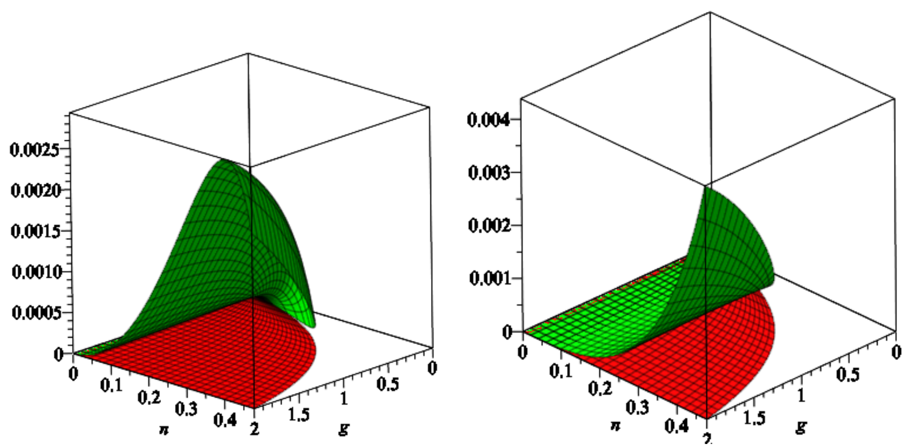
We have discussed about platform preferences between regimes AB and $\emptyset$ in §EC.2.5. Also in §EC.2.7, we showed that regime $J = B$ is not implementable. In this section, we will compare $J = A$ versus $J = \emptyset$, and $J = A$ versus $J = AB$ to get the full equilibrium structure.

Proof of Proposition 10: We first show that platform prefers $J = A$ over $J = \emptyset$ and then compare platform profit between $J = AB$ and $J = A$. To show that $\pi_O^A > \pi_O^{\emptyset}$, we have to find the sign of the following function:

$$\pi_O^A - \pi_O^{\emptyset} = \frac{(2\alpha_L - 1)\left[2\alpha_L\gamma_1(n(\alpha_H,\alpha_L),\gamma) + \gamma_0(n(\alpha_H,\alpha_L),\gamma)\right]}{9216(2+3\gamma)^2(2+\gamma)^2(\gamma+1)\gamma^2(1-n)^2\left[32\gamma(1+\gamma)\kappa_A^2 - 9n^3(4+4\gamma-\gamma^2)\right]\kappa\kappa_A^4}$$

Moreover, we have $\kappa = \kappa(n,\gamma,2)$ as in Eq. (EC.2.10), $\kappa_A = \kappa_A(n,\gamma,2)$ as in Eq. (EC.2.24), and $n = \frac{\alpha_H - \alpha_L}{\alpha_H + \alpha_L - 1}$. Note that $\pi_O^A - \pi_O^{\emptyset} > 0$ if and only if $2\alpha_L\gamma_1(n(\alpha_H,\alpha_L),\gamma) + \gamma_0(n(\alpha_H,\alpha_L),\gamma) > 0$. A detailed analysis can show that $\gamma_1(n(\alpha_H,\alpha_L),\gamma)$ can be positive or negative in Region 1. Consequently, if $\gamma_1(n(\alpha_H,\alpha_L),\gamma) > 0$ we have that $2\alpha_L\gamma_1(n(\alpha_H,\alpha_L),\gamma)+\gamma_0(n(\alpha_H,\alpha_L),\gamma) > 2\frac{1}{2}\gamma_1(n(\alpha_H,\alpha_L),\gamma)+\gamma_0(n(\alpha_H,\alpha_L),\gamma) = \gamma_1(n(\alpha_H,\alpha_L),\gamma)+\gamma_0(n(\alpha_H,\alpha_L),\gamma)$. On the other hand, if $\gamma_1(n(\alpha_H,\alpha_L),\gamma) < 0$ we have that $2\alpha_L\gamma_1(n(\alpha_H,\alpha_L),\gamma)+\gamma_0(n(\alpha_H,\alpha_L),\gamma) > 2\gamma_1(n(\alpha_H,\alpha_L),\gamma)+\gamma_0(n(\alpha_H,\alpha_L),\gamma)$. As the final step, we illustrate in Figure EC.2.13, both $2\gamma_1(n(\alpha_H,\alpha_L),\gamma)+\gamma_0(n(\alpha_H,\alpha_L),\gamma)$ and $\gamma_1(n(\alpha_H,\alpha_L),\gamma)+\gamma_0(n(\alpha_H,\alpha_L),\gamma)$ are positive for all n and γ in Region 1, indicating that $\pi_O^A > \pi_O^{\emptyset}$.

Figure EC.2.13 Platform profit comparison under $J = A$ and $J = \emptyset$.



(a) Sign of $\gamma_1(n,\gamma)+\gamma_0(n,\gamma)$

(b) Sign of $2\gamma_1(n,\gamma)+\gamma_0(n,\gamma)$

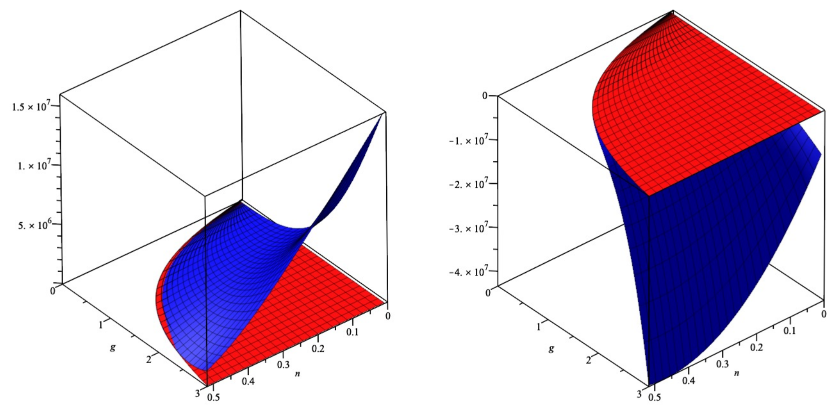
Platform Profit Comparison between $J = AB$ and $J = A$: First of all, from Proposition 2, inducing regime $J = AB$ is not feasible when $\alpha_H^{AB} < \alpha_H$. This readily means that when $\alpha_H^{AB} < \alpha_H^*$ (i.e., in Region 1), then platform chooses $J = A$ as offering CIS to both sellers is not even possible. Now, in what follows, we assume $\alpha_H < \alpha_H^{AB}$.

$$\begin{aligned}\Delta_O^{AB-A} &\equiv \pi_O^{AB} - \pi_O^A \\ &= \frac{n^2(2\alpha_L - 1)}{1024(2+\gamma)^2(1+\gamma)\gamma^2\kappa_A^4(n-1)^2(2+3\gamma)^2}\left[\Lambda_1(\kappa_A,\gamma,n)\alpha_L + \Lambda_2(\kappa_A,\gamma,n)\right]\end{aligned}$$

where $\Lambda_1(\kappa_A,\gamma,n)$ and $\Lambda_2(\kappa_A,\gamma,n)$ are omitted due to space limitations but is available from the authors upon request.

Moreover, we have $\kappa_A = \kappa_A(n,\gamma,2)$ as in Equation (EC.2.24), and $n = \frac{\alpha_H - \alpha_L}{\alpha_H + \alpha_L - 1}$. Note that $\pi_O^{AB} - \pi_O^{A} > 0$ if and only if $\Lambda_1(\kappa_A,\gamma,n)\alpha_L + \Lambda_2(\kappa_A,\gamma,n) > 0$. A detailed analysis can show that $\Lambda_1(\kappa_A,\gamma,n) \geq 0$ while $\Lambda_2(\kappa_A,\gamma,n) \leq 0$ in Region 1. Figure EC.2.14 shows the behaviour of these two functions across all feasible demand parameters in Region 1. Note that, one can verify that $\frac{\delta n}{\delta \alpha_H} > 0$. Therefore, for a given $\frac{1}{2} < \alpha_L < 1$, n increases as α_H increases. Consequently, as appeared in Figure EC.2.14, both $\Lambda_1(\kappa_A,\gamma,n)$, which is positive, and $\Lambda_2(\kappa_A,\gamma,n)$, which is negative, decrease at the same time. Therefore, there exists a threshold, namely $\alpha_H^{AB\to A}$, such that the sign of $\Lambda_1(\kappa_A,\gamma,n)\alpha_L + \Lambda_2(\kappa_A,\gamma,n)$ change from positive to negative.

Figure EC.2.14 Platform profit comparison under $J = AB$ and $J = A$.



(a) Sign of $\Lambda_1(\kappa_A,\gamma,n)$ (b) Sign of $\Lambda_2(\kappa_A,\gamma,n)$

The summary of equilibrium subscription fees, first- and second-period prices, and belief-updating rules are characterized in Table EC.2.5

Table EC.2.5 Characterization of Equilibrium Decisions Under Each Region

<table>
<tr><th rowspan="2">Regions</th><th colspan="2">$z_p > 1$</th><th>$z_p = 1$</th><th>$z_p < 1$</th></tr>
<tr><th>R_1</th><th>R_2</th><th>R_3</th><th>R_4</th></tr>
<tr><td>J^*</td><td>AB</td><td>A</td><td>$\emptyset$</td><td>$\emptyset$</td></tr>
<tr><td>ϕ</td><td>$(1-\beta)\left[\pi^{AB}_{C\mid B}\left(P^{AB}_B, P^{AB}_A\right) - \pi^{A}_{C\mid B}\left(P^{A}_B, P^{A}_A\right)\right]$</td><td>$(1-\beta)\left[\pi^{A}_{C\mid A}\left(P^{A}_A, P^{A}_B\right) - \pi^{\emptyset}_{C\mid A}\left(P^{\emptyset}_A, P^{\emptyset}_B\right)\right]$</td><td colspan="2">$\phi \in \left[\max\{S^{\emptyset}_B, S^{\emptyset}_A\}, +\infty\right)$</td></tr>
<tr><td>First period prices</td><td>$\begin{cases} p_{A,1} = \frac{\gamma+(2+\gamma)(\alpha_H+\alpha_L)}{2(2+\gamma)(2+3\gamma)} \\ p_{B,1} = \frac{4(1+\gamma)-(2+\gamma)(\alpha_H+\alpha_L)}{2(2+\gamma)(2+3\gamma)} \end{cases}$</td><td>$\begin{cases} p^A_{A,1} = \overline{p}^A_1 + z^A_p \Delta \\ p^A_{B,1} = \overline{p}^A_1 - z^A_p \Delta \end{cases}$</td><td>$\begin{cases} p^{\emptyset}_{A,1} = \overline{p}^{\emptyset} + \Delta \\ p^{\emptyset}_{B,1} = \overline{p}^{\emptyset} - \Delta \end{cases}$</td><td>$\begin{cases} p_{A,1} = \frac{\gamma+(2+\gamma)(\alpha_H+\alpha_L)}{2(2+\gamma)(2+3\gamma)} \\ p_{B,1} = \frac{4(1+\gamma)-(2+\gamma)(\alpha_H+\alpha_L)}{2(2+\gamma)(2+3\gamma)} \end{cases}$</td></tr>
<tr><td>Belief updat- ing</td><td>$\boldsymbol{\theta}_i = \boldsymbol{\theta}$ for both sellers</td><td>$\begin{cases} \boldsymbol{\theta}_A = \boldsymbol{\theta} \\ \boldsymbol{\theta}_B = \begin{cases} H & \text{if } \boldsymbol{\epsilon}_1 \leq \frac{2\Delta}{p_{A,1}-p_{B,1}} \text{ and } \boldsymbol{\theta} = H \\ L & \text{if } \boldsymbol{\epsilon}_1 \geq 1 - \frac{2\Delta}{p_{A,1}-p_{B,1}} \text{ and } \boldsymbol{\theta} = L \\ \bar{\boldsymbol{\theta}} & \text{otherwise,} \end{cases} \end{cases}$</td><td colspan="2">$\boldsymbol{\theta}_i = \boldsymbol{\theta}$ for both sellers</td></tr>
<tr><td>Second period prices</td><td>$\begin{cases} p_{A,2} = \frac{\gamma+(2+\gamma)\alpha_A(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)} \\ p_{B,2} = \frac{\gamma+(2+\gamma)\alpha_B(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)} \end{cases}$</td><td>$\begin{cases} \boldsymbol{\theta}_B = \boldsymbol{\theta} : \begin{cases} p_{A,2} = \frac{\gamma+(2+\gamma)\alpha_A(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)} \\ p_{B,2} = \frac{\gamma+(2+\gamma)\alpha_B(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)} \end{cases} \\ \boldsymbol{\theta}_B = \overline{\boldsymbol{\theta}} : \begin{cases} p_{A,2} = \frac{\gamma+(2+\gamma)\alpha_A(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)} + \frac{\gamma[\alpha_H+\alpha_L-2\alpha_A(\boldsymbol{\theta})]}{4(1+\gamma)(2+3\gamma)} \\ p_{B,2} = \frac{4(1+\gamma)-(2+\gamma)(\alpha_H+\alpha_L)}{2(2+\gamma)(2+3\gamma)} \end{cases} \end{cases}$</td><td colspan="2">$\begin{cases} p_{A,2} = \frac{\gamma+(2+\gamma)\alpha_A(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)} \\ p_{B,2} = \frac{\gamma+(2+\gamma)\alpha_B(\boldsymbol{\theta})}{(2+\gamma)(2+3\gamma)} \end{cases}$</td></tr>
</table>

Note: $\overline{p}^A_1 = \frac{1}{2(2+\gamma)} + \frac{\gamma(\alpha_H-\alpha_L)}{16(1+\gamma)(2+\gamma)(z^A_p)^2}$, $z^A_p = \frac{2\gamma(\alpha_H+\alpha_L-1)}{(2+3\gamma)(\alpha_H-\alpha_L)}\left[1 - 2\cos\left(\frac{1}{3}\arccos\left(-1+\frac{27}{2}\frac{(\alpha_H-\alpha_L)^3(4+4\gamma-\gamma^2)}{32\gamma(1+\gamma)(\alpha_H+\alpha_L-1)^3}\right)-\frac{4\pi}{3}\right)\right]$, $\Delta = \frac{\alpha_H-\alpha_L}{4\gamma}$

EC.3. Further Details for the Model Extensions

In addition to the analytical results derived in the main model, we examined four cases to test the robustness of our findings: (i) heterogeneous seller costs and (ii) state-dependent total market size, (iii) non-uniformly distributed demand noise and (iv) correlated demand noise. In each case, we conducted extensive numerical studies to investigate the model's behavior under these conditions. The details of each simulation model are provided below. All simulations were run on a device with the following specifications: 11th Gen Intel(R) Core(TM) i7-11370H, 3.30GHz, Python version 3.9.13, and RStudio version 2024.12.0+467.

EC.3.1. Heterogeneous Marginal Costs

To derive the self-learning criterion for sellers under heterogeneous marginal costs, we follow the same procedure as in the base model. The only difference is that seller A's prices must also satisfy the constraint $p_{A,t} \geq c_A$. This condition does not alter the learning scheme for sellers. However, deriving a closed-form equilibrium solution remains to be intractable. Therefore, we conducted extensive numerical studies to analyze how the model's behavior changes under this setting. For Figure 8 in the paper, we created 2,700 test cases with the following parameters: $\alpha_L = 0.8$, $\alpha_L \leq \alpha_H \leq 1$ in increments of 0.01, $0 \leq \gamma \leq 10$ with an average step size of 0.25 (because results are more sensitive at lower values of γ, we used step sizes of 0.1 for $0 \leq \gamma \leq 4$ and 1 thereafter.), and seller A's cost values $c_A \in 0,;0.05,;0.1$. We chose a higher α_L in this case because an increase in c_A disadvantages seller A, and we wanted to capture a broader range of the model's behavior.

In addition to the figure reported in the main paper, we also conducted a more detailed simulation across different values of seller A's cost, presented in Figure EC.3.1. For this simulation, we generated 2,160 test cases with the following parameters: $\alpha_L = 0.8$, $\alpha_L \leq \alpha_H \leq 1$ in increments of 0.005, $\gamma = 1$, and seller A's cost $0 \leq c_A \leq \alpha_H$ with an average step size of 0.015. Because results are more sensitive at lower values of c_A, we used step sizes of 0.005 for $0 \le c_A \le 0.1$, 0.01 for $0.1 \le c_A \le 0.2$, and 0.025 thereafter.

At high values of c_A, seller A must raise her price to such an extent that remaining in the market is no longer profitable. As a result, seller A exits the market, and the problem reduces to a single-seller learning setting. Because of these fundamental changes, in the main paper we restrict attention to cases where both sellers remain active in the market.

EC.3.2. State-dependent total demand

In this section we consider state-dependent total demand. Previously, total market available to sellers under both High and Low demand states was equal to 1. We relax this assumption in this section.

Model and First Best Solution: Total market size is now random. Demand in period $t = \{1,2\}$ is

$$d_{A,t}(p_{A,t}, p_{B,t}) = \alpha_A(\boldsymbol{\theta}) - p_{A,t} + 2\epsilon_t\gamma(p_{B,t} - p_{A,t}) \tag{EC.3.1}$$

$$d_{B,t}(p_{A,t}, p_{B,t}) = \alpha_B(\boldsymbol{\theta}) - p_{B,t} + 2\epsilon_t\gamma(p_{A,t} - p_B) \tag{EC.3.2}$$

If $\boldsymbol{\theta} = H$, then $\alpha_A(H) = \alpha_H$ and $\alpha_B(H) = 1 - \alpha_H$ and if $\boldsymbol{\theta} = L$, then $\alpha_A(H) = \zeta\alpha_L$ and $\alpha_B(L) = \zeta(1 - \alpha_L)$ where $\frac{1-\alpha_H}{1-\alpha_L} \le \zeta \le 1$. The first inequality $\frac{1-\alpha_H}{1-\alpha_L} \le \zeta$ guarantees that $\boldsymbol{\theta} = H$ is still the preferred case for seller A and that $\boldsymbol{\theta} = L$ is still the preferred case for seller B. We still assume that $\alpha_H \ge \alpha_L \ge \frac{1}{2}$ and that the random noise over the two periods, i.e., ϵ_1 and ϵ_2 are independent and both are uniformly distributed over $[0,1]$. Denote now $\overline{\alpha}_A = \frac{1}{2}\alpha_H + \frac{1}{2}\zeta\alpha_L$ and $\overline{\alpha}_B = \frac{1}{2}(1-\alpha_H) + \frac{1}{2}\zeta(1-\alpha_L)$.

The first best solution in this case will be derived with the same rules in EC.2.1:

$$p^{fb}_{A,1} = \frac{\alpha_H + \zeta\alpha_L + \gamma(1+\zeta)}{4(1+2\gamma)}, \quad p^{fb}_{B,1} = \frac{1 - \alpha_H + \zeta(1-\alpha_L) + \gamma(1+\zeta)}{4(1+2\gamma)}$$

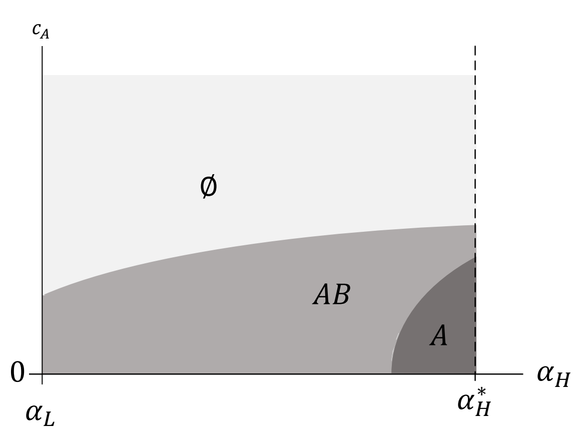
AB **Regime:** The analysis for *AB* regime with state-dependent total demand is similar to our base model. If platform shares the information with both sellers, then in the first period, sellers compete and set the prices based on the mean demand state and in the second period they will set the state-dependent prices.

$$p_{A,1}(\boldsymbol{\theta} = H) = \frac{(\alpha_H + \zeta\alpha_L)(2+\gamma) + \gamma(1+\zeta)}{2(3\gamma+2)(\gamma+2)} \quad p_{B,1}(\boldsymbol{\theta} = H) = \frac{(1 - \alpha_H + \zeta(1-\alpha_L))(2+\gamma) + \gamma(1+\zeta)}{2(3\gamma+2)(\gamma+2)}$$

$$p_{A,2}(H) = \frac{\gamma + \alpha_H(2+\gamma)}{(3\gamma+2)(\gamma+2)} \quad p_{B,2}(H) = \frac{\gamma + (1-\alpha_H)(2+\gamma)}{(3\gamma+2)(\gamma+2)}$$

$$p_{A,2}(L) = \frac{\zeta(\gamma + \alpha_L(2+\gamma))}{(3\gamma+2)(\gamma+2)} \quad p_{B,2}(L) = \frac{\zeta(\gamma + (1-\alpha_L)(2+\gamma))}{(3\gamma+2)(\gamma+2)}$$

Figure EC.3.1 Equilibrium Structure with State-Dependent Total Market Demand, $\alpha_L = 0.6, \gamma = 1$



No-CIS Regime: Suppose now that the platform does not share information with sellers. Each seller observes its own sales only. We identify the impact of the first period decisions on the second period information state of the sellers. For expositional brevity, we drop the time index and let (p_A, p_B) denote the first-period prices of sellers A and B, respectively. There will be no need to use the second period prices for belief updating.

Belief updating by seller A: Let S_A be the sales observed by seller A at the end of the first period. seller A knows that these sales come from $\alpha_H - p_A + 2\gamma\epsilon_1(p_B - p_A)$ or $\zeta\alpha_L - p_A + 2\gamma\epsilon_1(p_B - p_A)$ but she does not know which one. However, in some situations seller A can deduce the real value of α and ϵ_1. When seller A only observes its own sales, there are three possibilities: seller A identifies $\theta = H$, seller A identifies $\theta = L$, seller A cannot identify the demand state. In the following, we derive the probability distribution of each outcome.

Case I: $p_B \leq p_A$ Figure EC.3.2 facilitates our discussion for the case where $p_B \leq p_A$. The probability of each state can be derived as follows. If $\theta = H$, then seller A can identify this if sales S_A is sufficiently high, i.e.,

$$\begin{aligned}Pr\{\theta_A = H \big| \theta = H\} &= Pr\{S_A \geq \zeta\alpha_L - p_A \big| \theta = H\}\\ &= Pr\{\alpha_H - p_A + 2\gamma\epsilon_1(p_B - p_A) \geq \zeta\alpha_L - p_A\}\\ &= Pr\{\epsilon_1 \leq \frac{2\Delta_1}{p_A - p_B}\}\end{aligned}$$

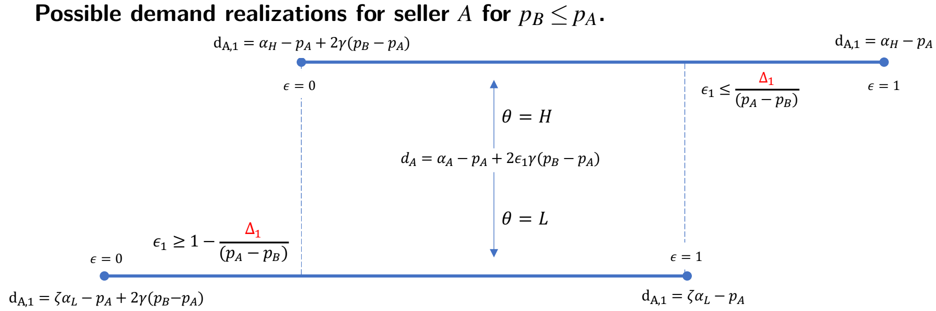
where $\Delta_1 = \frac{\alpha_H - \zeta\alpha_L}{4\gamma}$. Accordingly: $Pr\{\theta_A = \overline{\theta} \big| \theta = H\} = Pr\{\frac{2\Delta_1}{p_A - p_B} \leq \epsilon_1\}$.

If $\theta = L$, then seller A can identify this if sales S_A is sufficiently low, i.e.,

$$\begin{aligned}Pr\{\theta_A = L \big| \theta = L\} &= Pr\{S_A \leq \alpha_H - p_A + 2\gamma(p_B - p_A) \big| \theta = L\}\\ &= Pr\{\zeta\alpha_L - p_A + 2\gamma\epsilon_1(p_B - p_A) \leq \alpha_H - p_A + 2\gamma(p_B - p_A)\}\\ &= Pr\{1 - \frac{2\Delta_1}{p_A - p_B} \leq \epsilon_1\}\end{aligned}$$

and accordingly, $Pr\{\theta_A = \overline{\theta} \big| \theta = L\} = Pr\{\epsilon_1 \leq 1 - \frac{2\Delta_1}{p_A - p_B}\}$.

Figure EC.3.2 Possible demand realizations for seller A for $p_B \leq p_A$.



If $\zeta\alpha_L - p_A \leq \alpha_H - p_A + 2\gamma(p_B - p_A)$ (if and only if $1 \leq \frac{2\Delta_1}{p_A - p_B}$), then seller A can identify the value of true α for any realization of ϵ_1. If on the other hand, $\zeta\alpha_L - p_A \geq \alpha_H - p_A + 2\gamma(p_B - p_A)$ (if and only if $1 \geq \frac{2\Delta_1}{p_A - p_B}$), then there is always sufficient noise in the system such that the seller cannot automatically identify the ture demand state. Based on this, the posterior beliefs of seller A on θ by the end of the first period (following Bayesian updating) are as follows:

$$Pr\{\theta = H \big| \theta_A = \overline{\theta}\} = \frac{Pr\{\theta = H \text{ and } \theta_A = \overline{\theta}\}}{Pr\{\theta_A = \overline{\theta}\}}$$

$$
\begin{aligned}
&= \frac{Pr\{\theta_A = \overline{\theta} | \theta = H\}\frac{1}{2}}{Pr\{\theta_A = \overline{\theta} | \theta = H\}\frac{1}{2} + Pr\{\theta_A = \overline{\theta} | \theta = L\}\frac{1}{2}} \\
&= \frac{\left[1 - \frac{\alpha_H - \zeta\alpha_L}{2\gamma(p_A - p_B)}\right]\frac{1}{2}}{\left[1 - \frac{\alpha_H - \zeta\alpha_L}{2\gamma(p_A - p_B)}\right]\frac{1}{2} + \left[1 - \frac{\alpha_H - \zeta\alpha_L}{2\gamma(p_A - p_B)}\right]\frac{1}{2}} \\
&= \frac{1}{2}.
\end{aligned}
$$

In other words, if seller A cannot identify the true value of α, then the posterior distribution is identical to the prior.

Case II: $p_B \geq p_A$ If $\theta = H$, then seller A can identify this if sales S_A is sufficiently high, i.e.,

$$
\begin{aligned}
Pr\{\theta_A = H \big| \theta = H\} &= Pr\{S_A \geq \zeta\alpha_L - p_A + 2\gamma(p_B - p_A) \big| \theta = H\} \\
&= Pr\{1 - \frac{2\Delta_1}{p_B - p_A} \leq \epsilon_1\}
\end{aligned}
$$

and accordingly, $Pr\{\theta_A = \overline{\theta} \big| \theta = H\} = Pr\{\epsilon_1 \leq 1 - \frac{2\Delta_1}{p_B - p_A}\}$.

If $\theta = L$, then seller A can identify this if sales S_A is sufficiently low, i.e.,

$$
\begin{aligned}
Pr\{\theta_A = L \big| \theta = L\} &= Pr\{S_A \leq \alpha_H - p_A \big| \theta = L\} \\
&= Pr\{\epsilon_1 \leq \frac{2\Delta_1}{p_B - p_A}\}
\end{aligned}
$$

and accordingly, $Pr\{\theta_A = \overline{\theta} \big| \theta = L\} = Pr\{\frac{2\Delta_1}{p_B - p_A} \leq \epsilon_1\}$.

If $\alpha_L - p_A + 2\gamma(p_B - p_A) \leq \alpha_H - p_A$ (if and only if $1 \leq \frac{2\Delta_1}{p_B - p_A}$), then seller A can identify the value of true α for any realization of ϵ_1. If on the other hand, $\zeta\alpha_L - p_A + 2\gamma(p_B - p_A) \geq \alpha_H - p_A$ (if and only if $1 \geq \frac{2\Delta_1}{p_B - p_A}$), then there is always sufficient noise in the system such that the seller cannot automatically identify the value of true α. Based on this, we can now write the posterior beliefs of seller A on θ by the end of the first period.

$$
\begin{aligned}
Pr\{\theta = H \big| \theta_A = \overline{\theta}\} &= \frac{Pr\{\theta = H \text{ and } \theta_A = \overline{\theta}\}}{Pr\{\theta_A = \overline{\theta}\}} \\
&= \frac{Pr\{\theta_A = \overline{\theta} | \theta = H\}\frac{1}{2}}{Pr\{\theta_A = \overline{\theta} | \theta = H\}\frac{1}{2} + Pr\{\theta_A = \overline{\theta} | \theta = L\}\frac{1}{2}} \\
&= \frac{\left[1 - \frac{\alpha_H - \zeta\alpha_L}{2\gamma(p_B - p_A)}\right]\frac{1}{2}}{\left[1 - \frac{\alpha_H - \zeta\alpha_L}{2\gamma(p_B - p_A)}\right]\frac{1}{2} + \left[1 - \frac{\alpha_H - \zeta\alpha_L}{2\gamma(p_B - p_A)}\right]\frac{1}{2}} \\
&= \frac{1}{2}.
\end{aligned}
$$

In other words, if seller A cannot identify the true value of α, then the posterior distribution is identical to the prior. In other words, if seller A cannot identify the true value of α, then the posterior distribution is identical to the prior.

Belief updating by seller B Through a similar analysis, we can show that for $p_A \geq p_B$ we have:

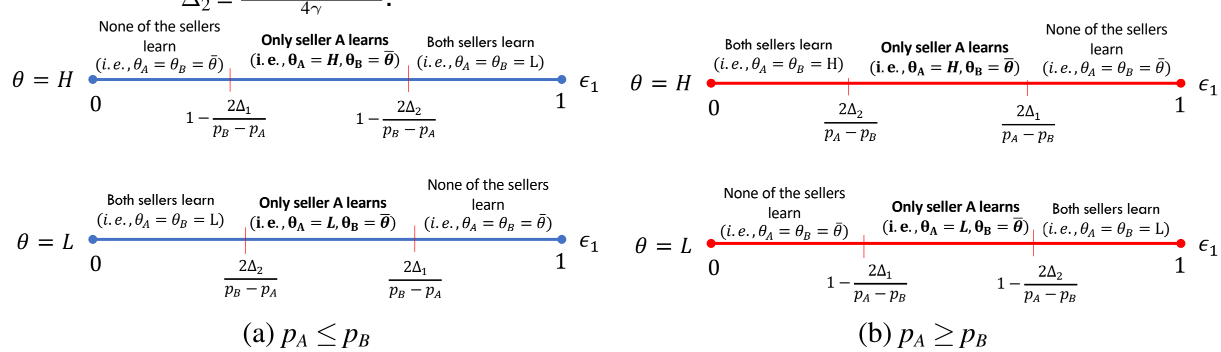
$$
\begin{aligned}
&Pr\{\theta_B = H \big| \theta = H\} = Pr\{\epsilon_1 \leq \frac{2\Delta_2}{p_A - p_B}\} \equiv Pr\{\epsilon_1 \leq \frac{\alpha_H - \zeta\alpha_L - 1 + \zeta}{4\gamma}\} \\
&Pr\{\theta_B = L \big| \theta = L\} = Pr\{1 - \frac{2\Delta_2}{p_A - p_B} \leq \epsilon_1\} \\
&Pr\{\theta_B = \overline{\theta} \big| \theta = H\} = Pr\{\frac{2\Delta_2}{p_A - p_B} \leq \epsilon_1\} \\
&Pr\{\theta_B = \overline{\theta} \big| \theta = L\} = Pr\{\epsilon_1 \leq 1 - \frac{2\Delta_2}{p_A - p_B}\}.
\end{aligned}
$$

Also, if $p_A \leq p_B$ the analysis is similar.

Posterior beliefs: Figure EC.3.3 provides an illustration of the posterior distribution for all possible realizations of α and ϵ_1 for the given first period prices (p_A, p_B). For $\zeta < 1$, we can look at the first period price decisions in two regions: (i) $|p_A - p_B| \leq 2\Delta_2$ and (i) $2\Delta_2 \leq |p_A - p_B|$. If $|p_A - p_B| \leq 2\Delta_2$, then both sellers identify the true state of the demand simultaneously - we refer to this as *co-learning*. However, if $2\Delta_2 \leq |p_A - p_B|$, then it is possible for seller A to identify the true state of the demand alone. In other words, depending on the realization of noise ϵ_1, it is possible that both sellers identify the true state of the demand simultaneously or only seller A learns the true state of the demand, or they are both in the dark. This region clearly provides an advantage to seller A and in addition to choosing p_A in this range, seller A can adjust p_A to increase the probability that it only learns the true state of the demand, which we refer to as *signal jamming*.

The key observation, in comparison with the case of $\zeta = 1$ in Figure EC.3.3 is the following. In our base case $\zeta = 1$, seller A and seller B were purely co-learning regardless of the pricing decisions. The implication of this was that sellers are not capable of signal jamming under the *no information sharing* scenario. For $\zeta < 1$, there exists an opportunity for seller A to *signal jam* even under no information sharing scenario. Our numerical investigation, presented in the main manuscript, illustrates that there are problem parameters under which the equilibrium prices satisfy $2\Delta_2 \leq |p_A - p_B|$, i.e., signal jamming occurs even under the no information sharing scenario and is detrimental to all parties.

Figure EC.3.3 Demand learning for each seller Based on the possible demand realization, $\Delta_1 = \frac{\alpha_H - \zeta\alpha_L}{4\gamma}$ and $\Delta_2 = \frac{\alpha_H - \zeta\alpha_L - (1-\zeta)}{4\gamma}$.



Comparison and Equilibrium: Since the equilibrium analysis is analytically intractable, we conducted extensive numerical studies and simulations with the following specifications to derive the equilibrium for state-dependent total demand. For Figure 9 in the paper, we created 2,400 test cases with the following parameters: $\alpha_L = 0.8$, $\alpha_L \leq \alpha_H \leq 1$ in increments of 0.01, $0 \leq \gamma \leq 10$ in increments of 0.25, and $\zeta \in 0.6, 0.8, 1$.

In addition to the figure provided in the main paper, we also performed more in-depth simulations across different values of total market size under the low-demand state. The results are presented in Figure EC.3.4. For this simulation, we considered 2,160 test cases with the following parameters: $\alpha_L = 0.8$, $\alpha_L \leq \alpha_H \leq 1$ in increments of 0.005, $\gamma = 1$, and $0 \leq \zeta \leq 1$ with an average increment of 0.025. Because the results are more sensitive for mid- and high-range values of ζ, we used uneven step sizes: 0.1 for $0 \leq \zeta \leq 0.7$, 0.02 for $0.7 \leq \zeta \leq 0.9$, 0.01 for $0.9 \leq \zeta \leq 0.95$, 0.005 for $0.95 \leq \zeta \leq 0.99$, and 0.001 for values above 0.99.

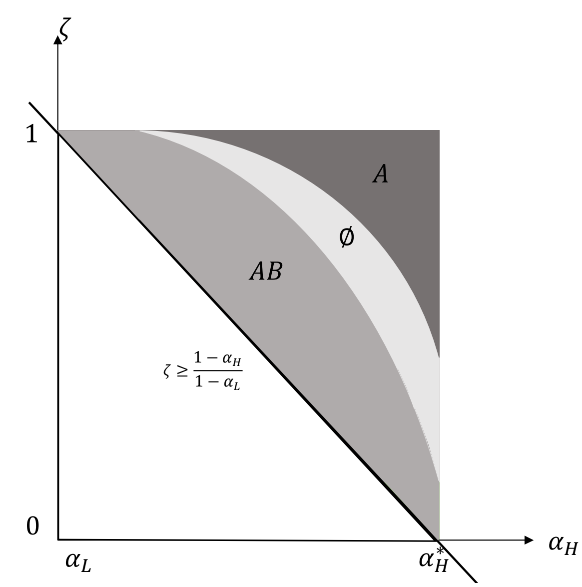
When total market demand is state dependent, the equilibrium regions change. In addition to the *AB* and *A* regimes, a no-CIS regime also emerges in equilibrium. In the base model (where $\zeta = 1$), the equilibrium is characterized by an α_H threshold: above this threshold, regime *A* dominates *AB*; at an even higher threshold, sellers can independently identify the true demand state, and regime $\emptyset$ becomes the equilibrium. Regime *B*, however, never arises.

However, as Figure EC.3.4 illustrates, when state-dependent total market demand is introduced, the no-CIS regime emerges in the transition between the *AB* and *A* regimes. This occurs because, under state-dependent total market demand, there is a possibility that only seller *A* learns the true demand under no-CIS regime. Consequently, within a certain range (after the α_H^{AB} threshold), seller *A* can identify the demand without subscribing to CIS, leaving her with no incentive to join. In this case, regime $\emptyset$ effectively behaves like regime *A*. As α_H increases further—widening the first-period price difference—the equilibrium shifts to regime *A*, as in the base model. This outcome is also consistent with the changes in belief updating shown in Figure EC.3.2. When the first-period price difference is moderate, only seller *A* is able to learn the true demand.

EC.3.3. Non-Uniform Noise Distribution

In the base model we assume that the demand noise follows an uniform distribution ($\epsilon_t \sim U[0,1]$). For this part, we consider non-uniform demand noise, wtih beta distribution over different parameters (α, β) values. Particularly, we have considered four cases shown in Figure 10 in the paper. We created 2800 test cases by fixing $\alpha_L = 0.65$, varying α_H from α_L to 1 in increments of 0.01, varying γ from 1 to 10 in increments of 0.5. We considered the following beta distribution parameters values $(\alpha, \beta) \in \{(2,8),(5,5),(1,1),(8,2)\}$. We produced random numbers from beta distribution using *scipy.stats* package in python. To determine equilibrium prices and identify the best response in each case, we searched seller *A*'s price range with a step size of 0.01 and seller *B*'s price range with a finer step size of 0.005. The results are presented in Figure 10 in the paper.

Figure EC.3.4 Equilibrium Structure with State-Dependent Total Market Demand, $\alpha_L = 0.6, \gamma = 1$



EC.3.4. Correlated Noise Distributions

In the base model, there is zero correlation between periods and full positive correlation between sellers within each period. In this section, we extend the analysis by considering non-uniform noise across sellers with varying correlation values. Specifically, in each period, the demand noise for sellers follows two correlated truncated normal distributions (bounded between 0 and 1). To generate random numbers from this distribution, we relied on the results of Manjunath and Wilhelm (2021), who also provide an R package for simulating doubly truncated multivariate normal random variables and their associated probability density functions. Using this package, we generated random numbers for correlation values $\{-0.5, 0, 0.5, 0.9, 1\}$ with seed value set as 918. Since the range of the final variable must lie between 0 and 1, we set the mean at $\frac{1}{2}$ and the standard deviation at $\sigma = \frac{1}{6}$ to capture a substantial portion of the normal distribution. We constructed 763 test cases with 100 scenarios each by fixing $\alpha_L = 0.65$, varying α_H from α_L to 1 in increments of 0.015, varying γ from 1 to 8 in increments of 1, and considering correlation values $\rho \in \{-0.5, 0, 0.5, 0.9, 1\}$. For each distribution, we discretized into 10 values, assigning to each value the total probability density of its corresponding 10-percent bin. This procedure yielded 100 combinations with corresponding marginal utilities for each parameter set, allowing us to compute expectations. To determine equilibrium prices and identify the best response in each case, we searched seller A's price range with a step size of 0.01 and seller B's price range with a finer step size of 0.005. The rationale for using larger steps for seller A is that she has a higher market share, and therefore, on average, her prices are higher than seller B's. The results are presented in Figure 11 of the paper.

EC.4. Non-uniform and Correlated Noise Distributions

This appendix provides the full analysis underlying the robustness checks summarized in Section 6.3.

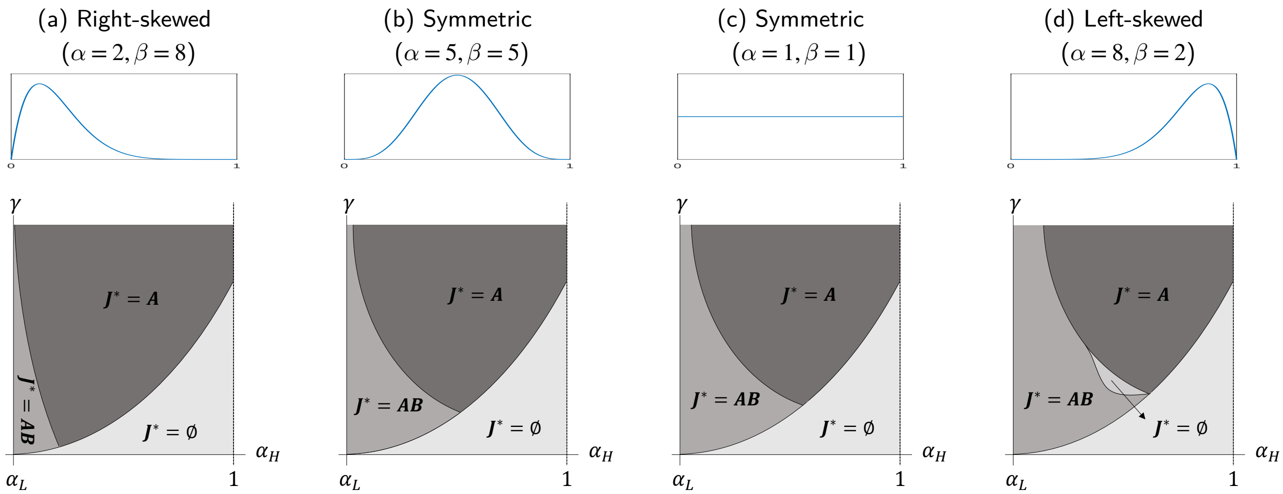
EC.4.1. Non-uniform noise distribution

In the baseline model, we assumed that ϵ_t follows a uniform distribution on $[0, 1]$. In this section, we relax this assumption and consider non-uniform cases by modeling ϵ_t with a parametrizable beta distribution, $\epsilon_t \sim \text{Beta}(\alpha, \beta)$. Since the beta distribution is bounded between 0 and 1, it provides a flexible way to capture different distributions between 0 and 1. By varying α and β, we examine right-skewed ($\alpha = 2, \beta = 8$), left-skewed ($\alpha = 8, \beta = 2$), and two symmetric distributions: the uniform case ($\alpha = 1, \beta = 1$) and a more concentrated symmetric case ($\alpha = 5, \beta = 5$). Figure EC.4.1 presents the results.

Note that the uniform distribution ($\alpha = 1, \beta = 1$) corresponds to our baseline model. For benchmarking, we replicate this case in panel (c) of Figure EC.4.1 above and focus our discussion on how equilibrium regimes change as the distributional shape varies. Increasing both α and β from 1 to 5 preserves the mean of ϵ_t at $\frac{1}{2}$ but reduces its variance. Comparing panels (b) and (c) of Figure EC.4.1, we observe that when ϵ_t becomes more concentrated around its mean, the equilibrium region associated with regime AB shrinks. This is intuitive: as demand noise diminishes, the need for information sharing between both sellers declines.

We next consider the two asymmetric cases. In the Right-skewed case (Figure EC.4.1(a)), the mean of ϵ_t shifts toward 0, which reduces demand noise. Consequently, the equilibrium region for regime AB contracts further, as sharing information becomes less valuable. In contrast, in the left-skewed case (Figure EC.4.1(d)), the mean shifts toward 1, amplifying demand noise. This has two effects. First, the equilibrium region for regime AB expands. Second, unlike the

Figure EC.4.1 Equilibrium information regimes under different demand shock distributions, $\epsilon \sim$ Beta(α, β).



baseline model where regime A always dominates regime $\emptyset$, we now observe that the platform sometimes prefers regime $\emptyset$ over both A and AB. The intuition is that although the mean shifts toward 1, the distribution is also more concentrated than under the uniform case, which reduces the overall uncertainty. Under these conditions, if the platform induces regime $\emptyset$, sellers can still learn the true demand state from their realized sales. As a result, regime $\emptyset$ emerges as an equilibrium, particularly when α_H is moderate and γ is relatively low, both of which enhance sellers' ability to learn demand.

EC.4.2. Correlated noise distribution

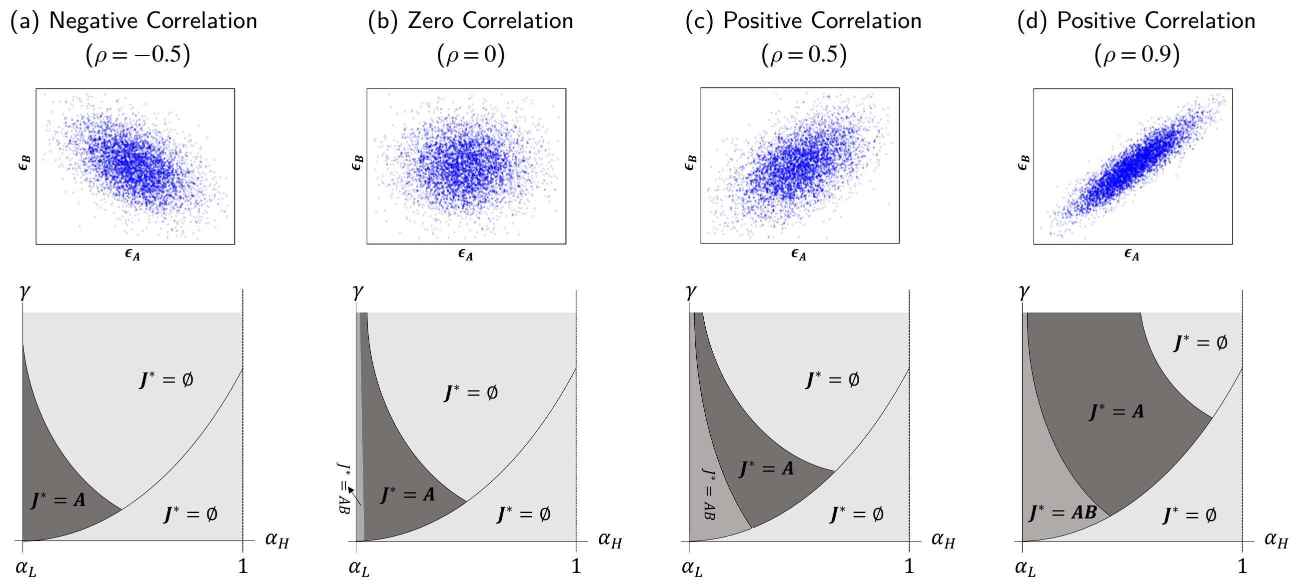
We now analyze how correlations between ϵ_A and ϵ_B (random shocks faced by seller A and seller B, respectively) shape the equilibrium regimes. To capture varying degrees of correlation, we model ϵ_A and ϵ_B using a truncated multivariate normal distribution with mean centered at 0.5 and variance-covariance matrix Σ given by $\Sigma = \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$, where σ^2 captures the variance around the mean and $\rho \in [-1, 1]$ controls the correlation between ϵ_A and ϵ_B. As ρ approaches -1 (resp. $+1$), the demand noise between seller A and seller B becomes more negatively (resp. positively) correlated. We consider three values: $\rho = -0.5$, $\rho = 0$, and $\rho = +0.5$. We also present $\rho = 0.9$ as a benchmark for the baseline model. The equilibrium regions are displayed in Figure EC.4.2.

As shown in Figure EC.4.2, equilibrium region $J = \emptyset$ emerges as the equilibrium information-sharing regime in the upper-right part of the parameter space (where both α_H and γ are high). The rationale for this outcome lies in the changes to the demand learning stage when the degree of correlation between demand shocks decreases. Specifically, as correlation weakens, the signals observed by the sellers diverge, leading to asymmetric learning outcomes.

Interestingly, when demand correlation is low or negative, two new cases arise: (i) seller A learns the true demand state while seller B does not, and (ii) seller B learns while seller A does not. Both cases induce signal-jamming behavior between the sellers. However, the consequences for the platform differ: when seller A engages in signal-jamming, it raises prices, which benefits the platform, whereas when seller B engages in signal-jamming, it depresses prices, which harms the platform.

In the upper-right region of the parameter space (where α_H is substantially larger than α_L), the platform benefits on average from seller A's signal-jamming. Consequently, regime $J = \emptyset$ becomes the platform's preferred equilibrium.

Figure EC.4.2 Equilibrium information regimes under correlation (ρ) between sellers' demand shocks (ϵ_A, ϵ_B).



By contrast, as α_H decreases, seller B's signal-jamming becomes too costly, prompting the platform to intervene and induce information sharing with seller A.

E-Companion References

Manjunath, BG, Stefan Wilhelm. 2021. Moments calculation for the doubly truncated multivariate normal density. *Journal of Behavioral Data Science* **1**(1) 17–33.